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Enclosing Problem on Two Dimensional Point Set

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Abstract: The problem of enclosing a planar point set by a minimum sized geometric object have been well studied in in computational geometry under the domain of facility location, VLSI, Pattern recognition and classification, to name a few. Unfortunately, there is no paper that contains all the works of enclosing problems, studied from early 80's to till date. Each of the problem discussed, have theoretical beauty as well as the practical importance in many application fields.

Keywords: Enclosing Problem; Facility Location; MER; Pattern Recognition and Classification.

I. INTRODUCTION

The problem of enclosing a planar point set by a minimum sized geometric object such as circle [29], rectangle [35], square, triangle [28], polygon have been well studied in computational geometry [5, 23]. In this survey we are mainly interested in *planar version* of enclosing problem which can be formally defined as follows. Given a set $P = \{p_1, p_2, \ldots, p_n\}$ of arbitrary distributed points in two dimensional plane, objective is to compute one or more geometric object of same type so that (i) union of all geometric objects encloses the n points of P and (ii) the total area(or parameter) of the union of all geometric objects is minimized. In some cases, the enclosing object is *orientation-invariant*, that is, the region bounded by the object remains same under rotation (for example circle). If the enclosing object is *orientation-dependent*, then it sometimes becomes more difficult to compute the optimal orientation over all possible orientations. Clearly, different geometric objects lead to different natures of *enclosing problem*.

Enclosing problem in circular domain, finds a circle of minimum radius (i.e. minimum enclosing circle) that contains all the points of P. In the military, this problem is referred as the *Bomb Problem*. If we assume each target on a map as a point in a plane, the center of the minimal enclosing circle of a map is a good position to drop a bomb for maximum destruction. Moreover, the radius of the minimal enclosing circle can be used as a good measure for estimating the required explosion area. In 1972, Elzinga and Hearn [16] gave an $O(n^2)$ algorithm. Shamos and Hoey [34], Preparata [30] (1977), and Shamos [33] independently proposed the *first* $O(n \log n)$ algorithms. Finally, and to everyone's surprise, in 1983 Nimrod Megiddo [22] showed that the minimal enclosing circle problem can be solve in $O(n)$ time using the prune-and-search techniques for linear programming. This landmark result is one of the most beautiful in the field of computational geometry. In this discussion, we mainly review about enclosing problems and its some variation where the enclosing object is either rectangle or square.

For orientation dependent object, the general problem in rectangular domain finds the minimum enclosing rectangle. This problem was previously studied and solved for both area and perimeter as the optimal criterion. The minimum enclosing rectangle of minimum area (or parameter) is referred as MER. It is straightforward to find the the minimum area enclosing rectangle that has sides parallel to the coordinate axes. Dropping the restriction of axis-parallelism and allowing arbitrarily oriented rectangles makes the problem of computing MER more complicated. One landmark result of Godfried Toussaint [35] computed the smallest area rectangle enclosing a collection of points. For this, first the convex hull of the points was computed. Let the resulting convex polygon be Q. It was known by the result of Freeman and Shapira [20] in 1975 that the minimum area rectangle enclosing Q must have one rectangle side flush with (i.e., collinear with and overlapping) one edge of Q. This geometric fact was used by Godfried Toussaint to develop the "rotating calipers" algorithm. The algorithm rotates a surrounding rectangle from one flush edge to the next, keeping track of the minimum area for each edge. It achieved $O(n)$ time after hull computation. The same approach [35] is applicable for finding the minimum-perimeter enclosing rectangle.

Many researchers used more that one geometric objects of same type to cover a planar set P on n points. In this context, Enclosing problems was divided into two kinds: the *discrete* and the *non-discrete* problems. In discrete version of this problem, the centers of the geometric o objects are points of P, whereas in the non-discrete problems the centers are not constrained. Hence, the discrete problems are somewhat more difficult than the non-discrete ones. Considering enclosing objects as squares, the problem is called *square-center* problem [21, 25]. Jaromczyk and Kowaluk [21] studied non-discrete square-center problem and proposed an $O(n^2)$ time algorithm that finds two parallel squares in arbitrary orientation that optimizes the sizes of the squares with respect to their side lengths. Katz et al. [25] studied discrete square-center problem with the area of the larger square is minimum for three cases. First they considered the squares as axis-parallel and compute them in $O(n\log^2 n)$ and $O(n)$ space. In case, squares were allowed to rotate but remain parallel, their algorithm computed these two squares in $O(n^2\log^4 n)$ time and uses $O(n^2)$ space. Finally, each square was allowed to rotate independently and proposed an algorithm that runs in $O(n^3\log^2 n)$ time and $O(n^2)$ space.

Bespamyatnikh and Segal [12] solved the problem of enclosing a set S of n points in d-dimensional space, $d \geq 2$, by two axis-parallel boxes such that the measure of the largest box is minimized where measure is a monotone function of the box. They proposed a simple algorithm to find boxes that runs in $O(n\log n + n^{d-1})$ time and $O(n)$ space. For the problem of enclosing a given point set P by pair of parallel rectangles in arbitrary orientation, Jaromczyk and Kowaluk [21] constructed an $O(n^2)$ time algorithm that decides whether two parallel rectangles with given side lengths can enclose P. Saha and Das [32] studied the problem of locating two parallel rectangles in arbitrary orientation to cover a planar point set P such that area of the larger rectangle is minimum. Their proposed algorithm requires $O(n^3)$ time and $O(n^2)$ space. For this problem, these two rectangles can be either disjoint or overlapping. In case two rectangles are only *disjoint*, Ahn and Bae [2] considered two variants of this problem: (i) the rectangles are free to rotate but must remain parallel to each other, and (ii) one rectangle is axis-parallel but the other rectangle is allowed to have an arbitrary orientation. For both of these variations, they proposed $O(n^2\log n)$ time and $O(n)$ space algorithms.

Another most important problems in this context, are *Rectilinear p-center Problem* and *Rectilinear p-Piercing Problem*. Given a set of n points in the plane, Rectilinear p-center Problem finds a set of p isothetic squares so that the union of the squares enclosed all the n points and the total area of the union of the squares is minimized. Rectilinear p-Piercing Problem is defined as follows. Given a set of n isothetic squares, determine if there exist a set of p points that together would intersect all the n squares. The above two problems are complementary to each other and both are NP-complete [26]. Moreover, their relative approximation problems are known to be NP-complete. When the squares are replaced by circles in the above two problems we get the *Euclidean p-center* and *p-piercing* problems, respectively. The Euclidean p-center and p-piercing problems are also known to be NP-complete [26]. So their respective relative approximation problems. There exists polynomial time solutions for both Rectilinear p-center Problem and Rectilinear p-Piercing Problem and when p is a constant and not given as a part of input. The same is true for Euclidean p-center and p-piercing problems. Many researchers have concentrated their efforts on finding efficient solutions with small values of p [13, 4, 15]. The general class of p-center and p-center problems are normally useful for the facility location type problems. A common example of facility location problem can be stated as follows: given n retail shops in a town, find the location of p warehouses so that the maximum distance (alternatively, sum of the distances) of the shops from the corresponding warehouse is minimized.

The problems above continue a list of Enclosing problems that deal with covering a point set P in the plane by two squares or rectangles. Recently, Bae et al. [6] considered the enclosing problems for computing two *non-convex* enclosing objects with the minimum area; the *L-shape* and the *rectilinear convex hull*. They computed a L-shape that encloses a given set of planar points P or a rectilinear convex hull of P with minimum area over all orientations. Their proposed algorithms computed both minimum enclosing shapes of arbitrary orientation in $O(n^2)$ time and $O(n)$ space. Díaz-Báñez et al. [11] proposed another $O(n\log n)$ time and $O(n)$ space algorithm to compute minimum area rectilinear convex hull of P over all orientations, improving the result of Bae et al. [6]. A nice survey of Enclosing problems that use different shapes can be found in [8, 9]. Another survey of obnoxious facility location and different covering location models can be found in [27, 7, 31]. Enclosing problem finds applications in facility location [10, 17, 24, 18], Pattern Recognition and Classification, etc [1, 3, 19].

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