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Design of Grassmannian Weightbooks and Binary weightbooks for MIMO Beamforming Systems

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Abstract: For exploiting the significant diversity in Multiple-Input Multiple-Output (MIMO) wireless systems requires either complete channel knowledge or knowledge of the optimal beamforming vector; both are hard to realize. Hence a quantized maximum Signal-to-Noise Ratio (SNR) Grassmannian beamforming and Binary Grassmannian beamforming for MIMO Wireless Systems are proposed where the receiver only sends the label of the best beamforming vector using predetermined codebook to the transmitter. Grassmannian weightbook gives optimal performance even in high noise environment. The designed Binary weightbook gives approximately same performance as that of Grassmannian weightbook but with less complexity.

Keywords: Multiple-Input Multiple-Output(MIMO), Transmitter diversity, Quantized feedback, Grassmannian Weightbook, Binary Weightbook

I. INTRODUCTION

Multiple-input multiple-output (MIMO) wireless systems make use of the spatial dimension of the channel to provide considerable capacity, increased resilience to fading , or combinations of the two[1]. In narrow-band Rayleigh-fading matrix channels, MIMO systems can provide a diversity in proportion to the product of the number of transmit and receive antennas. Diversity in a MIMO system can be obtained through the use of space–time codes or via intelligent use of channel state information at the transmitter. Transmit beamforming with receive combining[2] is one of the simplest approaches to achieving full diversity and has been of interest recently [3]–[4]. Beamforming and combining in MIMO systems are a generalization of the vector channel beamforming and combining methods found in Single-Input–Multiple-Output (SIMO) combiners and Multiple-Input–Single-Output (MISO) beamformers which provide significantly more diversity. beamforming and combining systems provide the same diversity order as well as significantly more array gain [5] at the expense of requiring channel state information at the transmitter in the form of the transmit beamforming vector. Unfortunately, in systems where the forward and reverse channels are not reciprocal, this requires coarsely quantizing the channel or beamforming vector to accommodate the limited bandwidth of the feedback channel.

Consider the problem of quantized beamforming for independent and identically distributed (i.i.d.) MIMO Rayleigh flat-fading channels when the transmitter has access to a low-bandwidth feedback channel from the receiver and the receiver employs Maximum Ratio Combining (MRC)[5]-[7]. To support the limitations of the feedback channel, we assume the use of a codebook of possible beamforming vectors known to both the transmitter and receiver. The codebook is restricted to have fixed cardinality N and is designed off-line. The receiver is assumed to convey the best beamforming vector from the codebook over an error-free, zero-delay feedback channel. A primary contribution of this correspondence is to provide a constructive method for designing a quantized beamforming codebook. We show, using the distribution of the optimal unquantized beamforming vector, that the codebook design problem is equivalent to the problem of packing one-dimensional subspaces known as Grassmannian line packing. These codebooks are a function of the number of transmit antennas and the size of the codebook but are independent of the number of receive antennas.

In today's hand-held devices with limited memory, size, and power , limited feedback [8] weightbooks with smaller memory footprint will help to reduce implementation costs. Furthermore, reduced search computation will ease stringent computational complexity requirements in real-time systems and allow systems to quickly adapt to highly mobile environments. The weightbooks adopted for recent standards demonstrate trends towards systematic finite alphabet weightbooks [9, p. 39], [10, pp. 457-466]. Among finite alphabet weightbooks, a binary weightbook is the simplest form. Thus, investigation of binary weightbooks provides performance degradation limits and complexity reduction limits compared with infinite alphabet weightbooks such as Grassmannian weightbooks.

We consider Grassmannian weightbook and binary weightbook design for MIMO beamforming systems using quantized feedback based on the Grassmannian beamforming criterion. Using binary weightbooks, the computational complexity for finding the optimum beamforming weight vector and the storage requirement for the weightbook can be reduced. We show that the Grassmannian criterion for binary weightbook design is to

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maximize the minimum Hamming distance of the corresponding block code. Thus, a block code that has a large minimum Hamming distance is an advantageous choice for binary weightbook design for MIMO beamforming systems using quantized feedback.

II. SYSTEM MODEL

Consider a MIMO beamforming system with M_t transmit antennas and M_r receive antennas. We assume that Maximal Ratio Combining (MRC) is used at the receiver. An M-array symbol s with unit average energy is pre-encoded by a beamforming weight vector

$$\mathbf{w} = [w_1, w_2, \ldots, w_{M_t}]^T$$

from a binary weightbook W, where $(.)^T$ denotes matrix transpose and

$$\sum_{i=1}^{M_t} |w_i|^2 = M_t$$

Note that wi = 1 or −1 for all i =1, 2...Mi, for a binary weightbook. The transmitted signal is given by

$$\sqrt{\frac{E_s}{M_t}}\mathbf{w}$$

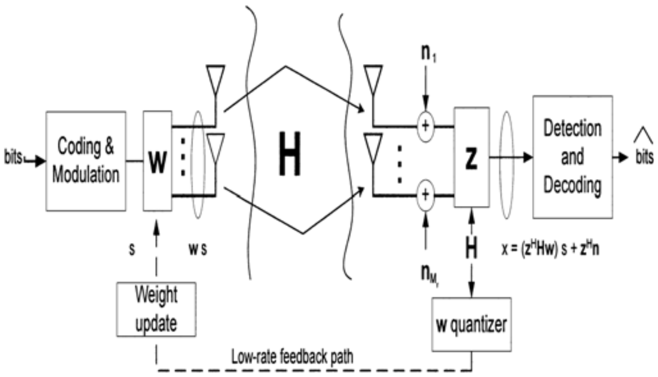

Fig 2.1 Block diagram for MIMO Systems

Where E_s is the transmit symbol energy. Thus, the transmit energy per antenna is E_s/M_t. We will denote the transmit bit energy which is equal to $E_s/\log_2 M$, by E_b. We will use a weightbook that consists of 2^F beamforming weight vectors, where F is the number of feedback bits. The receiver sends a feedback signal that the transmitter uses to select **w** from a weightbook W. The output signal of the maximal ratio combiner at the receiver is given by

$$r = \sqrt{\frac{E_s}{M_t}}\mathbf{z}^{\mathrm{H}}\mathbf{H}\mathbf{w}\cdot s + \mathbf{z}^{\mathrm{H}}\mathbf{n} = \sqrt{\frac{E_s}{M_t}}\mathbf{z}^{\mathrm{H}}\left(\sum_{l=1}^{M_t}\mathbf{h}_l w_l\right) s + \mathbf{z}^{\mathrm{H}}\mathbf{n}$$

Where **H** is the $M_r \times M_t$ channel gain matrix whose entries $h_{i,j}$ are independent and identically distributed (iid) circularly symmetric complex Gaussian fading gains with unit variance. The gain from the J th transmit antenna to the ith receive antenna is $h_{i,j}$, and **n** is a $M_r \times 1$ additive white Gaussian noise (AWGN) vector with variance $N_0/2$ per dimension. The combining weight vector at the receiver, **z**, is equal to **Hw**.

The instantaneous signal-to-noise ratio (SNR) at the receiver after MRC is given by

$$||\mathbf{H}\mathbf{w}||^2 \frac{E_s}{M_t N_0}$$

We use $(.)^H$ for matrix conjugate transpose, and $\|.\|$ for vector two-norm.

III. GRASSMANNIAN WEIGHT BOOK AND BINARY WEIGHT BOOK DESIGN

A. Grassmannian line packing : Grassmannian line packing is the problem of optimally packing one-dimensional subspaces[11] . It is similar to the problem of spherical code design with one important difference: spherical codes are *points* on the unit sphere while Grassmannian line packings are *lines* passing through the origin in a vector space. Grassmannian line packing forms the basis for our quantized beamforming codebook design.

***B. Grassmannian Beamforming Criterion*:** Design the set of codebook vectors $\{w_i\}_{i=1}^{N}$ [12],[13] such that the corresponding codebook matrix w maximizes

$$\delta(\boldsymbol{W}) = \min_{1\le k<l\le N} \sqrt{1 - |\boldsymbol{w}_k^H \boldsymbol{w}_l|^2}.$$

This criterion captures the essential point about quantized beamforming codebook design for Rayleigh-fading MIMO wireless systems: *Grassmannian line packings* are the key to codebook construction. Thus, beamforming codebooks can be designed without regard to the number of receive antennas by thinking of the codebook as an optimal packing of lines instead of a set of points on the complex unit sphere.

One benefit of making the connection between codebook construction and Grassmannian line packing is that it provides an approach for finding good codebooks, namely, leveraging work that has already been done on finding optimal line packings.

The codebook matrix W must satisfy

$$\boldsymbol{W} = \arg \max_{\boldsymbol{X}\in \mathcal{I}_{M_t}^{N}} \delta(\boldsymbol{X})$$

Where $I_{M_t}^N$ is given by the set of matrices in $U_{M_t}^N$ where each column can be represented as the normalized sum of unique column vectors of I_{M_t} . Since $I_{M_t}^N$ has finite cardinality, the global maximum [13]can be obtained by performing a brute-force search over all matrices in $I_{M_t}^N$. GSS codebooks provide better performance than selection diversity because additional vectors are included to allow a better quantization of the optimal beamforming vector.

C. *Binary Weight book design*: We derive the Grassmannian beamforming criterion for binary weightbook design. First, we define binary phase shift keying (BPSK) mapping, BPSK(.), as follows

BPSK(0) = 1

BPSK(1) = −1.

Then, we define inverse BPSK mapping, $BPSK^{-1}$(.), as follows

$$BPSK^{-1}(1) = 0$$

$$BPSK^{-1}(-1) = 1$$

Using the inverse BPSK mapping, we map a beamforming weight vector **w** = $[w_1, w_2, \ldots, w_{M_t}]^T$ from a weightbook W into **c** = $[c_1, c_2, \ldots, c_{M_t}]^T$ in a binary code C, where $c_i = BPSK^{-1}(w_i)$ for $i = 1, 2, \ldots, M_t$. For convenience, we will use the notations BPSK(.) and $BPSK^{-1}$ (.) also for vector inputs. Thus, BPSK(**c**) = **w** and $BPSK^{-1}$ (**w**) = **c**. Then,

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binary weightbook design for MIMO beamforming systems can be thought of as a corresponding binary block code design, where the block length of the block code is equal to $n = M_t$ and the number of codewords is equal to 2^F

The Grassmannian beamforming criterion for weightbook design is to maximize

$$\min_{\mathbf{w}_i \neq \mathbf{w}_j} \sqrt{1 - |\mathbf{w}_i^H \mathbf{w}_j|^2}$$

Where $\mathbf{w}_i, \mathbf{w}_j$ are beamforming weight vectors in a Weightbook $\mathcal{W}$. For binary weightbook design, above can be written as

$$\max_{\mathcal{W}} \min_{\mathbf{w}_i \neq \mathbf{w}_j} [1 - |\mathbf{w}_i^H \mathbf{w}_j|^2]$$

$$= \max_{\mathcal{W}} \min_{\mathbf{w}_i \neq \mathbf{w}_j} -|\mathbf{w}_i^H \mathbf{w}_j|^2$$

$$= \min_{\mathcal{W}} \max_{\mathbf{w}_i \neq \mathbf{w}_j} |(\text{number of terms with same sign}) - (\text{number of terms with different sign})|$$

$$= \min_{\mathcal{W}} \max_{\mathbf{c}_i \neq \mathbf{c}_j} [\max(M_t - 2d_H(\mathbf{c}_i, \mathbf{c}_j), 2d_H(\mathbf{c}_i, \mathbf{c}_j) - M_t)]$$

Where $\mathbf{w}_i$ = BPSK($\mathbf{c}_i$), $\mathbf{w}_j$ = BPSK($\mathbf{c}_j$), and $d_H(\mathbf{c}_i, \mathbf{c}_j)$ is the Hamming distance between $\mathbf{c}_i$ and $\mathbf{c}_j$ in the code C.

complementary code, $\bar{C}$, of the code C as follows

$$\bar{\mathcal{C}} = [1, 1, ..., 1]^T + \mathcal{C}$$
$$= \{[1, 1, ..., 1]^T + \mathbf{c} | \mathbf{c} \in \mathcal{C}\}$$

The Grassmannian beamforming criterion for binary eightbook design is to maximize the minimum distance of the unified code

$$d_{\min}(\mathcal{U}) \leq \lfloor M_t/2 \rfloor$$

D. Binary Weight book design for linear block codes:

We can design a binary weightbook using a given linear block code as follows. Suppose we have an (n,k) linear binary block code, where n is the block length and k is the dimension of the code.

If a (n, k) linear code C includes the codeword [1, 1, ..., 1]T, a generator matrix G can have [1, 1, ..., 1] as a row vector. Thus, any codeword in the (n, k) linear block code C can be represented as an unique linear combination of $\mathbf{b}_1$, $\mathbf{b}_2$, ..., $\mathbf{b}_{k-1}$, [1, 1,..., 1]T, where { $\mathbf{b}_1$, $\mathbf{b}_2$, ..., $\mathbf{b}_{k-1}$, [1, 1, ..., 1]T} constitute a basis of the block code. Half of the codewords have zero as the coefficient of [1, 1, ..., 1]T, and the other half of the codewords have one as the coefficient of [1, 1, ..., 1]T. By BPSK mapping of the former half of the codewords, we can have a binary weightbook for MIMO beamforming systems. The performance of this weightbook depends on the minimum Hamming distance of the (n, k) linear block code C, where $n = M_t$ and $k = F + 1$.

Given M_t and F, we can find a good weightbook using a well-known linear block code as follows. First, we find a $(M_t, F+1)$ linear block code that has [1, 1, ..., 1]T as a codeword and a large minimum Hamming distance. Then, the half of the codewords of the linear block code that start with zero can be used as beamforming weight vectors.

Binary weightbooks based on the Grassmannian beamforming criterion given the number of transmit antennas, M_t and the number of feedback bits, F. Given M_t and F, we will design a code C of which the column vectors are the 2^F codewords. Then, the corresponding binary weightbook can be found using BPSK mapping. The optimal codes

- $M_t = 2$, $F = 1$

$$\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

The minimum Hamming distance of the unified code, $d_{\min}(\mathcal{U})$, is one. This code is the optimum binary code since $d_{\min}(\mathcal{U})$ of other codes would be zero given $M_t = 2$ and $F = 1$. In general, for an arbitrary M_t and $F = M_t - 1$, the optimum binary code based on the Grassmannian beamforming criterion can be found by generating all the binary M_t-tuples of which the first element is zero

- $M_t = 3$, $F = 1$

$$\begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix}$$

The minimum Hamming distance for the unified code is one. Since this code achieves the bound, it is optimum.

- $M_t = 4$, $F = 2$

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix}$$

The code is linear. The minimum nonzero Hamming weight for the corresponding unified code for this code is two. Thus, the minimum Hamming distance for the unified code is two

- $M_t = 4$, $F = 3$

The optimum code consists of all the binary 4-tuples that start with zero.

- $M_t = 5$, $F = 1$

$$\begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 1 \end{pmatrix}$$

The minimum Hamming distance for the unified code is two $d_{\min}(\mathcal{U}) \leq 2$. Since this code achieves the bound, it is optimum.

We cannot use a binary weightbook for $F \geq M_t$. In these cases, the number of symbols in the code must be greater than two. For $F \geq M_t$, at least a ternary weightbook is required.

Note that that a binary weightbook has the all-zero vector as a member without loss of generality. Suppose we have a weightbook $\mathcal{W} = \{\mathbf{w}_1, \mathbf{w}_2, ..., \mathbf{w}_n\}$, where $\mathbf{w}_i = [w_{i1}, w_{i2}, ..., w_{iM_t}]$. Note that w_{ij} = 1 or −1 or all i and j for a binary weightbook. From a weightbook $\mathcal{W}$, we can make a weightbook $\mathcal{V}$ whose weight vectors {$\mathbf{v}_1$, $\mathbf{v}_2$, ..., $\mathbf{v}_n$ }are generated using $\mathbf{v}_i = \mathbf{w}(.)\ \mathbf{c}$, where (.) is the Schur product, i.e. element-wise product between two vectors, $\mathbf{c} = [c_1, c_2, ..., c_{M_t}]$ is an arbitrary vector, and c_i = 1 or −1 for all i. Then, the metric between $\mathbf{w}_i$ and $\mathbf{w}_j$, which is given

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by $|w_i^H w_j|$, is the same as the metric between the corresponding $\mathbf{v}_i$ and $\mathbf{v}_j$, which is given by $|v_i^H v_j|$. If we choose the vector **c** the same as **w**1, then **v**1 = **w**1 (.) **c** = **w**1 (.) **w**1 = [1, 1, ..., 1].

The weightbook $\mathcal{V}$ contains the weight vector [1, 1, ..., 1], which corresponds to the all zero codeword by the inverse BPSK mapping. Since we can find the weightbook $\mathcal{V}$ that contains [1, 1, ..., 1] and preserves the metric between two arbitrary weight vectors, we can assume that a binary weightbook has the all-zero vector as a member without loss of generality.

IV. SIMULATION RESULTS

The numerical performance results are represented for the Grassmannian beamforming weightbook and the binary Grassmannian weightbook . BPSK modulation is used for the results.

In Fig.4.1and Fig4.2., we show the bit error probability (BEP) vs. E_b/N_0 for M_t = 3, F = 1, 2, and M_r = 1, 2. For F = 2, it is shown that the binary Grassmannian weightbook and the Grassmannian weightbook have the same performance. For F = 1, the binary Grassmannian weightbook gives up compared with the Grassmannian weightbook.

In Fig. 4.3and Fig.4.4, we show the BEP vs. E_b/N_0 for M_t = 4 and M_r = 1, 2. For M_t = 3, 4 and F = 2, the BEP performance of the binary Grassmannian weightbook is the same as that of the Grassmannian weightbook.

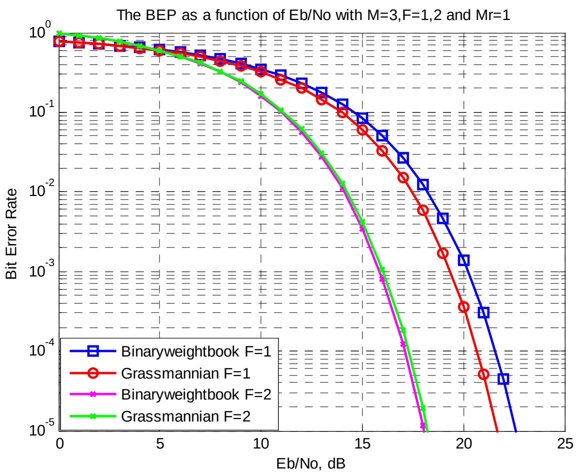

Fig 4.1.The BEP as a function of $\frac{E_b}{N_o}$ with Mt=3,F=1,2 and Mr=1

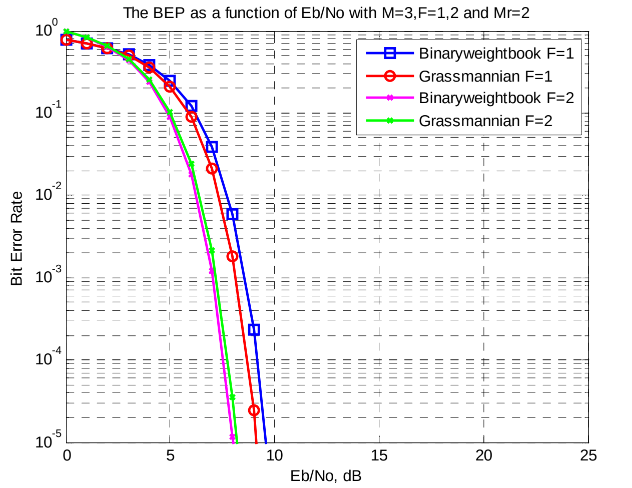

Fig 4.2.The BEP as a function of $\frac{E_b}{N_o}$ with Mt=3,F=1,2and Mr=2

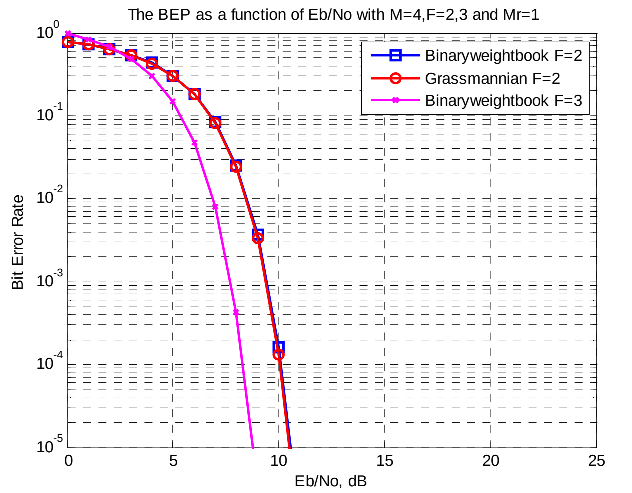

Fig 4.3.The BEP as a function of $\frac{E_b}{N_o}$ with Mt=4,F=1,2and Mr=1

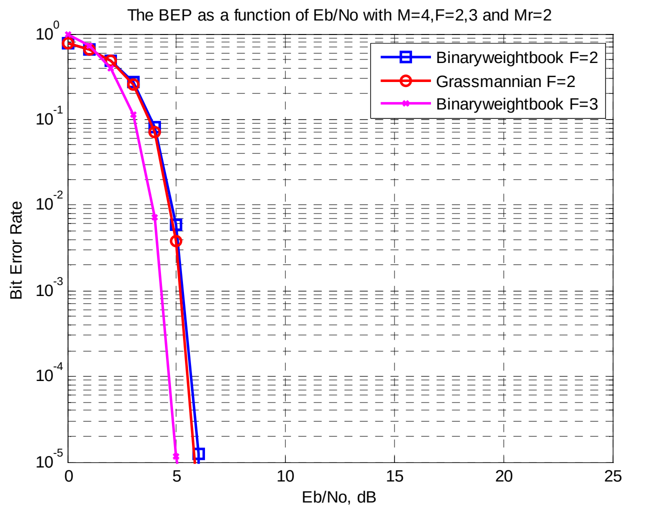

Fig 4.4The BEP as a function of $\frac{E_b}{N_o}$ with Mt=4,F=1,2and Mr=2

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V. CONCLUSION

The performance of Grassmannian weightbooks similar to that of binary weight book design . But the computational complexity for finding the optimum beamforming weight vector and the storage requirement for the weightbook for the Binary weightbook can be reduced compared with that of Grassmannian weight book.

VI. FUTHER WORK

Binary weight design using higher modulation schemes such as the QPSK,QAM can give higher performance. With the aid of the recent advances in the field of MIMO such as Neural Networks, a range other problem-solving methods have also emerged.

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