GLC and GLC** Continuous Functions: A Conceptual Flaw

P.L. Powar[1] Pratibha Dubey[2]

1 Department of Mathematics and Computer Science, R.D. University, Jabalpur

E-mail:pvjrdvv@rediffmail.com

2 Department of Mathematics, St. Aloysius College, Jabalpur

E-mail:dubeypratibha111@gmail.com

Abstract

The concept of generalized locally closed sets (glc-sets), GLC**-sets followed by the notion of GLC and GLC**-continuous maps was initiated by Balachandran et al. (Generalized locally closed sets and GLC-continuous functions, Indian J. pure appl. Math 27(3): 235-244, 1996).

In the present work, it has been established that the collection of glc-sets and the collection of GLC** -sets, each is exactly equal to the power set P (X) of X. Consequently, any arbitrary function with any choice of domain and range turns out to be GLC and GLC**-continuous function which is not desirable from analytic point of view.

Keywords: Topological spaces, locally closed sets, glc-set, GLC **-set, GLC-continuity, GLC**-continuity.

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1. Introduction.

The idea of locally closed set was introduced by Bourbaki [2] in 1966. (see also [3]). This concept of locally closed set had been used by Ganster and Reilly [4] for defining the generalized version of continuity viz. LC-irresolute, LC-continuity and sub-LC-continuity. Balachandran et al. [1] had extended the definition of locally closed sets and initiated the notion of "Generalized locally closed set", in particular, glc-set, GLC*-set and GLC**-set.

Since last few decades many topologist (cf. [4], [5], [6], [7], [8], [9], [10], [11]) are trying to explore the possibility of generalizing the classical phenomenon "continuity" of the function defined in the topological space. Following this trend Balachandran et al. [1] have also defined and explored the idea of GLC-irresolute maps and GLC-continuous maps. Extending the idea of Balachandran et al. [1], Park et al. [8] have defined semi generalized locally closed sets and locally-generalized closed sets along with SGLC-continuous functions and LδGLC-continuous functions respectively. (see also [9], [10], [11]).

Recently, Patil et al. [10] have further extended the concept of glc-sets and introduced the notion of g*wα-lc sets and g*wα^{*}-lc sets and g*wα^{**}-lc sets and have applied these concepts to define relevant different types of continuous functions.

In the present paper, authors have established that the respective collection of glc-sets, and the collection of GLC**-sets (cf. [1]) generated by the topology yield precisely the power set P(X) of X. This information leads to the conclusion that the corresponding GLC and GLC** - idea of continuity is not enhancing the class of continuous functions with some relaxed conditions however all functions with arbitrary domain and range turns out to be GLC and GLC**-continuous functions which is inadequate. In view of this observation, all the extensions turned out to be superfluous.

2. Pre-requisites.

The following notations have been referred throughout this work:

(X, τ) - Topological space with topology defined on the set X ($\neq \phi$).

cl(A) - Closure of A for the subset A of X with respect to (X, τ).

int(A)- Interior of A for the subset A of X with respect to (X, τ).

P(X) - Power set of X.

Definition 2.1. A subset B of (X, τ) is called g-closed [12] if $cl(B) \subseteq G$ whenever $B \subseteq G$ for an open set G in a topological space (X, τ). A subset C of (X, τ) is called **g-open** if its complement $X - C$ is **g-closed**.

Example 2.1. Consider a topological space X = {a, b, c, d} with the topology τ = {X, ϕ, {a, b, c}}, F_X = {X, ϕ, {d}}, where F_X is the collection of closed sets in (X, τ). Let A = {a, d} be a subset of X. There is only one open set say U = X containing A. Then it is easy to check that cl{a, d} = X which follows by the definition that cl{a, d} = X = U = X. Hence A = {a, d} is **g-closed**.

Remark 2.1. It is a direct consequence from the definition of g-closed sets that every open set is g-open and every closed set is g-closed but the respective converse is not true in general.

Definition 2.2. Let S be a subset of a topological space (X,τ). S is said to be generalized locally closed (glc-set) [1] if there exists g-open set G and g-closed set F such that S = G∩F . The collection of all generalized locally closed set is denoted by GLC (cf. [1]).

Example 2.2. Consider a topological space X = {a, b, c, d} with the topology τ = {X, τ, {b, c, d}, {a, c, d}, {c, d}}, F_X = {τ, X, {a}, {b}, {a, b}}. In view of Definition 2.1, the collection of g-closed sets = {X, τ, {a}, {b}, {a, b}, {a, b, c}, {a, b, d}} and the collection of g-open sets = {τ, X, {b, c, d}, {a, c, d}, {b, c, d}, {a, c, d}, {c, d}, {d}, {c}}. We now show that A = {b, c} ⊆ X is a glc-set.
Claim: A = {U ∩ V : U is g-open and V is g-closed}.
We now consider U = {b ,c, d} a g-open set and V = {a, b, c} a g-closed set. Then, U ∩ V = {b, c, d} ∩ {a, b, c} = {b, c} is a glc-set. It may be verified easily that the collection of all glc-sets is exactly equal to P (X).

Remark 2.2. It is clear that every g-closed set is glc-set and every g-open set is glc-set.

Definition 2.3. Consider a subset S of a topological space. Then S∈ GLC** if S∈ G ∩ F for any open set G and a g-closed set F of (X, τ) respectively (cf.[1]).

Definition 2.4. Let (X, τ) and (Y, σ) be two topological spaces. A function f :→ (X, τ) → (Y, σ) is said to be GLC-continuous (resp. GLC**-continuous) if f^{-1}(V) ∈ GLC (resp. f^{-1}(V) ∈ GLC**) for each V ∈ σ (cf.[1]).

Definition 2.5. A function : (X, τ) → (Y, σ) is said to be GLC-irresolute (resp. GLC** -irresolute) if $f^{-1}(V) \in$ GLC (resp.$f^{-1}(V)$ ∈ GLC) for each V ∈ GLC (resp. V ∈ GLC**) in (Y, σ) (cf.[1]).

3. Main Result.

We are now set to state the main result of this paper.

Theorem 3.1. Let (X, τ) be the topological space and GLC and GLC** be the collection of sets described in the Definition 2.2 and 2.3 respectively. Then

GLC ≅ GLC ≅ P (X)**

where P (X) is the power set of X.

Proof. Let X be any non empty set = {τ, X, $\{U_\alpha\}_{\alpha \in J}$}be the topology on X. Let A be any non empty proper subset of X. The following cases have been considered:

Case 1. A $\nsubseteq U_\alpha$ $(\neq$ X) for all $\alpha \in$ J and $U_\alpha \in \tau$ implies A $\subseteq$X only. It is clear that cl(A) $\subseteq$ X. Hence, A is g-closed. Referring Remark 2.2, we conclude that A is glc-set.

Case 2. A$\subseteq U_\alpha$ for some $\alpha \in$ J and CA $\subseteq$X but CA $\nsubseteq U_\alpha$ for each $\alpha \in$ J where C stands for the complement of A in X. It is obvious that cl(CA) $\subseteq$ X. Hence CA is g-closed which implies that A is g-open. In view of Remark 2.2, the set A is glc again.

Case 3. A $\subseteq U_\alpha$ for some $\alpha \in$ J and CA $\subseteq U_\delta$ for some $\delta \in$J where $U_\alpha, U_\delta \in \tau$. Let if possible that A is neither g-open nor g-closed.

Claim:

$$A = G_C \cap G_O \tag{3.1}$$

where G_C and G_O are g-closed and g-open sets in (X,τ) respectively. Since, A is not g-closed, there exists at least one index $\beta \in$ J such that

A $\subseteq U_\beta$ but cl(A) $\nsubseteq U_\beta$ (cf. Definition 2.1) Then

Either

(a) $$D(A) \subseteq CU_\beta \tag{3.2}$$

Or

(b) $$S\,(\neq \phi) \subseteq D(A) \text{ and } S \subseteq CU_\beta \text{ such that } A \cup D(A)_{\sim S} \subseteq U_\beta \tag{3.3}$$

Consider the set $A \cup CU_\beta$

Claim: $A \cup CU_\beta$ is g-closed.

There exists a family $\{U_\alpha^*\}_{\alpha \in J}$ of open sets such that

$$A \cup CU_\beta \subseteq U_\alpha^* \quad \text{for } \alpha \in \mathrm{J}$$

Since $A \subseteq A \cup CU_\beta$, the collection $\{U_\alpha^*\}_{\alpha \in J}$ is a sub-collection of $\{U_\alpha\}_{\alpha \in J}$ of open sets containing A.

i) Consider $\beta \in$ J such that $A \cup CU_\beta \subseteq U_\beta^*$ and $U_\beta \subseteq U_\beta^*$.

- If $U_\beta^* = X$, then $cl(A \cup CU_\beta) \subseteq X$ and $A \cup CU_\beta$ is g-closed.
- If U_β^* $(\neq \phi, X)$, then

$$
\begin{aligned}
cl(A \cup CU_\beta) &= cl(A) \cup cl(CU_\beta) \\
&= A \cup D(A)_{\sim S} \cup S \cup CU_\beta \quad (\because CU_\beta \ \ is\ closed) \\
&= A \cup D(A)_{\sim S} \cup CU_\beta \\
&\subseteq U_\beta^* \quad (\because CU_\beta \subseteq U_\beta^* ; U_\beta \subseteq U_\beta^* \text{ and using (3:3)})
\end{aligned}
$$

Therefore $A \cup CU_\beta$ is g-closed.

ii) We next consider $\alpha(\neq \beta) \in$ J such that $A \cup CU_\beta \subseteq U_\alpha^*$ for $U_\alpha \subseteq U_\alpha^*$ and $cl(A) \subseteq U_\alpha$

$$
\begin{aligned}
cl\left(A \cup CU_\beta\right) &= cl(A) \cup cl(CU_\beta) \\
&= cl(A) \cup CU_\beta \\
&\subseteq U_\alpha^*
\end{aligned}
$$

Thus, $A \cup CU_\beta$ is g-closed.

Claim: $A = \left(A \cup CU_\beta\right) \cap U_\beta$ where $G_C = A \cup CU_\beta$ is g-closed and $G_O = U_\beta$ is g-open (cf. Remark 2.1). Consider

$$
\begin{aligned}
G_C \cap G_O &= \left(A \cup CU_\beta\right) \cap U_\beta \\
&= \left(A \cap U_\beta\right) \cup (CU_\beta \cap U_\beta) \\
&= A
\end{aligned}
$$

Hence, (3.1) holds and finally we conclude that A is glc-set. Since, A was arbitrary subset of X, every subset of X is glc-set. Thus, the collection **GLC is precisely equal to P (X)**.

Since $G_O = U_\beta$ (open), it is direct by the Definition 2.3 that

$$\textbf{GLC**} \cong \textbf{P(X)}$$

This completes the proof. □

4. Conclusion.

- In view of Definitions 2.4 and 2.5, each function f defined from (X,τ) to (Y, σ) turns out to be GLC-continuous (irresolute) and GLC** -continuous (irresolute) which is not acceptable as a generalization of the classical concept of continuity in Topology.
- All generalizations of GLC-set and GLC** -set turned out to be stagnant and finally not desirable.

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