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An Evolutionary Approach to Image Denoising Using A Regularized L^1 TV Model

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Abstract: Total variation models are effective and popular in image reconstruction. In many papers a variation model with L^2 fidelity term was introduced and shown to be capable of removing Gaussian noise. For images corrupted with impulse noise or outliers, the total variation model with L^1 fidelity term exhibit good properties in restoring noise free pixels and in preserving contrast. However, this model is nonstrictly convex and nondifferentiable. Another research work proposed a regularized version of the L^1 model and an efficient semismooth algorithm which involves second order information was presented to solve the discretization of this model. This paper deals with denoising images corrupted with impulse noise using an evolutionary approach. Specifically, the Genetic Algorithm (GA) is employed to optimize the regularized L^1 model. Numerical results show the capability of GA in reconstructing *n x n* noisy images, with *n* = 256.

Keywords: Image Processing, Genetic Algorithms, Image Denoising, Impulse Noise Removal, Evolutionary Algorithm

I. INTRODUCTION

Total variation models have been shown to be effective in reconstructing degraded images. Many factors can cause image degradation and one of these factors is the presence of noise, which can either be Gaussian noise or Impulse noise [1, 2, 5, 11, 13].

A given noisy image $\bar{d}$ can be represented by

$$\bar{d} = s + \eta \tag{1}$$

where $s \in \mathbb{R}^n$ is the original image in stacked form and $\eta \in \mathbb{R}^n$ is the additive noise. A popular model in reconstructing images with impulse noise is the total variation model:

$$\min_{u \in BV(\Omega)} \int_{\Omega} |u - d| dx + \alpha \int_{\Omega} |\nabla u| dx \tag{2}$$

where $\alpha > 0, u, d \in L^1(\Omega)$ and $u, d : \Omega \to \Omega, \Omega \subset \mathbb{R}^2$. The first term is the data fidelity term which preserves important features in the image, while the second term is the total variation penalty term, responsible for removing or smoothing out noisy pixels in the image. Problem (2) is said to be contrast invariant as shown in [2]. That means if *u(x)* is a solution with observed image *d*, then *cu(x)* is a solution to the observed image *cd*, for $c > 0$. Moreover, the regularization of the L^1 model does not affect the contrast of the image, that is, the value of the fidelity parameter α does not affect the quality of the image in terms of contrast [2]. Even as the value of α is increased, good contrast is preserved until they completely disappear. In relation to this property, Nikolova [11] shows that the L^1 fidelity term is able to reconstruct the image exactly at some pixels.

However, the L^1 model is convex but not strictly. Thus, the solution is not unique. Also, (2) is nondifferentiable which implies that direct methods such as the gradient method, Newton's method, or Newton-type methods like CGM [3] cannot be applied. The Fenchel dual of (2) is also nonstrictly convex, thereby dual methods such as that in [1] cannot be used either. In [5], Dong, Hintermüller, and Neri developed a strictly convex regularized version of problem (2), namely,

$$\min_{u \in BV(\Omega)} \int_{\Omega} \Psi(s) + \alpha \int_{\Omega} \Phi(\nabla s) dx \tag{3}$$

where

$$\Psi(s) = \begin{cases} |Ks - d| - \dfrac{\lambda}{2} & if \ |Ks - d| \geq \lambda \\ \dfrac{1}{2\lambda} |Ks - d|^2 & if \ |Ks - d| < \lambda \end{cases}$$

and

$$\Phi(\nabla s) = \begin{cases} |\nabla s| - \dfrac{\gamma}{2} & if |\nabla s| \geq \gamma \\ \dfrac{1}{2\gamma} |\nabla s|^2 & if |\nabla s| < \gamma \end{cases}$$

where *K* is the blurring matrix, and λ, γ, α > 0. For strictly denoising problems, we take *K* = *I*, the identity matrix. They developed an efficient smooth primal-dual active set method to generate the solution for (3). This method is patterned after the algorithm in [9] suited for removing white noise.

A nonderivative based method in optimization is the Genetic Algorithm (GA). The GA is a heuristic which is effective in arriving at a near-optimal global solution. It employs concepts in evolution such as selection, crossover, and mutation. Also, it makes use of a fitness function that determines the quality of an individual in the population. Since GA is not gradient based, applications of it to problem (3) is less expensive in terms of memory storage than the Newton-based method. The discrete version of (3) is used as the objective function to determine the fitness value of an individual. GA has been successfully implemented in optimization problems in various disciplines. There have been numerous implementations of the GA in image restoration (see e.g.[10, 12]). In several resources available to the authors, the GA was applied on images with dimensions at most *128 x 128* [4, 7, 14]. In this paper, we propose to implement GA on problem (3) to reconstruct images with size *256 x 256* with salt-and-pepper noise and random-valued noise.

In the next section, the Genetic Algorithm (GA) is presented followed by the proposed GA-based approach to denoising images with salt-and-pepper noise and random-valued noise. Numerical results that exhibit the efficiency of the method in denoising images with salt-and-pepper noise and random-valued noise are presented in section 4. We finish the paper with some recommendations.

II. THE GENETIC ALGORITHM

The Genetic Algorithm (GA) is a powerful optimization technique which is known to be robust and problem-independent. It is also a nonderivative-based optimization method that is known to be capable of reaching the near-global, if not global, optimum of a problem. Furthermore, it belongs to a class of stochastic search algorithms, which is based on the laws of natural selection and survival of the fittest [8]. The GA starts with an initial population from which new populations evolve. Individuals of each population are estimates to the solution(s) of the problem. These estimates are called chromosomes or individuals in the population. A new population is made in every generation through the selection, crossover, and mutation operators.

Simple analysis of the Genetic Algorithm shows that the most important characteristics are as follow [6]:

1. Initial population: It represents the candidate (possible) solutions, called chromosomes, to the optimization problem.
2. Fitness function: It provides a quality measure to a chromosome and evaluates it.
3. Genetic operators: These ensure that diversity is maintained in the population.
    a. Selection: Several best fit chromosomes are selected from the parent population based on some selection criteria like the Roulette wheel selection.
    b. Mutation: This operator uses only one parent and creates one child by altering one of more genes in the chromosome. This operator prevents the population from stagnating at any local optima.
    c. Crossover: It is a process whereby a new chromosome (offspring) is produced from the information contained within two parent solutions (mates).

The general scheme of the GA used in this paper as outlined below is based on [8]:

```
Begin
[01] Initialize Population
[02] Evaluate Population
[03] Set generation=1
[04] Set gbest to 0
[05] Set Current Population to Initial Population
[06] do
[07]      Generate Parent Pool through Selection
[08]      Perform Crossover
[09]      Perform Mutation
[10]      Evaluate Current Population
[11]      Pick lbest, best individual in current generation
[12]      if lbest is better than gbest
[13]           lbest becomes new gbest
[14]      Replace worst individual in current generation
          with gbest
[15]      generation = generation + 1
[16] while(terrmination criterion is not satisfied)
End
```

III. GA-BASED APPROACH TO IMAGE DENOISING

In this section, a new approach to image denoising with salt-and-pepper noise and random-valued noise is described, which is based on the Genetic Algorithm presented in [8]. The researchers would evaluate the denoising efficiency of the GA method as it is applied to minimize (4). Thus, image denoising can be considered as an optimization problem.

The image denoising process carried out using the Genetic Algorithm can be summed up as follows.

A. *Image Coding*

An $N \ x \ N$ discrete image is stacked as an $N^2 \ x \ 1$ vector, which represents a chromosome in the population. A chromosome or individual u_l is defined as an estimate solution to the regularized L^1TV model. The k^{th} gene in the chromosome represents the $(i, j)^{th}$ pixel value of the image, where $k = i + N(j - 1), 1 \leq i, j \leq N, 1 \leq k \leq N^2$.

B. *Initial Population and Fitness Function*

The initial population is composed of randomly generated noisy images. Each chromosome is then evaluated with respect to the observed noisy image using the objective function, which is the discrete analogue of problem (3).

$$J(u) = min\frac{1}{2\lambda}\sum_{\{k:|u_k-d_k|<\lambda\}}|u_k-d_k|^2 + \sum_{\{k:|u_k-d_k|\geq\lambda\}}\left(|u_k-d_k|-\frac{\lambda}{2}\right) + \frac{\alpha}{2\gamma}\sum_{\{k:|(\nabla u)_k|2<\gamma\}}|(\nabla u)_k|_2^2 + \alpha\sum_{\{k:|(\nabla u)_k|vert_2<\gamma\}}\left(|(\nabla u)_k|-\frac{\gamma}{2}\right) \quad (4)$$

where $d \in \mathbb{R}^{N^2}$ corresponds to the observed noisy image, $u \in \mathbb{R}^{N^2}$ is the solution update or estimate, and α, λ, and $\gamma > 0$.

C. *Genetic Operators*

1) Selection: For the selection process, the Roulette Wheel Selection [4, 7, 8] is used as shown in Fig. 1. The probability that individual u_l is chosen is given by $P(u_l)$ where

$$P(u_l) = \frac{f_l}{\sum_{j=1}^{popSize} f_j} \quad (5)$$

where f_l is the fitness of the individual l, defined by

$$f_l = \frac{1}{1+J_l}$$

J_l is the cost given by (4) at u_l. The roulette wheel is created with slot sizes based on (5). The higher the fitness, the larger the slot in the wheel, thus, as stated in the Schema Theorem [4], the greater the probability of the individual to be chosen more than once. The selected individuals make up the parent pool for the next generation.

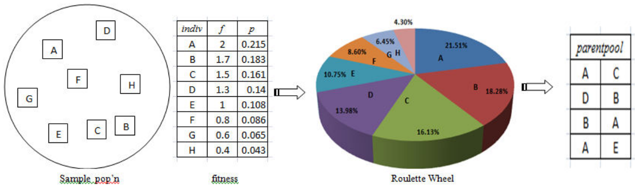

Figure 1. The Roulette Wheel Selection

2) Crossover: For crossover, new offsprings for the next generation are produced, with a crossover probability P_c. In this step, parent chromosomes from the parent pool exchange information by swapping gene values, expecting to produce better individuals for the next generation. The suggested range for P_c is [0.8, 1]. In this paper, we used the 2D-Single Point Crossover [4], where a crossover point *(i, j)* is randomly generated to produce 4 sets of blocks possible for exchange. Fig. 2 shows an example of a 2D-Single Point Crossover operator performed on parents *p1* and *p2*, producing the offsprings *c1* and *c2*.

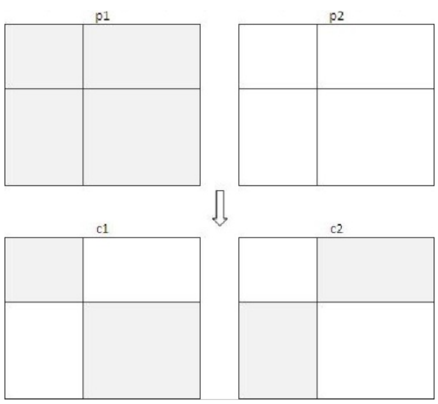

Figure 2. 2D-Single Point Crossover

3) Mutation: Chromosomes are slightly mutated, with a small mutation probability P_m, to allow variability in the population [4]. Chen, *et al* [4] developed a uniform mutation operator that determines the direction that will enhance the smoothing of pixels. A mutation point is randomly generated. The average pixel value of the eight neighboring pixels is taken, and is used to determine the mutated value of that selected point. In this paper, we used the median of the neighboring pixels since the median has been shown to be efficient in smoothing out outlier noise [11]. The value for P_m in our computations is within [0.005, 0.01]. Fig. 3 shows an example of a mutation using the modified uniform mutation operator.

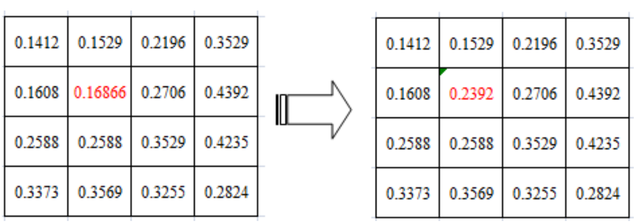

Figure 3. Modified Uniform Mutation Operator

D. Elitism

After performing the three genetic operators, a new set of population for the next generation is produced. Both the best individual per generation, l_{best}, and the best individual over generations, g_{best} are tracked. If $l_{best} > g_{best}$, then $g_{best} = l_{best}$ and g_{best} replaces the worst individual in the new generation.

IV. NUMERICAL RESULTS

Experiments were run to evaluate the denoising efficiency of the GA method as it is applied to minimize (4). Computations were done in MATLAB using a 1.76 GHz Intel core 2 duo processor with 0.99GB of RAM. The GA parameters used are shown in Table 1. The method terminated when maximum generation, *maxGen*, is reached.

Table I. The Genetic Algorithm Parameters

Parameters	***Value***
maxGen	500
PopSize	50
P_c	1
P_m	0.01

We tested the method on a grayscale *256 x 256* image (*see* Fig. 4). On the first implementation, the image was corrupted with *30%* salt-and-pepper noise (Fig. 5a). The second test had *30%* random-valued noise on the image (Fig. 5b).

Figure 4. Original image: "Cameraman"

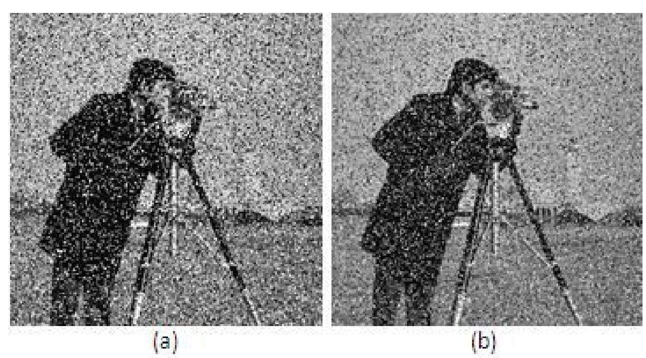

Figure 5. Noisy images corrupted with 30% (a) salt-and-pepper noise, (b) random-valued noise

The MSE (mean-squared error) and PSNR (peak-signal-to-noise ratio) are also computed to evaluate the fidelity of the denoised image with respect to the desired (clean) image, *s*. In our implementations, the desired image is known. The generation where the best solution first appeared is also noted. Refer to Table 2.

$$MSE = \sum \frac{|u-s|^2}{N^2} \qquad (6)$$

$$PSNR = 10 * \frac{log256^2}{MSE} \qquad (7)$$

Table II. Results for the L^1 TV Image Denoising

Noise	***Salt-and-Pepper***	***Random-Valued***
MSE	55.01	86.94
PSNR	30.76	28.77
t(sec)	706.82	843.01
gen	168	200

In terms of image appearance, the denoising procedure yielded fairly good results (*see* Fig. 6). Although, there is a number of noisy pixels that were not totally denoised, the rest of the reconstructed pixels are relatively noise-free compared to the observed image in Fig. 5. Also, we observe that the important features in the image, such as sharpness of edges and contrast, were preserved. Table 2 shows the numerical components of the experiment, where t is the running time in seconds and gen is the generation where the solution was first found.

Figure 6. Denoised images from noisy image with (a) salt-and-pepper noise, (b) random-valued noise

V. CONCLUSIONS

This is the first time known to the authors that the GA method was applied to the regularized L^1 total variation model (3) used to reconstruct images corrupted with impulse noise. The method worked very well in reconstructing our test images, without the need for computing first or second order information. At present, we are working on using GA in image deblurring using the model in [5].

VI. REFERENCES

[1] A. Chambolle, "An Algorithm for Total Variation Minimization and Applications," Journal of Mathematical Imaging and Vision, vol. 20.1/2, 2004, pp. 89-97.

[2] T. Chan and S. Esedoglu, "Aspects of Total Variation Regularized L1 Function Approximation," SIAM Journal on Applied Mathematics, vol. 65(5) , 2005, pp. 1817-1837.

[3] T. Chan, G. Golub, and P. Mulet, "A Nonlinear Primal-Dual Method for Total Variation-based Image Restoration. SIAM Journal on Scientific Computing, vol. 20, 1999, pp. 1964–1977.

[4] Y. W. Chen, T. Enokura, and Z. Nakao, "A Hybrid GA/SA Approach to Blind Deconvolution," In IEEE 2nd Int'l Conference on Knowledge-Based Intelligent Electronic System, vol. 3, 1998, pp. 144-149.

[5] Y. Dong, M. Hintermüller, and M. Neri, "An Efficient Primal-dual Method for L1 TV Image Restoration," SIAM Journal in Imaging Science, vol. 2(4), 2009, pp. 1168-1189.

[6] A E. Eiben and J.E. Smith. *Introduction to Evolutionary Computing*. Springer, 2003.

[7] T. Enokura, Y.W. Chen, and Z. Nako, "A Fast Algorithm for Image Restoration based on a Hybrid GA and SA," In Systems, Man, and Cybernetics, 1999. IEEE SMC '99 Conference Proceedings, vol. 4, 1999, pp. 891-894.

[8] D. E. Goldberg, Genetic Algorithms in Search, Optimization, and Machine Learning. Addison-Wesley, Reading, MA, 1989.

[9] M. Hintermüller and G. Stadler, "An Infeasible Primal-dual Algorithm for Total Bounded Variation-based inf-convolution-type Image Restoration," SIAM Journal on Scientific Computing, vol. 28(1), 2006, pp. 1-23.

[10] T. Jiang and D. J. Evans, "Image Restoration by Combining Local Genetic Algorithm with Adaptive Preconditioning," International Journal of Computer Mathematics, vol. 76(3), 2001, pp. 279-295.

[11] M. Nikolova, "A Variational Approach to Remove Outliers and Impulse Noise," Journal of Mathematical Imaging and Vision, vol. 20, 2004, pp. 99–120.

[12] S.J. Ribeiro and J.C. da Silva, "Genetic Algorithms and Biological Images Restoration," Preliminary report. Lecture Notes in Computer Science, Springer–Berlin, vol. 2527, 2002, pp. 340-349.

[13] L. Rudin, S. Osher, and E. Fatemi, "Nonlinear Total Variation based Noise Removal Algorithms," Physica D, vol. 60, 1992, pp. 259-268.

[14] D. Snyers and Y. Pétillot, "Image Processing Optimization by Genetic Algorithm with a New Coding Scheme," Pattern Recognition Letters, vol. 16(8), 1995, pp. 843-848.

