ISSN No. 0976-5697
Volume 2, No. 3, May-June 2011
International Journal of Advanced Research in Computer Science
RESEARCH PAPER
Available Online at www.ijarcs.info

Placement of K Disjoint Isothetic Unit Squares to Maximize Points Containment

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Abstract: Given a set P of n points in the two dimensional plane, we propose $O(k^2n^5)$ time and $O(kn^4)$ space algorithm to locate k isothetic unit squares which are pairwise disjoint and they together contain maximum number of points from P. Moreover, an $O(n^2 \log n)$ time and $O(n)$ space algorithm is demonstrated for $k = 2$.

Keywords: Isothetic; Space Partition Tree; Sliceable; Np-hard.

I. INTRODUCTION

Enclosing problems of many variations involving a planar point $P = \{p_1, p_2, p_3, \ldots, p_n\}$ set have been extensively studied in computational geometry [4, 14]. Problems of computing geometric object such as circle [17], triangle [19], rectangle and square [18] having smallest area or parameter are well known. Problem of finding the smallest enclosing convex polygon is the famous convex hull problem. The k-enclosing problem is an important variant of enclosure problem. The k-enclosing problem computes a smallest region of given type that contains at least k points of P. For example, a general problem in rectangular domain, computes smallest rectangle of arbitrary orientation that contains at least k points of a planar point set P. In other words, k-enclosing problem identifies a subset $P' \in P$ of size at least k that minimizes some closeness measure. Interest reader may read for different complexity results on k-enclosing problems [7]. A closely related problem is to locate one or more copies of a given region to maximize the size of the subset covered. In other words, instead of fixing k and computing an optimal (or smallest) enclosing region, the problem is to maximize the number of points covered by the given region(s) of fixed size and shape. These so called problems of maximal covering by convex objects such as circle, rectangle, square, polygon have also received attention of many researchers [5, 9, 12]. Maximal covering problem finds applications in facility location [8], Pattern Recognition and Classification, etc [1, 2, 11].

In the context of bichromatic planar point set, Díaz-Báñez et al. [10] consider the following problem: Given a set of red points and a set of blue points on the plane, find two isothetic unit squares S_R and S_B with disjoint interiors such that the number of red points covered by S_R plus the number of blue points covered by S_B is maximized. First they proposed $O(n^2)$ time algorithms to locate two disjoint isothetic unit squares S_R and S_B. Later, they improved the complexity to $O(n \log n)$ time [6]. It is interesting to generalize this problem by k, $k \geq 2$, isothetic unit squares. In this work this generalization is considered. We first solve the problem for $k = 3$. Other motivations for considering this problem are

as follows:

- In facility location type of problems of rectilinear kind, the areas need to be square.
- For un-constraint facility location type of problem with limited resources and, that is, with just k facilities available to be sited and with different populations at each of the demand nodes, the maximal covering problem seeks to locate the facilities in such a way that the largest possible population is covered.
- The requirement for disjoint interiors is relevant, for example, in facility location problems where facilities may interfere negatively with each other, or when their areas of influence are not allowed to overlap.

II.BASICS

An isothetic unit square is a square of unit size whose sides are parallel to one of the coordinate axes. We say that an isothetic unit square S encloses a point set Q if all the elements of Q lie on the boundaries of S or in the interior of S. For disjoint case, the above stated generalized problem can be stated as follows.

Problem 1 Given a set P of n points in the two dimensional plane, locate k isothetic unit squares which are pairwise disjoint and the together contain maximum number of points from P.

Here we use the following two results for constructing an algorithm to solve Problem 1.

Result 1 [6] Given a set of red points and a set of blue points on the plane, let two disjoint isothetic unit squares S_R and S_B be such that the number of red points covered by S_R plus the number of blue points covered by S_B is maximized. S_R and S_B can be located in $O(n\log n)$ time and $O(n)$ space.

Result 2 [16] Given a planar set P of n points, the isothetic unit square S containing maximum number of points from P can be computed in $O(n\log n)$ time using $O(n)$ space.

Note that the Result 2 can be used to locate two disjoint isothetic unit squares that together cover maximum number of points from P; where each point of set $P=\{p_1,p_2,p_3,\ldots,p_n\}$ has same color. The underlying algorithm requires $O(n\log n)$ time and linear space. Moreover, an isothetic unit square that contains maximum number of points P can be located in $O(n\log n)$ time and linear space using Result 2. These two results are follows.

In general case, Problem 1 is known to be NP-hard [15]. A set of rectangles on the plane is called sliceable if they can be recursively partitioned by a Space Partition Tree [20]. A Space Partition Tree is a tree structure, whose interior nodes denote either vertical or horizontal space partitions, and leaves denote the set of rectangles. Here we define a set of squares as k-sliceable in the following way.

Definition 1 Given a set of points P in two dimensional plane, a set of k isothetic unit squares is called k-sliceable if they can be recursively partitioned by $k-1$ horizontal or vertical line.

Given set of points in the plane and there exists a set of sliceable rectangles, the Mukherjee et al [13] considered the following problem.

Problem 2 Given n points $q_1,q_2,\ldots,q_n$ in the two dimensional plane, and some number p, where $p<\frac{n}{2}$, find p isothetic, non-intersecting rectangles $R_1,R_2,\ldots,R_p$ so that $R_i\cap R_j=\Phi$ for $i\neq j$, $q_i\in\bigcup_{j=1}^{p}R_j$ for $i=1,2,\ldots,n$ and $\sum_{j=1}^{p}Area(R_i)$ is minimized.

They used dynamic programming paradigm to construct $O(k^2n^5)$ time and $O(kn^4)$ algorithm to solve this problem.

Here we assume that there exists a set of k-sliceable isothetic squares and in next section demonstrates a method that solves Problem 1 for $k=3$.

A. Placement of a triplet of pairwise isothetic unit squares to maximize point containment

Let $P=\{p_1,p_2,p_3,\ldots,p_n\}$ be a set of points in two dimensional plane. Without loss of generality, assume that no two points have the same x- or y-coordinate. Consider two arrays Λ_x and Λ_y containing the points of P in ascending order with respect to their x and y-coordinate respectively. Let us denote the x-coordinate of the i-th entry of Λ_x by x_i and similarly the y-coordinate of the i-th entry of Λ_y by y_i, $1\le i\le n$.

Let $p_{x_{max}}$ and $p_{y_{min}}$ be the points with maximum x-coordinates and minimum y-coordinates among the points in P respectively. Similarly, $p_{y_{max}}$ and $p_{x_{min}}$ be the points with maximum y-coordinates and minimum x-coordinates among the points in P respectively. Observe that the minimum enclosing rectangle (MER) of the point set P is defined the above set of four points. We first propose an algorithm that computes three squares which are pairwise disjoint and they together contains maximum number of points from P. Observe that among these three squares, one square will be separated from other two squares by a horizontal or a vertical line. Without loss of generality, assume that the line separating one square from other two is horizontal. The other case where the line separation is vertical, can be handled in similar manner. Now we are describing the first pass of our proposed algorithm.

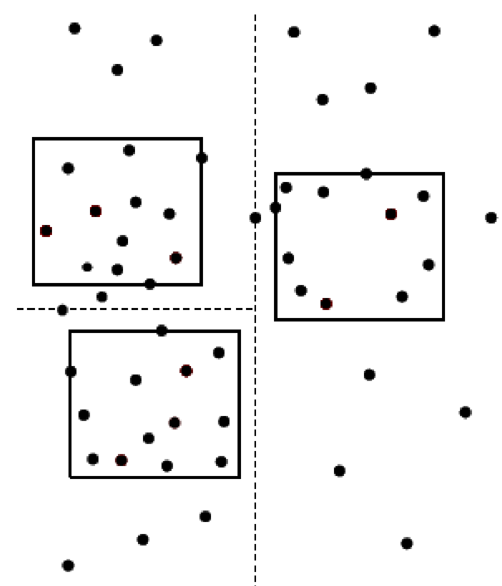

Figure.1 Sample point set and the corresponding triplet of isothetic unit squares which are pairwise disjoint and they together contain maximum number of points

Let the vertical line passing through the i-th point in array Λ_x divides the point set P into two sub-set Q_i and Q_i' respectively; the subset Q_i and Q_i' lie on the left and right side of this vertical line. For the position of the vertical line that passes through the i-point of Λ_x, use the Result 1 to locate a square that contains maximum number of points from Q_i and the Result 2 to compute a pair of disjoint squares containing maximum number of points from Q_i'. This triplate of squares is a potential candidate for position of the vertical line that passes through the i-th entry of Λ_x (See Figure 2.1). Observe that the time required to locate these triplet of squares is $O(n \log n)$. Now this process is repeated for each position of the vertical line that passes though a point $p \in \Delta_x$.

In second pass, the above process is invoked for each position of the horizontal line passing through the point in array Λ_y. In each pass, we keep the triplate of pairwise disjoint squares that together contains maximum number of point from P. Finally, the triplate of pairwise disjoint squares that together contains maximum number of point from P is reported. We thus have the following results.

Theorem 1 Given a set P of n points in two dimensional plane, three isothetic unit squares which are pairwise disjoint and they together contain maximum number of points from P can be located in $O(n^2 \log n)$ time and $O(n)$ space.

B. k Disjoint isothetic unit squares

If we naively extend this algorithm to solve Problem 1 then it is interesting to note that the solution would not have a polynomial time complexity in both n and k. Now to solve Problem 1, we propose an $O(n^5 \log n)$ time and $O(kn^4)$ algorithm algorithm that uses similar dynamic programming approach as proposed by Mukherjee et al [13] and the Result 1 as a subroutine.

Observe that placing horizontal and vertical partitioning lines among the points of P can generate $O(n^4)$ subsets of P. Let Q ($\in P$) be the subset of points enclosed by the minimum enclosing rectangle defined by the points (p_{x_i}, p_{y_k}) and (p_{x_j}, p_{y_l}), $i < j$ and $k < l$ as bottom-left and top-right corners respectively. Given a subset Q ($\in P$), let $Count(p_{x_i}, p_{y_k}, p_{x_j}, p_{y_l}, m)$ denote the maximum number of points from P jointly covered by m disjoint isothetic unit squares placed over the subset Q.

In the first step, the algorithm computes $Count(p_{x_i}, p_{y_k}, p_{x_j}, p_{y_l}, 1)$ for all possible subsets of P. Subsequently, it computes $Count(p_{x_i}, p_{y_k}, p_{x_j}, p_{y_l}, u)$ for all possible subsets of P using the results of the previous steps in similar dynamic programming approach as proposed by Mukherjee et al [13]. Finally, it reports $Count(p_{x_{min}}, p_{y_{min}}, p_{x_{max}}, p_{y_{max}}, k)$ and the k optimum squares. Observe that to compute $Count(p_{x_i}, p_{y_k}, p_{x_j}, p_{y_l}, 1)$ for all possible subsets, the Result 1 is used for each possible subset of P.

In view of the Result 1, computation of the first step requires $O(n^5 \log n)$. Complexity of subsequent steps, and hence, the over all time complexity of the algorithm is $O(k^2 n^5)$. Corresponding space complexity can also be shown to be $O(kn^4)$. Further details can be found in [13]. The result can be stated as,

Theorem 2 Given a set of n points in two dimensional plane and there exists a k-sliceable set of isotheic unit squares, k isothetic unit squares which are pairwise disjoint and they together cover maximum number of points from P can be located using $O(k^2 n^5)$ time and $O(kn^4)$ space.

III. CONCLUSIONS

Given a set P of n points in two dimensional plane and is there exists a set of k-sliceable isothetic unit squares, we have considered the problem of locating k isothetic unit squares which are pairwise disjoint and together contain maximum number of points from P. Moreover, a placement of triplet of pairwise disjoint squares are also computed so that they together contains maximum number of points from P. Generalization of the Problem 1 can allow these optimal k isothetic unit squares be (i) disjoint or overlapping with empty common zone and (ii) disjoint or overlapping with points in the common zone. For both two generalizations of Problem 1, the result of the work of Ahn et al. [3] can be used to show that each of the problem is NP-Hard. In particular, their NP-hardness gives a transformation that, even if you allow overlapping of squares, the squares of the optimal solution do not overlap. Hence, the generalization of both problems are NP-hard.

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