



A High-speed Residue Number Comparator for the 3-Moduli Set $\{2^n-1, 2^n, 2^n+1\}$

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Abstract: Carry-free property of Residue Number Systems (RNS) is very useful to achieve fast computing, parallelism and fault tolerant. However, there is no efficient general method for magnitude comparison in RNS. Most of the previous methods need redundant modulo or large moduli operations. In this paper, a new low cost method for magnitude number comparison in RNS has been presented that does not introduce any redundant modulo and uses small moduli operation.

Keywords: Residue Number System (RNS), comparator, parity, odd/even period, Chinese Remainder Theorem (CRT), Mixed Radix Converter (MRC).

I. INTRODUCTION

In a simple system classification, the residue numbers are in the weight-less number system which supports parallel numerical computing, limited carry propagation, low power consumption and secure communications. A complete computational unit should easily perform add, multiply, divide, root and comparison, while RNS is weak about division, root and comparison [1, 2, 3]. Division can be done using the subtraction but number comparison in this system, because of its weight-less property is very difficult [4]. Most of the proposed methods are based on reverse converters and compare numbers in weighted system [5-12]. Other methods for this purpose, use subtraction operator and then sign detection [1, 3, 13], in order to detect the sign, some reverse converting steps should be done [14, 15, 16]. One of the most important bottlenecks in RNS is when converting a residue number system to binary numeral system and vice versa, which greatly decreases system performance.

The other methods were used integer parity properties, in [1] number parity is stored in a table and in the comparison time, refers to the table in order to evaluate number parity. It means that memory loss is in a direct relation with dynamic range. In [13] additional hardware is used to detect parity which has high complexity.

The paper is organized as follows. Background is presented in the next section. Section 3 proposes a new algorithm for comparing the magnitude in RNS. Section 4 evaluates the proposed algorithm. Finally, section 5 concludes this paper.

II. BACKGROUND

Definition1 One bit, called parity, shows the number is even or odd. If the parity equals zero, the number is even. If the parity equals one, the number is odd.

Our main objective is that RNS comparison is fast, simple and one of the simplest ways to compare two numbers is by subtracting one number from the other number and examining the parity of their difference. Hence the comparison problem can be minimized into parity detection problem.

Axiom To compare two unsigned number X and Y is established by the following rules [1]:

1. Let X and Y be both odd or both even (be in the same parity) and $Z = (X - Y) \bmod M$ and M is odd, if and only if Z is an even integer, then $X \geq Y$. If and only if Z is an odd integer, then $X < Y$.

2. Let X and Y be in the different parities and $Z = (X - Y) \bmod M$ and M is odd, if and only if Z is an odd integer, then $X \geq Y$. If and only if Z is an even number, then $X < Y$.

Definition2 For modulo m , weighted numbers in the range $[0, m-1]$, $[2m, 3m-1]$, $[4m, 5m-1]$, ... are in the odd periods and weighted numbers in the range $[m, 2m-1]$, $[3m, 4m-1]$, $[5m, 6m-1]$, ... are in the even periods.

In [12] a new efficient direct and reverse converter for 3-moduli set $\{2^n-1, 2^n, 2^n+1\}$ has been introduced.

Being in even and odd periods the number $X=(x_{1(n-1)} \dots x_{11} x_{10}, x_{3n} \dots x_{31} x_{30})$ in moduli set $\{2^n-1, 2^{n+1}\}$ using the following formula is obtained:

$$X' = |x_{1-n} x_{3}|_2^{n-1}$$

$$Period \sim x'_0(x'_0 XOR x'_1) + x'_0(x'_0 XNOR x'_1) = x'_1 \quad (1)$$

Table 1: Compare proposed comparison technique with other technique

Comparator	OR/ AND	XOR/ XNOR	INV	MUX	HA	n-bit 1's complement adder	2n-bit 1's complement adder	n/n+1-bit comparator	2n-bit binary comparator	Delay
[8]	4n-2	4n	4n	1	16n	-	2	1	1	$t_{INV} + t_{MUX} + t_{ICA(2n)} + t_{BC(2n)}$
[9]	$[n \log n]$	-	2n	6n+2	8n	2	-	3	-	$[\log n]t_{OR} + t_{FA} + 5t_{MUX} + t_{ICA(n)} + t_{BC(n-1)}$
[13]	14 n	2	5n	6n+134	28n+30	-	-	1	-	$[\log n]t_{OR} + t_{INV} + (n+8)t_{FA} + 9t_{MUX}$
proposed	-	3	-	-	-	4	-	1	-	$t_{OR} + t_{AND} + t_{INV} + n t_{FA}$

Being even and odd the number X in moduli set $\{2^n-1, 2^{n+1}\}$ using the following formula is obtained:

$$Parity = X \text{ mod } 2 \text{ XOR period} = x_{30} \text{ XOR period} \quad (2)$$

III. DESIGNING THE NEW COMPARISON ALGORITHM

Comparison of A and B in the moduli set $\{2^n-1, 2^{n+1}\}$ can be done, if the result is equal, comparison in modulo 2^n as weighted system is done. Proposed method for the moduli set $\{2^n-1, 2^n, 2^{n+1}\}$ is as follows:

- To compare A and B in the moduli set $\{2^n-1, 2^n, 2^{n+1}\}$, Z is calculated:
 $Z = X - Y$.
- Parities of X, Y and Z using formulas (1) and (2) are calculated.
- If X and Y both are even or both are odd and Z is an odd number, then $X < Y$ otherwise $X > Y$. If one of X or Y is even and the other is odd and Z is even then $X < Y$ otherwise $X > Y$.
- If $Z=0$ then the comparison result is depend on comparing x_2 and y_2 .

IV. PERFORMANCE EVALUATION

In this section we present the performance evaluation for new algorithm and compare it with previous related algorithms. Table 1 shows the comparison between proposed method and previous ones. [9] is introduced one of the fastest RNS comparison for moduli set $\{2^n-1, 2^n, 2^{n+1}\}$ which uses n-bits modules.

V. CONCLUSION

In this paper a new efficient method for magnitude number comparison in RNS for the 3-moduli set $\{2^{n-1}, 2^n, 2^{n+1}\}$ is presented. In the proposed method, subtraction of integers and the parity of integer numbers and their period are used. This method is practical, efficient and fast without any redundant moduli and high complexity.

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