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A NEW CLASS OF OPEN AND CLOSED MAPPINGS

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Abstract : The purpose of this paper is to introduced and study new classes of open and closed functions called ${}_N D_\beta - open$ and ${}_N D_\beta - closed$ maps by using ${}_N D_\beta - open$ and ${}_N D_\beta - closed$ [10] sets and establish relationships of these maps with already existing generalized maps. Several properties of these new notions have been discussed and the connections between them are studied.

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1. INTRODUCTION

The present authors introduced the notion of ${}_N D_\beta - closed$, ${}_N D_\beta - open$ sets and ${}_N D_\beta - continuous$ functions in topological spaces and study some of their properties [10]. Different types of closed and open mappings were studied by various researchers. Generalized closed mappings were introduce and studied by Malghan [13]. Crosseley et.al [5] initiated and studied the notion of $\beta - open$ and $\beta - closed$ maps. Misser et al.[49] defined and investigated $semi^* - open$ and $semi^* - closed$ maps. Sayed et. al [20] devised the notion of $D_\alpha - open$ and $D_\alpha - closed$ maps and enlighten its properties

In this paper, we introduce new classes of maps ${}_N D_\beta - open$ and ${}_N D_\beta - closed$ maps. We also prove that the composition of two ${}_N D_\beta - open$ $(resp. {}_N D_\beta - closed)$ maps need not be ${}_N D_\beta - open$ $(resp. {}_N D_\beta - closed)$. We also establish some properties of ${}_N D_\beta - open$ and ${}_N D_\beta - closed$ maps.

In the whole paper (X, τ) and (Y, σ) (simply X and Y) represent the non-empty topological spaces on which no separation axioms are assumed, unless explicitly stated. Let $A \subseteq X$ the closure of A and interior of A will be denoted by $Cl(A)$ and $Int(A)$ respectively.

Here we recollect all the definitions which will be used in sequel.

2. PRELIMINARIES

Definition 2.1. Let (X, τ) be a topological space. A subset A of the space X is said to be,

(i) *preopen* [14] if $A \subseteq Int\,(C\ell(A))$ and *preclosed* if $C\ell(Int(A)) \subseteq A$.

(ii) *semiopen* [11] if $A \subseteq C\ell(Int(A))$ and *semiclosed* if $Int\,(C\ell\,(A)) \subseteq A.$.

(iii) $\alpha - open$ [16] if $A \subseteq Int\,(C\ell(\,Int\,(A)))$ and $\alpha - closed$ if $C\ell(Int(C\ell(A))) \subseteq A$.

(iv) $\beta - open$ [1] if $A \subseteq C\ell\,(\,Int(C\ell\,(A)))$ and $\beta - closed$ if $Int\,(C\ell\,(\,Int\,(A))) \subseteq A$.

(v) $generalized - closed$ (briefly $g - closed$) [12] if $C\ell(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X and $generalized - open$ (briefly $g - open$) if $X \setminus A$ is $g - closed$.

(vi) $semi^* - closed$ [17] if $Int^*(C\ell\,(A)) \subseteq A$ and $semi^* - open$ [18] if $A \subseteq C\ell*(\,Int\,(A))$.

(vii) $D_\alpha - closed$ [20] if $C\ell*(\,Int\,(C\ell*(A))) \subseteq A$ and $D_\alpha - open$ if $X \setminus A$ is $D_\alpha - closed$.

(viii) ${}_N D_\beta - closed$ [10] if $Int\,(C\ell^*(Int\,(A))) \subseteq A$ and ${}_N D_\beta - open$ if $X \setminus A$ is ${}_N D_\beta - closed$.

Definition 2.2. A function $f:(X, \tau) \to (Y, \sigma)$ is said to be,

(i) $\beta - continuous$ [1] if the inverse image of each open set in Y is β - *open* in X.

(ii) g - *continuous* [4] [15] if the inverse image of each open set in Y is g - *open* in X .

(iii) $semi^{*}$-*continuous* [19] if the inverse image of each open set in Y is $semi^{*}$-*open* in X.

(iv) $D_{\alpha}-continuous$ [20] if the inverse image of each open set in Y is $D_{\alpha}-open$ in X.

(v) ${}_{N}D_{\beta}-continuous$ [10] if the inverse image of each open set in Y is ${}_{N}D_{\beta}-open$ in X.

(vi) $\beta-open$ [1] (resp. $\beta-closed$) if the image of each open (resp. closed) set in X is $\beta-open$ (resp. $\beta-closed$) in Y.

(vii) $g-open$ [13] (resp. $g-closed$) if the image of each open (resp. closed) set in X is $g-open$ (resp. $g-closed$) in Y.

(viii) $semi^{*}$-*open* (resp. $semi^{*}$-*closed*) [19] if the image of each open (resp. closed) set in X is $semi^{*}$-*open* (resp. $semi^{*}$-*closed*) in Y.

(ix) $D_{\alpha}-open$ [20] (resp. $D_{\alpha}-closed$) if the image of each open (resp. closed) set in X is $D_{\alpha}-open$ (resp. $D_{\alpha}-closed$) in Y.

Definition 2.3. [12] A topological space (X,τ) is said to be $T_{1/2}$ if every g-closed set is closed.

The intersection of all $g-closed$ sets containing A [12] is called the $g-closure$ of A and denoted by $C\ell^{*}(A)$ and the $g-interior$ of A [12] is the union of all $g-open$ sets contained in A and is denoted by $Int^{*}(A)$. The intersection of all ${}_{N}D_{\beta}-closed$ sets containing A [10] is called the ${}_{N}D_{\beta}-closure$ of A and denoted by ${}_{N}D_{\beta}-C\ell(A)$ and the ${}_{N}D_{\beta}-interior$ of A [10] is the union of all ${}_{N}D_{\beta}-open$ sets contained in A and is denoted by ${}_{N}D_{\beta}-Int(A)$.

The collection of all ${}_{N}D_{\beta}-closed$ (resp. closed , D_{α}-closed , $g-closed$, $\beta-closed$, $semi^{*}$-closed) sets in (X,τ) denoted by $D_{\beta}C(X)$ (resp., C(X) , $D_{\alpha}C(X)$, $GC(X)$, $\beta C(X)$, $semi^{*}C(X)$). The collection of all D_{β}-open (resp. open , $D_{\alpha}-open$, $g-open$, $\beta-open$, $semi^{*}$-open) sets in (X,τ) denoted by $D_{\beta}O(X)$ (resp. O(X) , $D_{\alpha}O(X)$, $GO(X)$, $\beta O(X)$, $semi^{*}O(X)$). (c.f. [2], [10] [17] ,[19], [20])

3. ${}_{N}D_{\beta}-$ Closed Maps

We introduce the following definition

Definition 3.1. A map $f:(X,\tau)\to(Y,\sigma)$ is said to be ${}_{N}D_{\beta}-closed$ if $f(V)$ is ${}_{N}D_{\beta}-closed$ in Y for each closed set V in X.

Theorem 3.2. **(i)** Every $\beta-closed$ map is ${}_{N}D_{\beta}-closed$.

(ii) Every $g-closed$ map is ${}_{N}D_{\beta}-closed$.

(iii) Every $semi^{*}-closed$ map is ${}_{N}D_{\beta}-closed$.

(iv) Every $D_{\alpha}-closed$ map is ${}_{N}D_{\beta}-closed$.

Proof. (i) The proof follows from the definition and from the Theorem **3.3** of [10] that every $g-closed$ set is ${}_{N}D_{\beta}-closed$

(ii) The proof follows from the definition and from the Theorem **3.3** of [10] that every $semi^{*}-open$ set is ${}_{N}D_{\beta}-closed$.

(iii) The proof follows from the definition and from the Theorem **3.3** of [10] that every $\beta-$ open set is ${}_{N}D_{\beta}-closed$.

(iv) The proof follows from the definition and from the Theorem **3.3** of [10] that every $D_{\alpha}-closed$ set is ${}_{N}D_{\beta}-closed$.

Remark. 3.3. (i) ${}_{N}D_{\beta}-closed$ map need not be $\beta-closed$. (see the Example **3.4** below)

(ii) ${}_{N}D_{\beta}-closed$ map need not be $g-closed$. (see the Example **3.5** below)

(iii) ${}_{N}D_{\beta}-closed$ map need not be $semi^{*}-closed$ (see the Example **3.6** below)

(iv) ${}_{N}D_{\beta}-closed$ map need not be $D_{\alpha}-closed$. (see the Example **3.7** below).

Example 3.4. Let $X=\{a,b,c,d\}=Y$,

$\tau=\{X,\phi,\{a,b\},\{a,b,d\},\{c,d\},\{d\}\}$ and $\sigma=\{Y,\phi,\{a,b,d\},\{a\}\}$

then (X,τ) and (Y,τ) be a topological spaces.

$C(X)=\{Y,\phi,\{c,d\},\{c\},\{a,b\},\{a,b,c\}\}$ $C(Y)=\{Y,\phi,\{c\},\{b,c,d\}\}$,

$GC(Y)=\{Y,\phi,\{c\},\{b,c,d\},\{b,c\},\{c,d\},\{b,d\},\{a,c,d\},\{a,b,c\}\}$
$GO(Y)=\{Y,\phi,\{a,b,d\},\{a\}\},\{a,d\},\{a,b\},\{a,c\},\{b\},\{d\}\}$,
$\beta C(Y)=\{Y,\phi,\{b,c,d\},\{c\},\{b,d\},\{b,c\}\{c,d\},\{b\},\{d\}\}$,

${}_{N}D_{\beta}C(Y)=\{Y,\phi,\{b,c,d\},\{c\},\{b,d\},\{b,c\}\{c,d\},\{b\},\{d\},$
$\{a,c,d\},\{a,b,c\},\{a,d\},\{a,b\},\{a\}\}$

Let $f:(X,\tau)\to(Y,\sigma)$ be a map defined by $f(a)=d$, $f(b)=c$, $f(c)=a$ and $f(d)=b$ is ${}_{N}D_{\beta}-closed$ map, since the image of each closed set in (X,τ) is ${}_{N}D_{\beta}-closed$ in

(Y,σ). But map f is not $\beta-closed$, since $f(\{c,d\})=\{a,b\}$, which is not $\beta-closed$ in Y.

Example 3.5. Let $X=\{a,b,c\}$ be any set with topology $\tau=\{X,\phi,\{a\},\{a,b\},\{a,c\}\}$, then (X,τ) be a topological space. Let $Y=\{x,y,z\}$ with topology $\sigma=\{Y,\phi,\{y,z\},\{y\}\}$, then (Y,σ) be another topological space.

$C(X)=\{X,\phi,\{b,c\},\{c\}\},\{b\}\}$, $C(Y)=\{Y,\phi,\{x\},\{x,z\}\}$,

$GC(Y)=\{Y,\phi,\{x\},\{x,y\},\{x,z\}\}$

${}_N D_\beta C(Y)=\{Y,\phi,\{x\},\{z\},\{x,z\},\{y\},\{x,y\}\}$.

Let $f:(X,\tau)\to(Y,\sigma)$ be the function defined by $f(a)=x$, $f(b)=z$ and $f(c)=y$ is ${}_N D_\beta-closed$ map, since f image of each open set in (X,τ) is ${}_N D_\beta-closed$ in (Y,σ). But map f is not $g-closed$, since $f(\{b,c\})=\{y,z\}$, which is is not $g-closed$ in Y.

Example 3.6. Let $X=\{a,b,c,d\}$, $\tau=\{X,\phi,\{a\},\{b\},\{a,b\},\{a,b,c\}\}$ and $Y=\{1,2,3,4\}$, $\sigma=\{Y,\phi,\{2,4\},\{2,3,4\}\}$. Then (X,τ) and (Y,σ) be any two topological space. $C(X)=\{X,\phi,\{b,c,d\},\{a,c,d\}\},\{c,d\},\{d\}\}$,

$C(Y)=\{Y,\phi,\{1,3\},\{1\}\}$.

$GC(Y)=\{Y,\phi,\{1,3\},\{1\},\{1,4\},\{1,2\},\{1,2,3\},\{1,2,4\},\{1,3,4\}\}$,

$GO(Y)=\{Y,\phi,\{2,4\},\{2,3,4\},\{2,3\},\{3,4\},\{4\},\{3\}\}$,

$Semi*C(Y)=\{Y,\phi,\{1,3\},\{1\},\{3\}\}$,

${}_N D_\beta C(Y)=\{Y,\phi,\{1,3\},\{1\},\{1,4\},\{1,2\},\{1,2,3\},\{1,2,4\},\{2,4\},\{2,3\},\{2\},\{3\},\{4\},\{1,3,4\},\{3,4\}\}$.

Define a function $f:(X,\tau)\to(Y,\sigma)$ by, $f(a)=2$, $f(b)=3$, $f(c)=4$, $f(d)=1$, which is a ${}_N D_\beta-closed$ map, since f image of each closed set is ${}_N D_\beta-closed$ in (Y,σ). But f is not $semi^*-closed$ map, since $f(\{b,c,d\})=\{1,3,4\}$, which is not $semi^*-closed$ in (Y,σ).

Example 3.7. Let $X=\{x,y,z\}$ and $\tau=\{X,\phi,\{y,z\},\{z\}\}$, then (X,τ) be a topological space. $C(Y)=\{Y,\phi,\{x\},\{x,y\}\}$. Let $Y=\{r,s,t\}$ and $\sigma=\{Y,\phi,\{s,t\},\{s\},\{t\}\}$, then (Y,σ) be a topological space. $C(Y)=\{Y,\phi,\{r\},\{r,t\},\{r,s\}\}$,

$GC(Y)=\{Y,\phi,\{r\},\{r,t\},\{r,s\}\}$,

$GO(Y)=\{Y,\phi,\{s,t\},\{s\},\{t\}\}$

$D_\alpha C(Y)=\{Y,\phi,\{r\},\{r,t\},\{r,s\}\}$,

$D_\alpha O(Y)=\{Y,\phi,\{s,t\},\{s\},\{t\}\}$

${}_N D_\beta(Y)=\{Y,\phi,\{r\},\{r,t\},\{r,s\},\{s\},\{t\}\}$,

${}_N D_\beta O(Y)=\{Y,\phi,\{s,t\},\{s\},\{t\},\{r,s\},\{r,t\}\}$.

Let function $f:(X,\tau)\to(Y,\sigma)$ defined by $f(x)=s$, $f(y)=t$, $f(z)=r$ is ${}_N D_\beta-closed$ map, since the image of each closed set in X is ${}_N D_\beta-closed$ in Y, but f is not $D_\alpha-closed$, Since $f(\{x,y\}=\{s,t\}$, which is not $D_\alpha-closed$ in Y.

Interrelationship

From the above discussions and known results, we have the following implications.

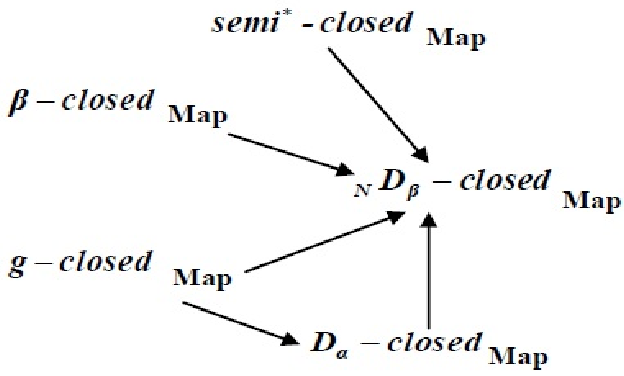

Figure 1

Remark. 3.8. The composition of two ${}_N D_\beta-closed$ maps need not be ${}_N D_\beta-closed$ in general. This is shown by the following example.

Example 3.9. Let $X=Y=Z=\{b,c,d\}$ be the sets with the topology $\tau=\{X,\phi,\{c\},\{c,d\}\}$, $\sigma=\{Y,\phi,\{b,c\}\}$ and $\eta=\{Z,\phi,\{b,d\},\{d\}\}$, respectively. Then (X,τ), (Y,σ) and (Z,η) be the topological spaces. $C(X)=\{X,\phi,\{b,d\},\{b\}\}$, $C(Y)=\{Y,\phi,\{d\}\}$, $C(Z)=\{Z,\phi,\{b,c\},\{c\}\}$. We define a map $f:(X,\tau)\to(Y,\sigma)$ as $f(b)=d$, $f(c)=b$ and $f(d)=c$ the map $g:(Y,\sigma)\to(Z,\eta)$ as $g(b)=c$, $g(c)=d$ and $g(d)=b$. Then f and g are ${}_N D_\beta-closed$ maps, but their composition $g\circ f:(X,\tau)\to(Z,\sigma)$ is not $D_\beta-closed$. For, if $A=\{b,d\}$ be any closed set in (X,τ) and $g\circ f(A)=g(f(\{b,d\}))=g(\{c,d\})=\{b,d\}$, which is not a ${}_N D_\beta-closed$ set in (Z,η).

Theorem 3.10. If $f:(X,\tau)\to(Y,\sigma)$ is a closed map and $g:(Y,\sigma)\to(Z,\eta)$ is $D_\beta-closed$ map, then their composition $g\circ f:(X,\tau)\to(Z,\sigma)$ is ${}_N D_\beta-closed$ map.

Proof. Let G be any closed set in (X,τ). Since f is a closed map, $f(G)$ is closed in (Y,σ). Since g is ${}_N D_\beta-closed$ map, $g(f(G))$ is ${}_N D_\beta-closed$ set in (Z,η). Therefore $g\circ f(G)=g(f(G))$ is ${}_N D_\beta-closed$ set in (Z,η).

Theorem 3.11. If the space is $T_{1/2}$, then every $D_\alpha-closed$ (resp. ${}_N D_\beta-closed$) set is $\alpha-closed$ (resp. $\beta-closed$).

Proof. Let A be any $D_\alpha-closed$ (resp. ${}_N D_\beta-closed$) subset of the space X, then we have $(C\ell^*(Int(C\ell^*(A)))\subseteq A$ (resp. $Int(C\ell^*(Int(A)))\subseteq A$). Since X is $T_{1/2}-space$, every $g-closed$ set is closed, consequently $C\ell^*(A)=C\ell(A)$. Thus, we get $C\ell(Int(C\ell(A)))\subseteq A$ (resp. $Int(C\ell(Int(A)))\subseteq A$)

Theorem 3.12. Let $f:(X,\tau)\to(Y,\sigma)$ and $g:(Y,\sigma)\to(Z,\eta)$ be any two mappings such that their composition $g\circ f:(X,\tau)\to(Z,\eta)$ is ${}_N D_\beta-closed$ mapping. Then the following statements are true :

1. If f is continuous map and surjective, then g is ${}_N D_\beta-closed$ mapping
2. If g is ${}_N D_\beta-irresolute$ map and injective, then f is ${}_N D_\beta-closed$ mapping.
3. If f is $g-continuous$ map , surjective and (X,τ) is $T_{1/2}-space$, then g is ${}_N D_\beta-closed$ mapping.

Proof: 1. Let A be any closed set in (Y,σ). Since f is continuous, $f^{-1}(A)$ is ${}_N D_\beta-closed$ in (X,τ) and therefore $g\circ f(f^{-1}(A))$ is ${}_N D_\beta-closed$ in (Y,σ). Since f is surjective, g is ${}_N D_\beta-closed$ map.

2. Let F be any closed set in (X,τ). Since $g\circ f$ is ${}_N D_\beta-closed$ mapping, $g\circ f(F)$ is ${}_N D_\beta-closed$ in (Z,η). Since g is ${}_N D_\beta-irresolute$ map, $g^{-1}(g\circ f(F))$ is ${}_N D_\beta-closed$ in. (Y,σ). Since f is injective, f is ${}_N D_\beta-closed$ mapping.

3. Let G be any closed set in (Y,σ). Since f is $g-continuous$, $f^{-1}(G)$ is $g-closed$ in (X,τ). Since (X,τ) is $T_{1/2}-space$, $f^{-1}(G)$ is closed in (X,τ) consequently $g\circ f(f^{-1}(G))$ is ${}_N D_\beta-closed$ in (Z,η). i.e. $g(G)$ is ${}_N D_\beta-closed$ in (Z,η), since f is surjective. It implies that is g is ${}_N D_\beta-closed$ mapping..

Theorem 3.13. Let $f:(X,\tau)\to(Y,\sigma)$ be any ${}_N D_\beta-closed$ mapping. Then ${}_N D_\beta-Closure(f(A))\subset f(Cl(A))$.

Proof: Suppose f is ${}_N D_\beta-closed$ map and let A be any subset of X . Since $Cl(A)$ is the closed set in (X,τ), $f(Cl(A))$ is ${}_N D_\beta-closed$ in (Y,σ). We have $f(A)\subset f(Cl(A))$, therefore by Theorem **3.10** of [10] ${}_N D_\beta-Closure\ (f(A))\subset {}_N D_\beta-Closure\ (f(Cl(A)))\to(1)$. Since $f(Cl(A))$ is ${}_N D_\beta-closed$, ${}_N D_\beta-Closure\ (f(Cl(A))=f(Cl(A)$, we have from (1), ${}_N D_\beta-Closure\ (f(A))\subset\ (f(Cl(A)))$.

Remark 3.14. However, the converse of the above Theorem **3.13** need not be true by the following example.

Example 3.15. Let $X=\{a,b,c\}$, $\tau=\{X,\phi,\{a,b\},\{a\}\}$ and $Y=\{1,2,3\}$, $\sigma=\{Y,\phi,\{2\},\{2,3\}\}$. Then (X,τ) and (Y,σ) be topological spaces. $C(X)=\{X,\phi,\{b,c\},\{c\}\}$, $C(Y)=\{Y,\phi,\{1,3\},\{1\}\}$.

$D_\beta C(X)=\{X,\phi,\ \{c\},\{a,c\},\{a\},\ \{b,c\},\{b\}\}$,

$D_\beta C(Y)=\{Y,\phi,\ \{1.3\},\{1,2\},\{1\},\ \{2\},\{3\}\}$. Let $f:(X,\tau)\to(Y,\sigma)$ be a function defined by $f(a)=1$, $f(b)=2$, $f(c)=3$. Then ${}_N D_\beta-Closure(f(A))\subset(f(Cl(A)))$ for every subset A of X . But f is not a ${}_N D_\beta-closed$ mapping, since $f(\{b,c\})=\{2,3\}$, which is not ${}_N D_\beta-closed$ in (Y,σ).

4. ${}_N D_\beta-$ *Open Map*

Definition 4.1. A function $f:(X,\tau)\to(Y,\sigma)$ is said to be ${}_N D_\beta-open$ if $f(V)$ is ${}_N D_\beta-open$ in Y for each open set V in X.

Theorem 4.2. (i) Every $\beta-open$ map is ${}_N D_\beta-open$.

(ii) Every $g-open$ map is ${}_N D_\beta-open$.

(iii) Every $semi^*-open$ is ${}_N D_\beta-open$.

(iv) Every $D_\alpha-open$ map is ${}_N D_\beta-open$.

Proof;- It is obvious.

Theorem 4.3. Let $f:(X,\tau)\to(Y,\sigma)$ be any bijective map, then the following statements are equivalent;

(i) f^{-1} is ${}_N D_\beta-continuous$.

(ii) f is ${}_N D_\beta-open$ map.

(iii) f is ${}_N D_\beta-closed$ map.

Proof. (i) $\Rightarrow$ (ii): Let V be any open set in (X,τ). By assumption $(f^{-1})^{-1}(V)=f(V)$ is ${}_N D_\beta-open$ in (Y,σ). This shows that f is ${}_N D_\beta-open$ map.

(ii) $\Rightarrow$ (iii): Let G be any closed set in (X, τ). Then G^c is open set in (X, τ), therefore by assumption $f(G^c) = (f(G))^c$ is ${}_N D_\beta - open$ in (Y, σ), consequently $f(G)$ is ${}_N D_\beta - closed$ in (Y, σ). Hence the map f is ${}_N D_\beta - closed$.

(iii) $\Rightarrow$ (iv): Let G be any closed set in (X, σ). Then by assumption $f(G)$ is ${}_N D_\beta - closed$ in (Y, σ) and therefore $f(G) = (f^{-1})^{-1}(G)$ is ${}_N D_\beta - closed$ in (Y, σ), therefore f^{-1} is ${}_N D_\beta - continuous$.

Theorem 4.4. If a map $f:(X, \tau) \to (Y, \sigma)$ *is* ${}_N D_\beta - open$, then $f(Int(A)) \subset {}_N D_\beta - Int(f(A))$ for every subset A of (X, τ).

Proof. Let $f:(X, \tau) \to (Y, \sigma)$ be any ${}_N D_\beta - open$ map and suppose A be any open set in (X, τ), then $Int(A)$ is open in (X, τ). Therefore $f(Int(A))$ is ${}_N D_\beta - open$ in (Y, σ) and therefore ${}_N D_\beta - Int(f(Int(A))) = f(Int(A)) \to (1)$. Since $Int(A) \subseteq A$, $f(Int(A)) \subseteq f(A)$, consequently ${}_N D_\beta - Int(f(Int(A))) \subset {}_N D_\beta - Int(f(A))$. By (1), we have $f(Int(A)) \subset {}_N D_\beta - Int(f(A))$.

Remark 4.5. However, the converse of the above Theorem **4.4** need not be true by the following example.

Example 4.6. Let $X = \{a, b, c\}$, $\tau = \{X, \phi, \{a, b\}, \{a\}\}$ and $Y = \{1,2,3\}$, $\sigma = \{Y, \phi, \{2\}, \{2,3\}\}$. Then (X, τ) and (Y, σ) be any two topological spaces.

$D_\beta O(Y) = \{Y, \phi, \{2\}, \{3\}, \{2,3\}, \{1,3\}, \{1,2\}\}$. Define a function $f:(X, \tau) \to (Y, \sigma)$ by $f(a) = 1$, $f(b) = 2$, $f(c) = 3$. Then $f(Int(A)) \subset {}_N D_\beta - Int(f(A))$ for every subset A of (X, τ). But the map f is not ${}_N D_\beta - open$.

Conclusion- The notion of lower separation axioms, closed graphs and strongly closed graphs, which are defined in terms $D_\beta - open$ and $D_\beta - closed$ sets [9] can also be established by using ${}_N D_\beta - open$ and ${}_N D_\beta - closed$ sets.

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