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Application of Undecimated Wavelet Transform in Breast Cancer Detection: Decomposition and Reconstruction

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Abstract: In this research paper, undecimated wavelet transform (UDWT) is focused on with respect to its decomposition and reconstruction and its applicability in enhancing mammographic images. It is established that UDWT eliminates decimation process, it is shift invariant and each coefficient is up sampled by a factor, hence facilitating a high degree of freedom. The proposed method, UDWT is discussed in the context of *Ã trus* algorithm and the two ways in which it can be applied to enhance mammograms –the 2-level UDWT, 3-Level UDWT and 4-Level UDWT. As such the main aim of this research paper is to demonstrate how undecimated wavelet transform differs considerably by eliminating decimation process, shift insensibility and absence of phase information.

Keywords: undecimated wavelet transform, shift invariance, wavelet transform

I. INTRODUCTION

The Undecimated Wavelet Transform (UDWT) is a paradigm shift from the traditional Discrete Wavelet Transform (DWT). Undecimated wavelet transform has in the last few decades been researched on under diverse names for instance redundant wavelet transform, stationary wavelet transform, and translation/shift invariant wavelet transform [6]. The traditional discrete wavelet transform has received criticism from various quarters with regard to its admissibility to analyzing mammographic images; reason being it makes use of decimators to after the filtering process. This leads to having both approximation and detail signal to be "**half as much as original signal in length after transformation** [1]. Although it has in the past been widely applied in mammography based on its efficiency from a computational perspective, it does not entail shift invariance [10, 11]. It is for this purpose that undecimated wavelet transform is applied to overcome the aforesaid drawback and give additional complete characteristics of each signal under analysis. In undecimated wavelet transform, this is achieved by removing decimators in order to eliminate decimation process after filtering and provide more precise information [1]. Based on the precision offered by the undecimated wavelet transform, this technology is applied in denoising images and detecting weak information, data especially in mammography. In the same note, undecimated wavelet transform has overcomes the shift insensibility; low directionality that results from down-sampling; and inexistence of phase information [7].

Therefore, UDWT is characterized by redundancy, non-down sampling and fulltime invariant version of WT [7]. In this research article, undecimated wavelet transform is covered to detail in regard to decomposition (analysis) and reconstruction (synthesis) of up to 4 – level. Besides the use of undecimated wavelet transform in denoising monographic images is discussed.

II. DECOMPOSITION (ANALYSIS) AND RECONSTRUCTION (SYNTHESIS)

Discrete wavelet transform facilitates in eliminating frequency components at definite times of the data. This necessitates having a robust capacity to do away with unnecessary as well as keep the good part of data for denoising or compression. Another advantage that comes along with discrete wavelet transform is the ability to easily compute inverse transform in order to reconstruct signals after they identifying and removing superfluous data and/or noise [3]. Furthermore, discrete wavelet transforms have the capacity to stretch and shift by powers of 2. In addition discrete wavelet transform makes use of quit a few wavelet filter compared to the single wavelet filter of continuous wavelet transform [3]. Undecimated discrete wavelet transform is single and it provides a clear learning bridge compared to decimate wavelet transform. In another perspective, it does not encompass aliasing problems such as those encountered in the convectional decimate wavelet transform.

Illustration of undecimated wavelet transform is shown in the figure below

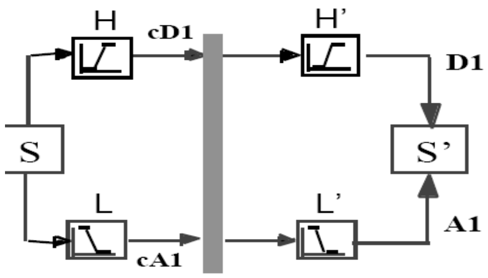


Figure 1. A single-level UDWT

As shown in the above figure, the signal flow diagram of UDWT contains four filters referred to as filter banks. In respect to decomposition, this is represented by the left section which involves forward transformation of signals.

The second section which is to the right entails synthesis or inverse transform of the signals but is preferably called reconstruction portion. Additional levels of both decomposition and reconstruction may be in the vertical bar section as shown in the diagram in order to produce undecimated wavelength transform of higher levels [3]. To effectively apply undecimated discrete wavelet transform in mammography [5] argues that it is best to use ***Algorithme a Trous***. In other words, this would entail an employment of this transform discretely [5]. This algorithm was pioneered by Holschneider in music synthesis and is said to work the same as monorthonormal multiresolution algorithm [6].

In this respect, putting into consideration an original signal of the form x[n] = do [n], the UDWT'S filter-bank decomposition formula would be;

A_j+1 [n] = a_j [n] x hd_j [n],

D_j+1[n] = a_j [n] x gd_j [n],

The above formula is derived from the decomposition coefficients at a level *j* which is greater than zero within a continuous time signal x (*t*) [5]. At the initial stages of decomposition, there are two paths which can be used to decompose signals. Assuming there is a signal, S, it will be first filtered by the high pass decomposition filter, H, in order to give coefficients CDI [3]. At this point, every sequence of coefficients at each decomposition level corresponds in length to the original coefficients [5]. Assuming that initially signal contains N samples, an L-level UDWT representation of the form

$$a_j[n] = \frac{1}{2}\left(a_{j+1}[n] * h_{r_j}[n] + d_{j+1}[n] * h_{r_j}[n]\right)$$

would result in having size N (L+I) [5]. This renders UDWT to be highly redundant. At the first stage when coefficients are generated further decomposition is done to up to 2nd, 3rd or 4th level by making use of high pass and filters. In other words, additional filters are applied to decompose each of the high pass and low pass filters. For instance, further splitting of the filters would result into the following as argued by [5].

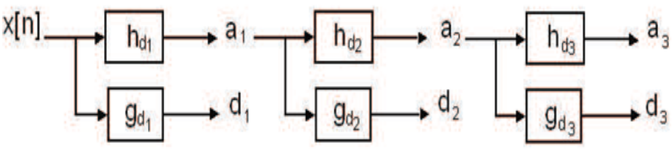


Figure 2. Three-level UDWT decomposition

In between filters at each level of decomposition, that correspond to a pair of consecutive level of UDWT decomposition, a relation of the form illustrated below prevails [5]

In referring to figure 1.1 above, the decomposition coefficients CDI, generated by the high pass filter is moved into the next portion which entails synthesis. This is the reconstruction section in which the signals that were split into many parts are combined [3]. Assuming an initial signal of the form x (n) = ao (n), the reconstruction of this previously decomposed signal would be of the following form [5],

$$a_j[n] = \frac{1}{2}\left(a_{j+1}[n] * h_{r_j}[n] + d_{j+1}[n] * h_{r_j}[n]\right)$$

This gives a reconstruction scheme as illustrated below [5].

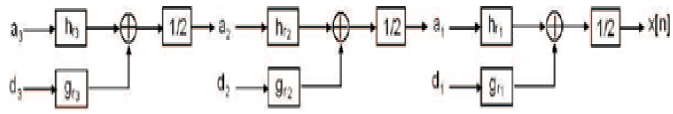


Figure 3. Three level UDWT reconstruction

As indicated in the above sector, High pass reconstruction filter, H, produces the detail coefficients D. this process is repeated even in the lower sections by the low pass decomposition and pass reconstruction filters to produce coefficients CA1 and Approximation Al respectively. Approximation represents the fine – tuned signal after undergoing low pass filtering. High pass decomposition and High pass Reconstruction filters collectively produce high pass half band filters whereas low pass decomposition and low pass reconstruction filters collectively produces low pass half-band filters.

The decomposition and reconstruction stage levels are illustrated in the below diagram, which is a 2-level UDWT [3].

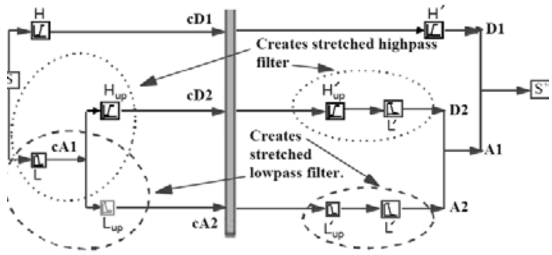


Figure 4. 2-level UDWT decomposition and reconstruction

An extra level of decomposition and reconstruction is achieved by splitting CA1 into CD2 and CA2. Therefore, it is possible to carry out additional filter stretching but this is specifically executed by powers of 2 dynamically. This is demonstrated in the figure above where a 2 – level UDWT is achieved one advantage of UDWT is that there is no down sampling and as such up-sampling is achieved by a factor of 2 which is represented by H_{up} in the figure above -this result in a stretched high-pass filter.

In comparison, CA2 coefficient results in a stretched low pass filter. At the end, the signal, S, which results during synthesis, is reconstructed by combining D1 and A1. Based on the fact that results from combining A2 and D2, S then results from A1 + D1 which is equal to A2 +D2+D1 [3]. It is at this stage that denoising of the signal is done which subsequently results in smoothened signals. It is important to note that, if there is no major interest with some of the coefficients for instance D2, it is zeroed thus resulting into S' = A2 + 0 + D1 = A2 + D1.in the same regard, if all coefficients are set at zero, D2 would be equal to zero; in other words, filtering zeros produces zeros and this is the fundamental concept applied by '*Ã Trous*' algorithm. It for that reason Undecimated Discrete Wavelet also called Redundant Discrete Wavelet Transform, is a system of inserting zeros in order to stretch the filters and therefore called '*Ã Trous*' meaning 'with holes'.

Splitting the stretched filters more frequently gives a 4-level Undecimated Wavelet Transform which has got additional stretched filters thereby splitting the wavelet signals into 5 frequency sub-bands as illustrated in the figure below.

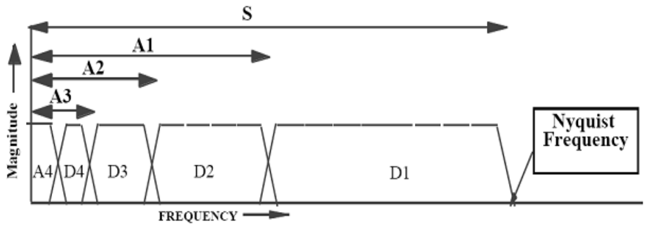


Figure 5. Four-level UDWT frequency allocation

In the above 4-Level UDWT, the signal, S, is decomposed into D1 and A1 coefficients; whereas A1 is decomposed into D2 and A2, A2 is decomposed into D3 and A3; and A3 is decomposed into D4 and A4. 4-Level UDWT is highly complex in terms of computations and size. One major characteristic of high pass and low pass filters in 4-Level UDWT is that filter stretching is done by a power of 2, 4 and 8 in order to derive the extra filter sub-bands. Thus the functions of 4-Level UDWT are very much similar to those of 2-level UDWT as explained above [3].

III. PROPOSED METHOD: UNDECIMATED WAVELET TRANSFORM

The concept of wavelet transform, as explained by [12, 16] is that mammograms consists of structures varied in contrasts and sizes. For instance, a particular mammogram may consist of small structures referred to as microcalcifications as well as large structures called masses. Therefore such regions within the mammogram are surrounded by normal vessels and dense tissues which complexes proper identification of carcinomas [17]. Min this respect, to identify microcalcifications effectively, it is essential to apply a method that focuses on localized fine structure whilst coarse background structures are removed.

This application of wavelet is essential to analyze images that consist of both fine and coarse patterns. In particular, undecimated wavelet transform, explained in the literature above, decomposes and synthesizes mammographic images into distinct scale components. Such analysis and synthesis of the original mammographic images is base on a fine probe represented by a high-level wavelet which is sufficiently localized within the space domain. In this regard, the undecimated wavelet transform enhance, to a large extent the size of the microcalcification as well as mass. In this research as it allows an input mammographic image to be analyzed and synthesized into sects of independent coefficients which correspond to every basic vector and also prove to be experimentally efficient by applying the basic UDWT transform -standard UDWT

IV. EXPERIMENTAL RESULTS & DISCUSSION

A. *Two-dimensional Standard UDWT:*

$$c_{j+1}[l] = (\bar{h}^{(j)} * c_j)[l] = \sum_k h[k] c_j[l + 2^j k]$$

$$w_{j+1}[l] = (\bar{g}^{(j)} * c_j)[l] = \sum_k g[k] c_j[l + 2^j k]$$

Assuming a UDWT , W that uses a filter bank (h, g) of a I-D signal Co that gives a set W={W1……., Wj, Cj}. In this case, wj refers to wavelet coefficients within the scale j whereas cj is the coefficient of coarse resolution [9]. As previously mentioned, the wavelet transform from particular resolution to the next can be obtained by making use of 'ã *trus*' algorithm [15, 20] as follows:……..……………… (1)

In this case, according to [9], h^j [L] = h [L] if $L/2^j$ is an integer and/or zero. For instance incase of the following decomposition,

h= (……., h [-2], 0, h [-1], 0, h [0], 0, h [1], 0, h [2]…)

Reconstruction would be given by

C_j [L] =1/2 {($\hat{h}^{(j)}$ x c_j + 1) [L] + ($\hat{g}^{(j)}$ x w_j +1) [L]} ……. (2)

In that case, a filter bank of the form (h, g, $\hat{h}$,g^2) would be required only to verify the definite reconstruction condition as illustrated below;

H (z^{-1}) H (z) + G (z^{-1}) (G^2 (z) =1 ………….…………… (3)

According to [9], the above analysis and synthesis avails a wide range of coefficients especially in designing the synthesis prototype filter bank [9].

To extend the ã *trous* algorithm up to a 2–level, the following can be used.

$$c_{j+1}[l] = (\bar{h}^{(j)} * c_j)[l] = \sum_k h[k] c_j[l + 2^j k]$$

$$w_{j+1}[l] = (\bar{g}^{(j)} * c_j)[l] = \sum_k g[k] c_j[l + 2^j k]$$

In the above case, h_g x c would be the convolution of c multiplied by the separable filter hg. Therefore at every scale, the wavelet images would be realized W^1, W^2 and W^3 that have a redundancy factor 3(J-1) + 1 [3]. The figure 6 below, [9] demonstrates a sample back projection of a wavelet coefficient in a 2-level undecimated wavelet transforms.

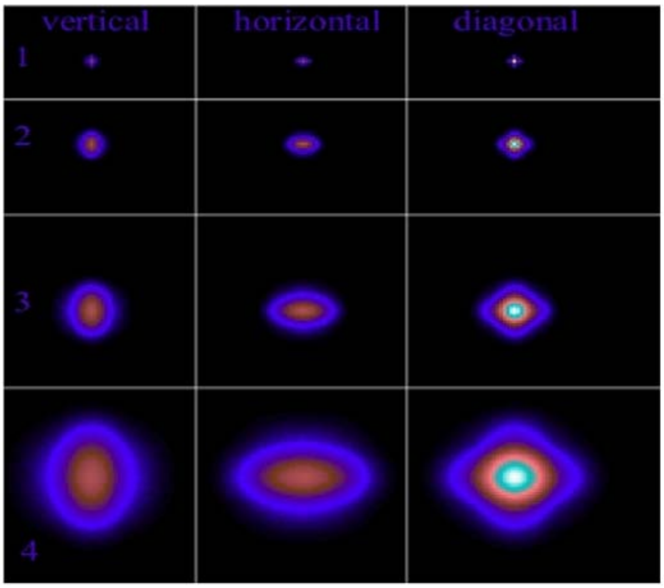


Figure. 6 Back Projections

Source: Starck Jean-Luc et al., 2007

Each of the above image matches up with the back projection of a single wavelet coefficient all the images have positive values in which case the coefficient closely relate to the diagonal, horizontal and vertical directions as their magnitude increases. It is worth nothing that all the wavelet coefficients are set to zero apart from one of them which belongs to another filter band and positive synthesis function are realized [9]. The positive synthesis functions are based on the fundamental concept that UDWT analysis is non-sub-sampled, hence providing various ways to synthesize the original mammographic image. According to [20], any filter bank of the form (ĥ, ĝ) that complies with reconstruction condition 3 above would result in exact reconstruction.

B. Detection of microcalfication using UDWT:

By applying a four-level UDWT decomposition, it is possible to overcome the inconveniency of losing particular details in the mammographic image particularly when details are in high band frequencies; By analyzing and synthesizing another wavelets, for instance Deubechies (*db*4), detection of microcalfications is highly improved a illustrated in figure 7b in the diagram below.

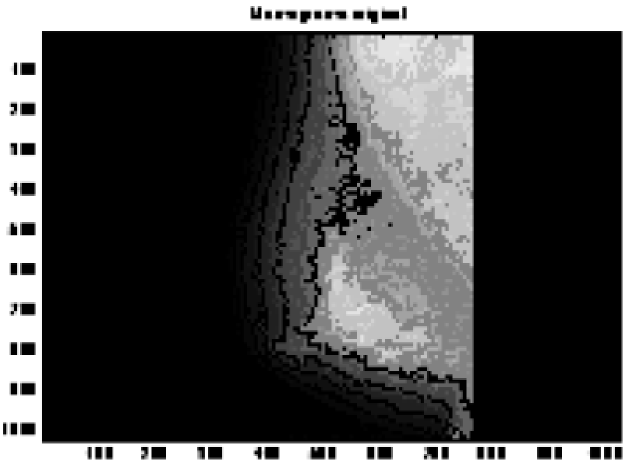
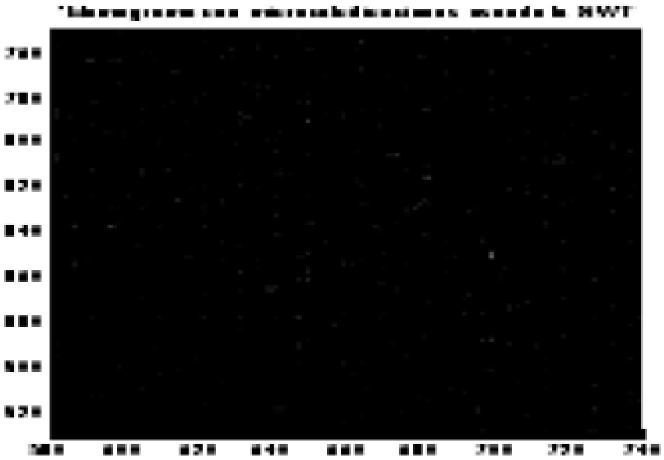


Figure 7. a. Original mammogram



b. Enhanced mammogram

Source: Alarcon-Aquino et al., 2009

Undecimated wavelet transform is non-redundant, time invariant and non-down sampling. For that reason, the length of coefficient is the same at every level of decomposition and its redundancy eliminates shift-invariance. Such property is ideal for data fusion, edge detection and image enhancement. Therefore as illustrated in the figure above detection of microcalfications increase significantly –UDWT performs considerable in detecting microcalfications [13]. Applying undecimated wavelet transform in mammography enhances the image quality to a greater extent as evidenced in the above section. According to [4] the use of ã *trous* algorithm modifies the discrete wavelet transform decomposition levels by up sampling both the low pass and high pass filters at every consecutive level [14]. This is achieved by inclusion of zeros, up sampling, between each and every filter coefficient levels.

Subsequently, by eliminating decimation stage, wavelet transform via UDWT is attained by omission of both down sampling during decomposition and up sampling during reconstruction. Rather, wavelet transform is executed at every point of the image hence facilitating saving the detail coefficients as well as using low frequency coefficients at each consecutive level. This is illustrated in figure 2 above. It is worth noting, that by making use of all coefficient at every analysis and synthesis level, information of high frequency is attained.

V. CONCLUSION AND FUTURE WORK

In this research paper, it is clear that analysis and synthesis of undecimated wavelet transform coefficients can be applied to improve detection of microcalfications in mammographic images. Reason being every coefficient is made use of to refine the image details and elimination of decimation process provides a certain degree to design both the high pass and low pass filters. Besides, this research paper has elaborated on the decomposition and reconstruction using, it is clear that the higher the level of analysis and synthesis, the more fine image results are. Therefore, undecimated wavelet transform is a good approach for detection microcalfications in mammographic images. It is thus good basis for developing an automatic diagnostic system and facilitate in mammogram interpretation as well as establishing early breast cancer diagnosis.

VI. ACKNOWLEDGEMENTS

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