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Analysis and Performance Evaluation of Power Amplifier Linearization Techniques

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Abstract: Linearity performance has become a defining characteristic as it affects power efficiency, channel density, signal coverage, and adjacent channel power ratio. The non-linearity of Power Amplifier generates inter-modulation (IMD) components, also referred to as out-of-band emission or spectral re-growth, which interfere with adjacent channels. To attain high energy-efficiency, PAs should be operated at their output saturation regions but this operational mode could not provide high bandwidth-efficiency for a single-carrier high-order quadrature amplitude modulation (QAM) signals as well as multi-carrier orthogonal frequency division multiplexing (OFDM) signals. It is therefore difficult to compensate the nonlinearity of the power amplifier (PA) in the design of a wireless system. There are several techniques for power amplifier linearization. This paper is based on the mathematical analysis of these linearization techniques and their comparision.

Keywords: Power Amplifier; feedforward linearization; feedback linearization; digital predistortion

I. INTRODUCTION

As the wireless communication is gaining its widespread popularity, the increased complexity of the devices and wireless protocols produce an unrelenting neccesity for linear radio frequency (RF) components and systems. These ubiquitous wireless devices require high performance test systems to characterize linearity. Power amplifiers are the important part of the bandpass communication channel system. The simple polynomial approximation of nonlinear transfer function based upon the Taylor series expansion [1] can be used to define the nonlinear Power amplifiers. Typically the odd terms are considered. If even-power nonlinear processes are present within the device, there will be variations in the dc conditions. In particular, a device which has significant even-degree distortion will show a low-frequency ac component on its dc supply [2].

II. MATHEMATICAL ANALYSIS OF POWER AMPLIFIER

Power amplifier device nonlinearity can be modeled by a polynomial

$$V_0(t) = a_1 V_i(t) + a_3 V_i^3(t) + a_5 V_i^5(t)...... \qquad (1)$$

Table1: Coefficients of the intermodulation terms.

Coefficient order	n=3	n=5	n=7	n=9	n=11
Fundamental	9/4	25/4	1225/64	3969/64	53361/256
3^{rd} order	3/4	25/8	735/64	1323/32	38115/128
5^{th} order	-	5/8	245/64	567/32	38115/512
7^{th} order	-	-	35/64	567/128	12705/512
9^{th} order	-	-	-	63/128	2541/512
11^{th} order					231/512

Applying a two carrier RF signal to the power amplifier transistor

$$V_{in}(t) = v\cos(\omega_c t - \omega_m t) + v\cos(\omega_c t + \omega_m t) = 2v\cos\omega_m t\cos\omega_c t \qquad (2)$$

Where $\omega_m t$ represent amplitude modulation. $\omega_c t$ represent carrier waveform

Writing out the response $V_0(t)$ by performing trigonometric expansion

$$V_0(t) = a_1 2v\cos\omega_m t\cos\omega_c t + a_3(2v\cos\omega_m t\cos\omega_c t)^3 + a_5(2v\cos\omega_m t\cos\omega_c t)^5 + ... + a_n(2v\cos\omega_m t\cos\omega_c t)^n \qquad (3)$$

The De Moivre's theorem states that

$$\cos^{2n+1} x = \frac{1}{2^{2n}}\sum_{k=0}^{n}\binom{2n+1}{k}\cos\left(2n+1-2k\right)x \qquad (4)$$

for odd powers of cos x, where 1<k< 1/(n-1)

Putting equation (4) in eq (3) we get the nth degree output modulated only on fundamental carrier as

$$V_0(t) = a_n 2^n v^n \frac{n!}{2(\frac{n-1}{2})!(\frac{n+1}{2})!}\frac{1}{2^{n-1}}\{\cos n\omega_m t + n\cos(n-2)\omega_m t + \frac{n!}{(n-k)!k!}\cos(n-2k)\omega_m t\}\cos\omega_c t \qquad (5)$$

From equation 5, we can find the power series coefficients of third, fifth, seventh and all higher order of intermodulation terms as shown in the table1. Note that the coefficients in the equation (5) and (6) are for both upper and lower sidebands so in the table they are divided by two for a single sidelobe. The individual spectral outputs are divided into separate components, each coming from a different degree of nonlinearity. For example, the third-order IM product has two components, one from the third-degree nonlinearity and another from the fifth degree. Each fundamental output has three components: the linear term, and a third- and a fifth-degree nonlinear contribution.

Practically, the fundamental component of AM-PM displays the phase shift Δ with respect to the AM-AM response. Thus the PA output can be written as

$$V_0(t) = [g_1 \cos(\omega_m t + \Delta) + g_3 \cos 3(\omega_m t + \Delta) + g_5 \cos 5(\omega_m t + \Delta)...] [\cos\{\omega_c t + \Phi \cos(2\omega_m t)\}] \quad (6)$$

Where

$$g_1 = 2av + \frac{9}{2}a_3 v^3 + \frac{25}{2}a_5 v^5 + \frac{1225}{32}a_7 v^7 + \frac{3969}{32}a_9 v^9 + \frac{53361}{128}a_{11} v^{11} \quad (6a)$$

$$g_3 = \frac{3}{2}a_3 v^3 + \frac{25}{4}a_5 v^5 + \frac{735}{32}a_7 v^7 + \frac{1323}{16}a_9 v^9 + \frac{38115}{128}a_{11} v^{11} \quad (6b)$$

$$g_5 = \frac{5}{4}a_5 v^5 + \frac{245}{32}a_7 v^7 + \frac{567}{16}a_9 v^9 + \frac{38115}{256}a_{11} v^{11} \quad (6c)$$

$$g_7 = \frac{35}{32}a_7 v^7 + \frac{567}{64}a_9 v^9 + \frac{12705}{256}a_{11} v^{11} \quad (6d)$$

$$g_9 = \frac{63}{64}a_9 v^9 + \frac{2541}{256}a_{11} v^{11} \quad (6e)$$

$$g_{11} = \frac{231}{256}a_{11} v^{11} \quad (6f)$$

Δ = phase angle b/w the AM-AM and AM-PM, Φ =peak amplitude of AM-PM distortion. $\omega_m t$ is the two carrier beat frequency which is half of the carrier frequency. The frequency of AM-PM response is double of the envelope modulation frequency as it has two peaks in every cycle.

Typically the third-order inter-modulation distortion product (IM3) are of most concern since distortion products which are far away in frequency from the desired output can be removed by filtering. But typical higher power devices, such as LDMOS, which are carefully biased and tuned in order to present favorable nulls in the IM characteristics. Such devices shows flat gain characteristics and a very abrupt compression. These devices requires higher order terms, say, up to the ninth order. So from the equation (6) we can find the intermodulation terms as:

$$IM3_{AM} = [g_3 \cos 3(\omega_m t + \Delta)] \cos \omega_c t \quad (7)$$

$$IM3_{PM} = [g_1 \cos(\omega_m t + \Delta)][\frac{\Phi}{2} (\sin(\omega_c + 2\omega_m t) + (\sin(\omega_c - 2\omega_m t)] \quad (8)$$

$$IM5_{AM} = [g_5 \cos 5(\omega_m t + \Delta)] \cos \omega_c t \quad (9)$$

$$IMn_{PM} = [g_{n-2} \cos(n-2)(\omega_m t + \Delta)][\frac{\Phi}{2} (\sin(\omega_c + 2\omega_m t) + (\sin(\omega_c - 2\omega_m t)] \quad (10)$$

$$IMn_{AM} = [g_n \cos n(\omega_m t + \Delta)] \cos \omega_c t \quad (11)$$

$$IMn_{PM} = [g_{n-2} \cos(n-2)(\omega_m t + \Delta)][\frac{\Phi}{2} (\sin(\omega_c + 2\omega_m t) + (\sin(\omega_c - 2\omega_m t)] \quad (12)$$

Equation (11) and (12) are the generalized equations to find the nth order inter-modulations for AM-AM and AM-PM respectively.

III. LINEARIZATION TECHNIQUES

There are many linearization techniques for minimizing power amplifier nonlinear distortion. But broadly we have feedback, feedforward and digital predistortion technique for linearization of power amplifier. The following subsections discusses these techniques with their advantages and limitations.

A. RF Feedback

In radio frequency (RF) feedback the output signal is fed back without detection or down-conversion[6]. RF feedback is illustrated in Fig 1. The RF signal is input to a subtractor on the left side in Fig.1.An amplifier can be used for an active feedback network or resistors or transformers can be deployed as passive feedback networks[3]. The feedback network can reduce distortion appearing at the output of the nonlinear amplifier in Fig1.

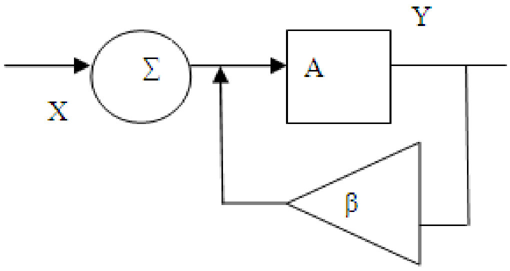

Figure 1. RF feedback linearization method

The output of a simple RF feedback circuit is given as

$$Y = \frac{AX}{1 + A\beta}$$

Voltage-controlled current feedback and current-controlled voltage feedback are commonly used as they are simple and their distortion is predictable.[4] However, due to the time delays in the feedback network, there is a drawback of loop stability problem in this design. Thus, RF feedback use is limited to narrowband systems.[5]

B.Envelope Feedback

A second feedback linearization method is envelope feedback.[6] Fig.2 shows an envelope feedback scheme. In Fig.2, a portion of input and output signals is sampled by a coupler and the envelopes of the two sampled signals are detected by means of an envelope detector i.e.peak detector for amplitude and phase detector for phase, respectively. Using a differential amplifier both envelope signals are subtracted which results into an error-signal which is further controlled by a FET/PIN attenuator, which modifies the envelope to remove the error of the RF signal. Furthermore, envelope feedback can be applied to either a transmitter or a single PA. However, a disadvantage of the envelope feedback is that it only accounts for distortion in signal-amplitude and not in signal-phase. To get the correct error signal, the delay of the circuit need to be controlled such that both the inputs of differential amplifier should reach at the same time. For that the following expression should follows:

$$\Delta_{IN} = \Delta_{PA} + \Delta_{OUT} \tag{13}$$

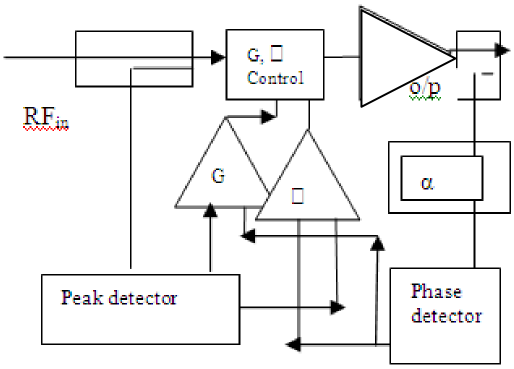

Figure 2. Envelope feedback linearization.

The attenuator chosen is such that at zero drive voltage there is a centered value of attenuation, say α_0. The value of α_0 must be chosen to permit for sufficient variation one of the side of this value to overcome any gain compression or expansion over the projected operating range.

The attenuator characteristic, at envelope domain time τ, can be expressed in the form[2]

$$\alpha(\tau) = 1 + G(V_0(\tau - \Delta_{out} - \Delta_{VID}) - V_{in}(\tau - \Delta_{in} - \Delta_{VID})) \tag{14}$$

Where Δ_{out} and Δ_{in} are output and input path delays, respectively. And Δ_{VID} is the gain amplifier path delay. The output of main PA can be characterized as

$$V_0(\tau) = a_1(\alpha V_{in}(\tau - \Delta_{PA})) + a_3 (\alpha V_{in}(\tau - \Delta_{PA}))^3 + a_5(\alpha V_{in}(\tau - \Delta_{PA}))^5 \ldots \tag{15}$$

Thus if there are more delays and are not controlled, they will obviously generate the non linear output. In order to provide AM-PM correction as well in the envelope domain feedback we require a method of RF differential phase detection. A simple multiplier as can be most easily realized using the square-law response of a diode.

So if input is

$$V_{in}(t) = A(t)\cos(\omega_m t + \theta(t)) \tag{16}$$

Then the output will be

$$V_0(t) = (1-\delta)A(t)\cos(\omega_m t + \theta(t) + \sigma) \tag{17}$$

Where δ and σ are the gain compression and AM-PM at the envelope input level. Thus the output of the multiplier is represented as

$$V_m(t) = (1-\delta)A(t)A(t) \cos(\omega_m t + \theta(t))\cos(\omega_m t + \theta(t) + \sigma) = (\frac{1}{2})(1-\delta)A(t)A(t)\{\cos\sigma + \cos(2(\omega_m t + \theta(t)) \tag{18}$$

which will be reduced by the IF filtering to the video signal,

$$V_m(t) = (\frac{1}{2})(1-\delta)A(t)A(t)\cos\sigma \tag{19}$$

A 90° phase shifter is required so that phase lead and lag are distinguished.

C.RF Power Amplifier Feedforward Linearization

The feedforward linearization technique was invented by H. S. Black [7] and applied to many communication systems. The feedforward linearization architecture is shown in Fig.3 and it is based on dividing the input signal into two branches. In the main branch the input signal is amplified by the main power amplifier yielding the PA output [8]. In the second branch the PA output is scaled and compared with the original input.

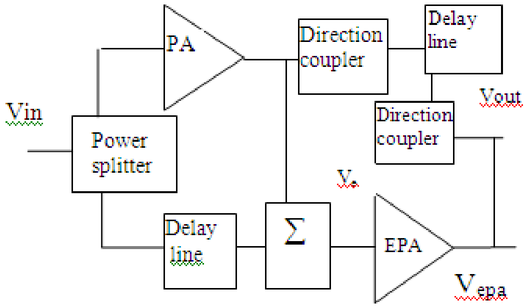

Figure 3. RF power amplifier feedforward linearization

The resulting error signal goes through a second PA known as the error PA. After the error signal is obtained it is amplified and added to the delayed output of the main PA. Since the error signal is the nonlinear distortion, adding it from the PA output linearizes the PA. The following equations describes the feedforward linearization.

$V_{in}(t) = v\cos\omega t$ and

After passing sampling coupler the output will be

$$\alpha V_{pa}(t) = a_1\alpha(v\cos\omega t) + a_3\alpha(v\cos\omega t)^3 \tag{20}$$

The input of error amplifier will be

$$V_e(t) = v\cos\omega t - \alpha V_{pa} = v\cos\omega t - \alpha a_1(v\cos\omega t) - \alpha a_3(v\cos\omega t)^3 = -\alpha a_3(v\cos\omega t)^3$$

if $\alpha = 1/a_1$

The distortion can be amplified with the auxiliary error PA and added from the original PA output as shown in Fig.3

$$V_{out}(t) = a_1(v\cos\omega t) + a_3(v\cos\omega t)^3 + (1/\alpha)(\alpha a_3(v\cos\omega t)^3) = a_1(v\cos\omega t) \tag{21}$$

which is linearized. The gain of the error amplifier has to be greater than 1/α in order to compensate for the (voltage) coupling factor β of the output coupler which is being used to achieve the necessary addition in the PA output. The output of error amplifier EPA can be written as

$$v_{epa} = b_1 v_e + b_3 {v_e}^3 = b_1\alpha a_3(v\cos\omega t)^3 + b_3(\alpha a_3(v\cos\omega t)^3)^3 \tag{22}$$

The transmission coefficient of coupler γ is related to coupling coefficient β as $\gamma = \sqrt{1-\beta^2}$

Analyzing the third order expansion from equation (6), the PA output can be re written as

$$V_{pa}(t)=(a_1v+\frac{3}{4}a_3v^3)$$
$$(\cos\omega t)+\frac{1}{4}a_3v^3(\cos 3\omega t)$$

thus the error amplifier input and output is

$$V_e(t)=[v-\alpha(a_1v+\frac{3}{4}a_3v^3)]\cos\omega t$$
$$v_{epa}=b_1[v-\alpha(a_1v+\frac{3}{4}a_3v^3)]\cos\omega t$$
$$+b_3([v-\alpha(a_1v+\frac{3}{4}a_3v^3)]^3\cos^3\omega t$$
$$=b_1[v-\alpha(a_1v+\frac{3}{4}a_3v^3)]\cos\omega t+\frac{3}{4}b_3$$
$$([v-\alpha(a_1v+\frac{3}{4}a_3v^3)]^3\cos\omega t$$

from equation (6)
When $\alpha a_1=1$ then

$$v_{epa}=[-\{\frac{3b_1\alpha a_3}{4}\}v^3-\{\frac{81b_3\alpha^3a_3^3}{256}\}v^9]\cos\omega t \qquad (23)$$

The final output

$$V_o(t)=\gamma V_{pa}(t)+\beta v_{epa}$$
$$=[\gamma(a_1v+\frac{3}{4}a_3v^3)+\beta(-\{\frac{3b_1\alpha a_3}{4}\}v^3$$
$$-\{\frac{81b_3\alpha^3a_3^3}{256}\}v^9)]\cos\omega t$$

Thus to cancel the third order distortion we put $\gamma=\beta b_1\alpha$ then the output becomes

$$V_o(t)=[\gamma a_1v-\beta\frac{81b_3\alpha^3a_3^3}{256}\}v^9]\cos\omega t \qquad (24)$$

Feedforward linearization is stable technique but it suffers from poor efficiency as an auxiliary error PA is required. Feedforward can be used over a wide bandwidth of about 10-100 MHz. However, mismatching of devices in amplitude and phase can lead to impairment and reduce the performance of the feedback system. Since this system is feedforward in nature, alterations in aging and temperature degrade the correction of linearity. To reduce these effects, we can use multiple feedforward loops with a single feedforward loop as the main amplifier. However, this configuration increases the complexity of a feedforward system. In addition, external devices such as delay lines are necessary.

D. Adaptive Baseband Predistortion

The adaptive baseband predistortion, also called digital predistortion is basically a Cartesian feedback with digital signal processor. The primary disadvantages of digital predistortion is relative complexity and bandwidth limitations but it has accuracy and computational rate of the specific DSP [9]. Furthermore, power consumption is also increased due to the digital signal processor[10]. In addition, digital predistortion has storage and processing overhead for the lookup tables[11].

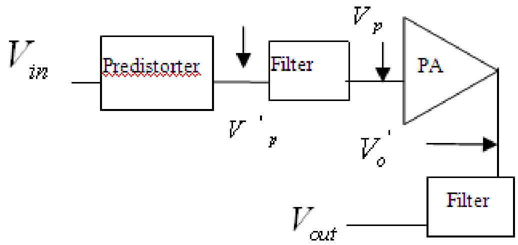

Fig.4 Adaptive baseband predistortion linearization.

The analysis of predictor will be defined by following equations:

$V_{in}(t)=v(\tau)\cos\omega t$ where τ is the time in envelope domain

$$V'_p(t)=b_1v(\tau)\cos\omega t+b_3(v(\tau)$$
$$\cos(\omega t+\varphi_3))^3+b_5(v(\tau)\cos(\omega t+\varphi_5))^5 \qquad (25)$$
$$+b_7(v(\tau)\cos(\omega t+\varphi_7))^7\ldots$$

Using equation (6)

$$V_p(t)=b_1v(\tau)\cos\omega t+\frac{3}{4}b_3v^3(\tau)\cos(\omega t+\varphi_3)$$
$$+\frac{5}{8}b_5v^5(\tau)\cos(\omega t+\varphi_5)+\frac{35}{64}b_7v^7(\tau) \qquad (26)$$
$$\cos(\omega t+\varphi_7)\ldots$$

And the PA output will be

$$V_o'(t)=a_1V_p(\omega t)+a_3V_p^{\,3}(\omega t+\theta_3)+a_5V_p^{\,5}$$
$$(\omega t+\theta_5)\ldots=a_1[b_1v(\tau)\cos\omega t+\frac{3}{4}b_3v^3(\tau)$$
$$\cos(\omega t+\theta_3)+\frac{5}{8}b_5v^5(\tau)\cos(\omega t+\theta_5)+\frac{35}{64}b_7v^7(\tau)$$
$$\cos(\omega t+\theta_7)\ldots]+a_3[b_1v(\tau)\cos\omega t+\frac{3}{4}b_3v^3(\tau)$$
$$\cos(\omega t+\theta_3)+\frac{5}{8}b_5v^5(\tau)\cos(\omega t+\theta_5)+\frac{35}{64}b_7v^7(\tau)$$
$$\cos(\omega t+\theta_7)..]^3+a_5[b_1v(\tau)\cos\omega t+\frac{3}{4}b_3v^3(\tau) \qquad (27)$$
$$\cos(\omega t+\theta_3)+\frac{5}{8}b_5v^5(\tau)\cos(\omega t+\theta_5)+\frac{35}{64}b_7v^7(\tau)$$
$$\cos(\omega t+\theta_7)..]^5$$

From the above equation we can extract the third order term as

$$[a_1(\frac{3}{4})b_3\cos(\omega t+\theta)+a_3b_1^{\,3}(\frac{3}{4})$$
$$\cos(\omega t+\theta_3)]v^3(\tau)$$

In order to remove third order distortion the above equation should be zero when

$a_1=b_1=1, b_3=a_3/a_1$ and $\varphi_3=\theta_3+\pi$

In the similar way fifth order distortion will be null when

$$\frac{5}{8}b_5\cos(\varphi_5)=(\frac{9}{16})a_3^{\,3}+(\frac{9}{8})a_3^{\,2}\cos 2\theta_3$$
$$-(\frac{5}{8})a_5\cos\theta_5$$

and $\frac{5}{8}b_5\sin(\varphi_5)=(\frac{9}{8})a_3^{\,2}\sin 2\theta_3-(\frac{5}{8})a_5\sin\theta_5$

Thus proper choice of coefficient could remove the distortion in the predistortion technique.

E. Envelope elimination and restoration (EER)

EER is a technique which increases linearity and power efficiency simultaneously [12]. The basic method of EER is to split the RF input signal into phase and envelope separately and then combine them after amplification [13]. The basic principle of EER is that any narrow-band signal can be produced by simultaneous amplitude (envelope) and phase modulation. The architecture in Fig.5 shows an EER technique with limiter and envelope detector to seperate the phase and envelope information, respectively.

The limiter eliminates the envelope and thus makes it possible for a high-efficient nonlinear Power Amplifier to amplify the constant-envelope signal. Finally, the envelope amplifier creates an amplified replica of the input signal at the output. EER gives better linearity with high efficiency. However, it has limited bandwidth of the class-S modulator and thus difficulty in correct alignment of the envelope and phase signals.a jitter or high ACPR could be the issues. Furthermore, high envelope variations can drive the RF power amplifier in to cutoff and thus significant distortion. However, the resulting feedback poses a problem in bandwidth limitation and loop stability. Furthermore, complexity of the circuit is also increased. The modulator is usually a class-S amplifier

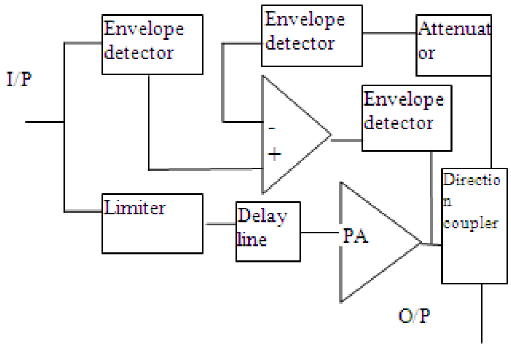

Figure 5. Envelope elimination and restoration.

IV. CONCLUSION

The linearization techniques discussed so far has a tradeoff between high efficiency, complexity and cost. The additional circuitry also need delay lines to compensate the synchronization of multiple inputs. The digital predistorter technique also need additional circuitry and complexity but the efficient adaptive algorithm can be used to give high efficiency power amplifier. A comparison of different RF power amplifier linearization techniques is shown in Table 2.

Table 2. Comparison of different RF power amplifier linearization techniques

Technique	Performance	Band width	cost	Control	Drawback
Cartesian	Moderate	Narrow	Moderate	Control at input	Low gain & stability
Polar	Moderate	Wide	Moderate	Control at input	Low gain
Analog RF Predistortion	Low	Wide	Low	Control at input	Comparativily Low gain
Digital Predistortion	Moderate	Wide	Moderate	Control at input	Easy to control but depends on DSP
Feed forward	Good	Wide	High	Control at output	Low efficiency

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