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SOME FUNCTIONS DEFINED BY D_β-CLOSED SET

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Abstract: In [16] present authors introduced and studied a new class of generalized closed sets called D_β-closed sets, D_β-open sets, D_β-continuous and D_β-irresolute functions in topological spaces. In this paper we introduce and investigate new class of open and closed functions called D_β-open and D_β-closed functions by using D_β-open and D_β-closed sets and explore their fundamental properties. The concept of D_β-closed graph, strongly D_β-closed graph and some weak separation axioms are also extend by using the notion of D_β-open sets.

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1. INTRODUCTION

Monsef et al .[1] devised and investigated a new notion of open set which is one of the member of class of basic open sets called β-open set. This kind of set discussed by Andrijevic [3] under the name, semi-preopen sets. The concept of generalized closed (briefly g-closed) sets in a topological space and a class of topological spaces called $T_{1/2}$-space was introduced by Levine [18] and these sets were further considered by Dunham and Levine [11]. Dunham (See also [10]) continued the study of $T_{1/2}$-spaces. In 1982 Dunham [12] derived a new closure operator C^* by using g-closed sets in such a way that for any topological space (X,τ), $C^*(E) = \cap\{A : E \subseteq A \in D\}$, where $D = \{A : A \subseteq X,\ \ A \text{ is g-closed}\}$ and he also proved that C^* is a Kurtowski closure . operator in X. Dunham [11] also proved that (X,τ) is always a T_0-space and by improving the result, he established that (X,τ^*) is $T_{1/2}$-space, for any topological space (X,τ) . Munshi et al. [25] introduced the notion of g-continuous functions. Balchandran et al. [4] also studied the notion of g-continuity. Malghan [21] introduced the generalized closed maps and discussed its fundamental properties. β-open (β-closed) functions defined and discussed by Monsef et al. [1]. Mahmoud et al. [20] initiated and studied the lower separation axioms via β-open sets. Caldas (see [8], [5], [6]) discussed some properties of functions with strongly α-closed graphs by utilizing α-open sets and the α-closure operator. Long [19] investigated the required condition for a graph G(*f*) to be a closed subset of the product space $X \times Y$. Herrington et al. [14] introduced the concept of strongly closed graph. Noiri [27] gave some characterization of strongly closed graph. Sayed et al. [29] introduced D_α-closed sets and in topological spaces by using the generalized closure operator C^* due to Dunham [12] and also established D_α-continuous, D_α-open and D_α-closed, D_α-closed graph and strongly D_α-closed graph functions in topological spaces. In this paper, in Section 1, we give some basic definitions and results which will use in the sequel. In Section 2, we define D_β-open and D_β-closed functions and characterize its fundamental properties. In Section 3, we devise and elaborate some lower separation axioms using D_β-open sets. In Section 4, we discuss the concepts of D_β-closed graphs and strongly D_β-closed graphs.

1.1. Preliminaries. Throughout this paper (X,τ) will always denote a topological space on which no separation axioms are assumed, unless explicitly stated. If *A* is a subset of the space (X,τ), $C\ell(A)$ and $Int(A)$ denote the closure and the interior of *A* respectively. Here we recall the following known definitions and properties.

Definition 1.1. Let (X,τ) be a topological space. A subset *A* of the space *X* is said to be,

(i) preopen [22] if $A \subseteq Int(C\ell(A))$ and preclosed if $C\ell(Int(A)) \subseteq A$.

(ii) semiopen [17] if $A \subseteq C\ell(Int(A))$ and semi-closed if $Int(C\ell(A)) \subseteq A$..

(iii) α-open [20] if $A \subseteq Int(C\ell(Int(A)))$ and α-closed if $C\ell(Int(C\ell(A))) \subseteq A$.

(iv) β-open [1] if $A \subseteq C\ell(Int(C\ell(A)))$ and β-closed if $Int(C\ell(Int(A))) \subseteq A$.

(v) generalized closed (briefly g-closed) [16] if $C\ell(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X and generalized open (briefly g-open) if $X \setminus A$ is g-closed.

(vi) pre*-closed set [30] if $C\ell^*(Int(A)) \subseteq A$ and pre*-open set if $A \subseteq Int^*(C\ell(A))$.

(vii) semi*-closed set [28] if $Int^*(C\ell(A)) \subseteq A$ and semi*-open set [23] if $A \subseteq C\ell^*(Int(A))$.

(viii) D_α-closed [29] if $C\ell^*(Int(C\ell^*(A))) \subseteq A$ and D_α-open if $X \setminus A$ is D_α-closed.

(ix) D_β-closed [16] $Int^*(C\ell^*(Int^*(A))) \subseteq A$ and D_β-open if $X \setminus A$ is D_β-closed.

Definition 1.2. A function $f:(X,\tau) \to (Y,\sigma)$ is said to be,

(i) β-continuous [1] if the inverse image of each open set in Y is β-open in X.

(ii) g-continuous [4] if the inverse image of each open set in Y is g-open in X.

(iii) pre*-continuous [30] if the inverse image of each open set in Y is pre*-open in X.

(iv) semi*-continuous [23] if the inverse image of each open set in Y is semi*-open in X.

(v) D_α-continuous [29] if the inverse image of each open set in Y is D_α-open in X.

(vi) D_β-continuous [16] if the inverse image of each open set in Y is D_β-open in X.

(vii) β-open [1] (resp. β-closed) if the image of each open (resp. closed) set in X is β-open (resp. β-closed) in Y.

(viii) g-open [21] (resp. g-closed) if the image of each open (resp. closed) set in X is g-open (resp. g-closed) in Y.

(ix) semi*-open (resp. semi*-closed) [24] if the image of each open (resp. closed) set in X is semi*-open (resp. semi*-closed) in Y.

(x) D_α-open [29] (resp. D_α-closed) if the image of each open (resp. closed) set in X is D_α-open (resp. D_α-closed) in Y.

Definition 1.3. Let $f:(X,\tau) \to (Y,\sigma)$ be a function:

(i) the subset $\{(x, f(x)) \mid x \in X\}$ of the product space $(X \times Y)$ is called the graph of f [19] and is usually denoted by G(f).

(ii) has a closed graph [19] if its graph G(f) is closed sets in the product space $X \times Y$.

(iii) has a strongly closed graph [14] if for each point (x, y) not belongs to G(f), there exists open sets $U \subset X$ and $V \subset Y$ containing x and y, respectively such that $(U \times C\ell(V)) \cap G(f) = \phi$.

(iv) has D_α-closed graph [29] if for each $(x, y) \in (X \times Y) \setminus G(f)$, there exists $U \in D_\alpha O(X, x)$ and $V \in GO(Y, y)$ such that $(U \times C\ell^*(V)) \cap G(f) = \phi$.

(v) has a strongly D_α-closed graph [29] if for each $(x, y) \in (X \times Y) \setminus G(f)$, there exists a α-open set U in X and $V \in O(Y)$ such that $(U \times C\ell(V)) \cap G(f) = \phi$.

Definition 1.4. A topological space (X,τ) is said to be,

(i) $T_{1/2}$ [18] if every g-closed set is closed.

(ii) $g\text{-}T_0$ [7] if for any distinct pair of points x and y in X, there exists a g-open set U in X containing x but not y or containing y but not x.

(iii) $g\text{-}T_1$ [7] if for any distinct pair of points x and y in X, there exists g-open set U in X containing x but not y and an g-open set V in X containing y but not x.

(iv) $g\text{-}T_2$ [7] if for any distinct pair of points x and y in X, there exists g-open sets U and V in X containing x and y, respectively such that $U \cap V = \phi$.

(v) $\beta\text{-}T_0$ [20] if for any distinct pair of points x and y in X, there exists β-open set U in X containing x but not y or containing y but not x.

(vi) $\beta\text{-}T_1$ [20] (resp. $D_\alpha\text{-}T_1$ [24]) if for any distinct pair of points x and y in X, there exists D_α-open (resp. D_α-open) set U in X containing x but not y and an β–open (resp. D_α-open) set V in X containing y but not x.

(vii) $\beta\text{-}T_2$ [20] (resp. $D_\alpha\text{-}T_2$ [29]) if for any distinct pair of points x and y in X, there exists β–open (resp. D_α-open) sets U and V in X containing x and y, respectively such that $U \cap V = \phi$.

The intersection of all g-closed sets containing A [11] is called the g-closure of A and denoted by $C\ell^*(A)$ and the g-interior of A [12] is the union of all g-open sets contained in A and is denoted by $Int^*(A)$. The family of all D_β-closed (resp. closed , D_α-closed, g-closed, β-closed, semi*-closed) sets of X denoted by $D_\beta C(X)$ (resp., $C(X)$, $D_\alpha C(X)$, $GC(X)$, $\beta C(X)$, semi* C(X)). The family of all D_β-open (resp. open set of, D_α-open, g-open, β-open, semi*-open) sets of X denoted by $D_\beta O(X)$ (resp. $O(X)$, $D_\alpha O(X)$, $GO(X)$, $\beta O(X)$, semi* O(X)).

Lemma 1.5. [19] A function $f:(X,\tau)\to(Y,\sigma)$ has a closed graph if for each $(x,y)\in(X\times Y)\setminus G(f)$, there exists $U\in O(X,x)$ and $V\in O(Y,y)$ such that $f(U)\cap V=\phi$.

Lemma 1.6. [27] The graph G(*f*) is strongly closed if and only if for each point $(x,y)\in G(f)$, there exists open sets $U\subset X$ and $V\subset Y$ containing x and y, respectively such that $f(U)\cap C\ell(V)=\phi$.

2. $D_\beta-$ OPEN AND $D_\beta-$CLOSED FUNCTIONS

Definition 2.1. A function $f:(X,\tau)\to(Y,\sigma)$ is said to be $D_\beta-$open (resp. $D_\beta-$closed) if the image of each open (resp. closed) set in *X* is $D_\beta-$open (resp. $D_\beta-$closed) in *Y.*

Theorem 2.2. **(i)** Every β-open function is $D_\beta-$open.

(ii) Every g-open function is $D_\beta-$open.

(iii) Every semi*-open function is $D_\beta-$open.

(iv) Every $D_\alpha-$open function is $D_\beta-$open.

Proof. **(i)** The proof follows from the definition and from the Theorem **2.3** of [16] that every g-open set is $D_\beta-$open.

(ii) The proof follows from the definition and from the Theorem **2.3** of [16] that every semi*-open set is $D_\beta-$open.

(iii) The proof follows from the definition and from the Theorem **2.3** of [16] that every $\beta-$open set is $D_\beta-$open.

(iv) The proof follows from the definition and from the Theorem **2.3** of [16] that every $D_\alpha-$open set is $D_\beta-$open.

Remark. (i) $D_\beta-$open function need not be $\beta-$open. (see the Example 2.3 below)

(ii) $D_\beta-$open function need not be g-open. (see the Example 2.4 below)

(iii) $D_\beta-$open function need not be semi*-open (see the Example 2.4 below)

(iv) $D_\beta-$open function need not be $D_\alpha-$open. (see the Example 2.5 below).

Example 2.3. Let $X=\{a,b,c\}$, $\tau=\{X,\phi,\{b,c\},\{c\}\}$, then (X,τ) be a topological space. Let $Y=\{1,2,3\}$ and $\sigma=\{Y,\phi,\{2,3\},\{3\}\}$, then (Y,τ) be a topological space.

$C(Y)=\{Y,\phi,\{1\},\{1,2\}\}$,

$\beta C(Y)=\{Y,\phi,\{1,2\},\{1\},\{2\}\}$,

$\beta O(Y)=\{Y,\phi,\{2,3\},\{3\},\{1,3\}\}$,

$D_\beta C(Y)=\{Y,\phi,\{1\},\{1,2\},\{2\},\{1,3\}\{3\}\}$,

$D_\beta O(Y)=\{Y,\phi,\{2,3\},\{3\},\{1,3\},\{2\}\{1,2\}\}$.

Let $f:(X,\tau)\to(Y,\sigma)$ be a function defined by *f*(a) = 3, *f*(b) = 1, *f*(c) = 2 is $D_\beta-$open function, since the image of each open set is $D_\beta-$open in *Y.* But *f* is not a $\beta-$open function, since $f(\{b,c\})=\{1,2\}$, which is not $\beta-$open in *Y.*

Example 2.4. Let $X=\{a,b,c\}$ be any set and $\tau=\{X,\phi,\{a,b\}\}$, then (X,τ) be a topological space. Let $Y=\{x,y,z\}$ and $\sigma=\{Y,\phi,\{y,z\},\{z\}\}$, then (Y,σ) be a topological space.

$C(Y)=\{Y,\phi,\{x\},\{x,y\}\}$,

$GC(Y)=\{Y,\phi,\{x\},\{x,y\},\{x,z\}\}$,

$GO(Y)=\{Y,\phi,\{y,z\},\{z\},\{y\}\}$,

$D_\beta C(Y)=\{Y,\phi,\{x\},\{x,y\},\{x,z\},\{y\},\{z\}\}$,

$D_\beta O(Y)=\{Y,\phi,\{y,z\},\{z\},\{y\},\{x,z\},\{x,y\}\}$,

$S^*O(Y)=\{Y,\phi,\{y,z\},\{z\},(x,z\}\}$. Let $f:(X,\tau)\to(Y,\sigma)$ be the function defined by *f*(a) = x, *f*(b) = y, *f*(c) = z is D_β-open function, since *f* image of each open set is D_β-open in *Y.* But *f* is not g-open, since $f(\{a,b\})=\{x,y\}$, which is is not g-open in *Y* and not semi*-open in *Y.*

Example 2.5. Let $X=\{x,y,z\}$ and $\tau=\{X,\phi,\{y,z\},\{z\}\}$, then (X,τ) be a topological space. Let $Y=\{a,b,c\}$ and $\sigma=\{Y,\phi,\{b,c\},\{b\},\{c\}\}$, then (Y,σ) be a topological space. $C(Y)=\{Y,\phi,\{a\},\{a,c\},\{a,b\}\}$,

$GC(Y)=\{Y,\phi,\{a\},\{a,c\},\{a,b\}\}$,

$GO(Y)=\{Y,\phi,\{b,c\},\{b\},\{c\}\}$

$D_\alpha C(Y)(Y)=\{Y,\phi,\{a\},\{a,c\},\{a,b\}\}$,

$D_\alpha O(Y)=\{Y,\phi,\{b,c\},\{b\},\{c\}\}$

$D_\beta C(Y)=\{Y,\phi,\{a\},\{a,c\},\{a,b\},\{b\},\{c\}\}$,

$D_\beta O(Y)=\{Y,\phi,\{b,c\},\{b\},\{c\},\{a,b\},\{a,c\}\}$.

Let function $f:(X,\tau)\to(Y,\sigma)$ defined by *f*(x) = b, *f*(y) = a, *f*(z) = c is D_β-open function, since the image of each open set in *X* is D_β-open in *Y*, but *f* is not D_α-open, Since $f\{(y,z)\}=\{a,c\}$, which is not D_α-open in *Y.*

Remark. The composition of two D_β-open maps need not be D_β-open in general. This is shown by the following example.

Example 2.6. Let $X=Y=Z=\{a,b,c\}$ be the sets with the topology $\tau=\{X,\phi,\{a,b,d\},\{b,d\}\}$, $\sigma=\{Y,\phi,\{a,b\},\{a,b,c\},\{b,d\},\{b\},\{a,b,d\}\}$ and $\eta=\{Z,\phi,\{b,c\},\{b,c,d\},\{c\},\{c,d\},\{a,b\},\{b\},\{a,b,c\}\}$, respectively. Then (X,τ), (Y,σ) and (Z,η) be the topological spaces. We define $f:(X,\tau)\to(Y,\sigma)$ as *f*(a) = b, *f*(b) = c, *f*(c) = d, f(d) = a and the map $g:(Y,\sigma)\to(Z,\eta)$ as *g*(a) = d, *g*(b) = c, *g*(c) = a and *g*(d) = a. Then *f* and *g* are D_β-open maps, but their composition $g\circ f:(X,\tau)\to(Z,\sigma)$ is not D_β-open map, because be any open set in (X,τ) and $g\circ f(A)=g(f(\{b,d\}))=g(\{a,c\})=\{a,d\}$, which is not a D_β-open set in (Z,η).

Theorem 2.7. If $f:(X,\tau)\to(Y,\sigma)$ is a open map and $g:(Y,\sigma)\to(Z,\eta)$ is D_β-open map, then their composition $g\circ f:(X,\tau)\to(Z,\sigma)$ is D_β-open map.

Proof. Let *G* be any open set in (X,τ). Since *f* is open map, *f*(G) is open in (Y,σ). Since g is D_β-open map, *g*(*f*(G)) is D_β-open set in (Z,η). Therefore $g\circ f(G)=g(f(G))$ is D_β-open set in (Z,η).

Theorem 2.8. (i) Every g-closed function is D_β-closed.

(ii) Every semi*-closed function is D_β-closed.

(iii) Every β-closed function is D_β-closed.

(iv) Every D_β-closed function is D_β-closed.

Proof. It is obvious.

Theorem 2.9. If the space is $T_{1/2}$, then every D_α-closed (resp. D_β-closed) set is α-closed (resp. β-closed).

Proof. Let *A* be any D_α-closed (resp. D_β-closed) subset of the space *X*, then we have $(C\ell^*(Int(C\ell^*(A)))\subseteq A$ (resp. $Int^*(C\ell^*(Int^*(A)))\subseteq A)$. Since the space *X* is $T_{1/2}$, every g-closed set is closed, consequently $C\ell^*(A)=C\ell(A)$. Therefore, we get $C\ell(Int(C\ell(A)))\subseteq A$ (resp. $Int(C\ell(Int(A)))\subseteq A$).

Theorem 2.10 If $f:(X,\tau)\to(Y,\sigma)$ is a g-closed map, $g:(Y,\sigma)\to(Z,\eta)$ is D_β-closed map and (Y,σ) be $T_{1/2}$-space, then their composition $g\circ f:(X,\sigma)\to(Z,\eta)$ is D_β-closed and therefore $\beta-$closed.

Proof. Let *A* be any closed set in (X,τ). Since map *f* is g-closed, *f*(A) is g-closed in (Y,σ). Since (Y,σ) is $T_{1/2}$-space, *f*(A) is closed in (Y,σ). Since *g* is D_β-closed, *g*(*f*(A)) is D_β-closed set and therefore β-closed set in (Z,η) and $g\circ f(A)=g(f(A))$. Hence the composition map $g\circ f$ is D_β-closed and therefore $\beta-$closed.

Theorem 2.11. Let $f:(X,\tau)\to(Y,\sigma)$ be any bijective map, then the following statements are equivalent;

(i) f^{-1} is D_β-continuous.

(ii) *f* is D_β-open map.

(iii) *f* is D_β-closed map.

Proof. (i) $\Rightarrow$ (ii): Let *U* be any open set in *X*. By assumption $(f^{-1})^{-1}(U)=f(U)$ is D_β-open in *Y*. This shows that *f* is D_β-open map.

(ii) $\Rightarrow$ (iii): Let *F* be any closed set in *X*. Then F^c is open set in *X*, therefore by assumption $f(F^c)=(f(F))^c$ is D_β-open in *Y*, consequently *f*(F) is D_β-closed in *Y*. Hence the map *f* is D_β- closed.
(iii) $\Rightarrow$ (iv): Let *F* be any closed set in *X*. Then by assumption *f*(F) is D_β-closed in *Y* and therefore $f(F)=(f^{-1})^{-1}(F)$ is D_β-closed in *Y*, therefore f^{-1} is D_β-continuous.

3. LOWER SEPARATION AXIOMS

Separations are very useful concepts in topological spaces. They can be used to define the more restricted classes of topological spaces. The separation axioms exhibit the knowledge about the points and sets that are distinguishable or separated in some weaker sense or some stronger sense. Here we define and study some new types of lower separation axioms, namely $D_\beta-T_n$ for n= 0,1 and 2.

Definition 3.1. A topological space (X,τ) is said to be ,

(i) $D_\alpha-T_0$ if for any distinct pair of points x and y in *X*, there exists a D_α-open set *U* in *X* containing x but not y or containing y but not x.
(ii) $D_\beta-T_0$ if for any distinct pair of points x and y in *X*, there exists a D_β-open set *U* in *X* containing x but not y or containing y but not x.
(iii) $D_\beta-T_1$ if for any distinct pair of points x and y in *X*, there exists a D_β-open set *U* in *X* containing x but not y and a D_β-open set *V* in *X* containing y but not x.
(iv) $D_\beta-T_2$ if for any distinct pair of points x and y in *X*, there exists D_β-open sets *U* and *V* in *X* containing x and y, respectively such that $U\cap V=\phi$.

Remark. Every $D_\beta-T_0$ space need not be $D_\beta-T_1$.

Example 3.2. Let $X=\{a,b,c\}$ be any set and $\tau=\{X,\phi,\{a,b\},\{b\},\{b,c\}\}$, then (X,τ) be a topological space. $C(X)=\{\phi,X,\{c\},\{a,c\},\{a\}\}$, $D_\beta C(X)=\{X,\phi,\{c\},\{a,c\},\{a\}\}$, $D_\beta O(X)=\{X,\phi,\{a,b\},\{b\},\{b,c\}\}$. Thus the space X is

$D_\beta - T_0$ but it is not $D_\beta - T_1$. Since for a pair of distinct points a and b, there exists two D_β-open sets $\{a,b\}$ and $\{b\}$ in which $a \in \{a,b\}$ and $a \notin \{b\}$ but $b \in \{a,b\}$ and also $b \in \{b\}$.

Remark. Every $D_\beta - T_1$ space need not be $D_\beta - T_2$.

Example 3.3. Let $X = \{a,b,c\}$ be any set and $\tau = \{X, \phi, \{a,c\}, \{c\}\}\}$, then (X,τ) be a topological spaces. $C(X) = \{\phi, X, \{b\}, \{a,b\}\}$, , $D_\beta C(X) = \{X, \phi, \{b\}, \{a,b\}, \{b,c\}, \{a\}, \{c\}\}$, $D_\beta O(X) = \{X, \phi, \{a,c\}, \{c\}, \{b,c\}, \{a\}, \{a,b\}\}$. Thus the space *X* is $D_\beta - T_1$, but not $D_\beta - T_2$. Since for the distinct pair of points a and b, there are two D_β-open sets $\{a,c\}$ and $\{b,c\}$ such that $a \in \{a,c\}$ and $b \in \{b,c\}$, but these sets are not disjoint.

Remark. (i) Every $D_\alpha - T_0$ space is $D_\beta - T_0$.

(ii) Every $D_\alpha - T_1$ space is $D_\beta - T_1$.

(iii) Every $D_\alpha - T_2$ space is $D_\beta - T_2$.

Proof. It is obvious.

Theorem 3.5. A space *X* is $D_\beta - T_0$ space if and only if for each pair of distinct points x, y of *X*, $C\ell_{D_\beta}(\{x\}) \neq C\ell_{D_\beta}(\{y\})$.

Proof. *Necessity* - Let (X,τ) be any topological space and let x, y be any two points in *X* with $x \neq y$. Since *X* is $D_\beta - T_0$, then there exists a D_β-open set *A* such that $x \in A$ but $y \notin A$. Therefore $x \notin X \setminus A$ and $y \in (X \setminus A)$, where $X \setminus A$ is D_β- closed in *X*. Since $C\ell_{D_\beta}(\{y\})$ is the smallest D_β-closed set containing $\{y\}$ and therefore $y \in C\ell_{D_\beta}(\{y\})$ and $C\ell_{D_\beta}(\{y\}) \subseteq X \setminus A$. Therefore $x \notin C\ell_{D_\beta}(\{y\})$. Hence $C\ell_{D_\beta}(\{x\}) \neq C\ell_{D_\beta}(\{y\})$.

Sufficiency - Suppose for any disjoint pair points p , q of *X* with $C\ell_{D_\beta}(\{p\}) \neq C\ell_{D_\beta}(\{q\})$ then there exists at least one point $r \in X$ such that $r \in C\ell_{D_\beta}(\{p\})$ but $r \notin C\ell_{D_\beta}(\{q\})$. On contrary suppose $p \in C\ell_{D_\beta}(\{q\})$, then $C\ell_{D_\beta}(\{p\}) \subseteq C\ell_{D_\beta}(\{q\})$, consequently $r \in C\ell_{D_\beta}(\{q\})$, which is the contradiction of the fact that $r \notin C\ell_{D_\beta}(\{q\})$. Hence $p \in (X \setminus C\ell_{D_\beta}(\{q\})$ and $q \notin (X \setminus C\ell_{D_\beta}(\{q\})$. This implies that *X* is $D_\beta - T_0$.

Theorem 3.6. Let $f : (X, \tau) \to (Y, \sigma)$ be a bijective and D_β-continuous and Y is T_0 space, then *X* is $D_\beta - T_0$.

Proof. Let x_1, x_2 be any two distinct elements of *X*. Since *f* is bijective map, then there exists $y_1, y_2 \in Y$ with $y_1 \neq y_2$ such that $y_1 = f(x_1)$ and $y_2 = f(x_2)$. Hence $x_1 = f^{-1}(y_1)$ and $x_2 = f^{-1}(y_2)$. Since *Y* is T_0 space, therefore there exists an open set *G* in *Y* such that $y_1 \in G$ but $y_2 \notin G$. Since *f* is D_β-continuous, $f^{-1}(G)$ is D_β-open in *X*. Therefore $y_1 \in G$ implies that $f^{-1}(y_1) \in f^{-1}(G)$ and $y_2 \in G$ implies that $f^{-1}(y_2) \in f^{-1}(G)$ consequently $x_1 \in f^{-1}(G)$ but $x_2 \notin f^{-1}(G)$. Therefore for any two distinct points y_1, y_2 in *Y* with $y_1 \in G$ and $y_2 \notin G$ and *G* is open in *Y*, then there exists a D_β-open set $f^{-1}(G)$ in *X* such that $x_1 \in f^{-1}(G)$ but $x_2 \notin f^{-1}(G)$. This shows that *X* is $D_\beta - T_0$.

Theorem 3.7. Let $f : (X, \tau) \to (Y, \sigma)$ be a bijective and D_β-open map and (X,τ) is T_0 space, then (Y,σ) is $D_\beta - T_0$ space.

Proof. Let y_1, y_2 be any pair of distinct points of *Y* and *f* is one-one and onto, therefore there exists two distinct points x_1, x_2 in *X* such that $f(x_1) = y_1$ and $f(x_2) = y_2$. Since (X,σ) is T_0, therefore there exists an open set *G* in *X* such that $x_1 \in G$ but $x_2 \notin G$. Since *f* is D_β-open map, then f(G) is D_β-open in *Y* such that $f(x_1) = y_1 \in f(G)$ but $f(x_2) = y_2 \notin f(G)$. Hence *Y* is $D_\beta - T_0$.

Theorem 3.8. Let $f : (X, \tau) \to (Y, \sigma)$ be a bijective and D_β-irresolute mapping and *Y* is $D_\beta - T_0$ space. Then *X* is also $D_\beta - T_0$.

Proof. Suppose x_1, x_2 be any two distinct points of *X*, then there exists $y_1, y_2 \notin Y$ with $y_1 \neq y_2$ such that $f(x_1) = y_1$ and $f(x_2) = y_2$. Since *Y* is $D_\beta - T_0$, $x_1 = f^{-1}(y_1)$ and $x_2 = f^{-1}(y_2)$. Since *Y* is $D_\beta - T_0$, there exists a D_β-open set *G* in *Y* such that $y_1 \in G$ and $y_2 \notin G$. Since *f* is D_β-irresolute, $f^{-1}(G)$ is D_β-open in *X*. We have $y_1 \in G$ implies $f^{-1}(y_1) = x_1 \in f^{-1}(G)$ but $y_2 \notin G$ implies $f^{-1}(y_2) = x_2 \notin f^{-1}(G)$. Then, for $x_1, x_2 \in X$ with $x_1 \neq x_2$, there exists a D_β-open set $f^{-1}(G)$ in *X* such that $x_1 \in f^{-1}(G)$ but $x_2 \notin f^{-1}(G)$. Hence *X* is $D_\beta - T_0$.

Theorem 3.9. Let $f:(X, \tau) \to (Y,\sigma)$ be bijective and D_β-continuous and Y is T_1 space, then X is D_β - T_1 space.

Proof. Let x_1, x_2 be any two distinct points of *X*. By assumption there exists two points $y_1, y_2 \in Y$ with $y_1 \neq y_2$ such that $y_1 = f(x_1)$ and $y_2 = f(x_2)$. Therefore $x_1 = f^{-1}(y1)$ and $x_2 = f^{-1}(y_2)$. Since *Y* is T_1 space, there exists open sets *M* and *N* in *Y* such that $y_1 = f(x_1) \in M$ but $y_2 = f(x_2) \notin M$ and $y_2 = f(x_2) \in N$ but $y_1 = f(x_1) \notin N$. Since *f* is being D_β-continuous, $f^{-1}(M)$ and $f^{-1}(N)$ are D_β-open sets in *X* such that $x_1 = f^{-1}(y_1) \in f^{-1}(M)$ but $x_2 = f^{-1}(y_2) \notin f^{-1}(M)$ and $x_2 = f^{-1}(y_2) \in f^{-1}(N)$ but $x_1 = f^{-1}(y_1) \notin f^{-1}(N)$. This shows that *X* is D_β - T_1.

Theorem 3.10. $f:(X, \tau) \to (Y,\sigma)$ be a bijective, D_β-open map and (X,τ)) is T_1 space, then (Y,σ) is D_β - T_1.

Proof. Let $f:(X, \tau) \to (Y,\sigma)$ be a bijective, D_β-open map and *X* is T_1. Let y_1, y_2 be any pair of distinct points of *Y*, then there exists two distinct points x_1, x_2 in *X* such that $f(x_1) = y_1$ and $f(x_2) = y_2$. Since (X,τ) is T_1, there exists open sets *G* and *H* in *X* such that $x_1 \in G$ but $x_2 \notin G$ and $x_2 \in H$ but $x_1 \notin H$. Since *f* is D_β-open map, *f*(G) and *f*(H) in *Y* such that $f(x_1) = y_1 \in f(G)$ but $f(x_2) = y_2 \notin f(G)$ and $f(x_1) = y_1 \notin f(H)$ but $f(x_2) = y_2 \notin f(H)$. Therefore *Y* is D_β - T_1.

Remark: By referring the above mentioned results we can conclude that, a space being D_β - T_i (for i = 0,1) is a topological property.

Lemma 3.11. The set *G* is D_β-open in the space (X,τ) if and only if for each $x \in G$, there exists a D_β-open set *F* such that $x \in F \subseteq G$.

Proof. Suppose G is any D_β-open set in (X,τ), then for each $x \in G$, let G = F such that $x \in F$ and therefore $x \in F \subseteq G$. Conversely, assume that for each $x \in G$, there exists a D_β-open set *F* such that $x \in F \subseteq G$. Therefore $G = \bigcup \{F_x : F_x \in D_\beta O(X)$, for each x}. Hence *G* is D_β-open in (X,τ).

Theorem 3.12. A space (X,τ) is D_β - T_1 space if and only if the singletons are D_β-closed sets.

Proof. *Necessity*- Let (X,τ) be any D_β - T_1 space and suppose x be any point of *X*. We show that {x} is D_β-closed set. Suppose $y \in \{x\}^c$, then $x \neq y$. Since *X* is D_β - T_1, therefore there exists two D_β-open sets *G* and *H* such that $x \in G, y \notin G$ and $y \in H$, $x \notin H$. This implies that $y \in H \subseteq \{x\}^c$, therefore by Lemma 3.11, the set $\{x\}^c$ is D_β-open. i.e. {x} is D_β-closed.

Sufficiency- Let $x, y \in X$ such that $x \neq y$. It implies that {x} and {y} two disjoint D_β-closed sets in *X*. Therefore $\{x\}^c, \{y\}^c$ are two D_β-open sets such that $y \in \{x\}^c$ but $y \notin \{y\}^c$ and $x \in \{y\}^c$ but $x \notin \{x\}^c$. Hence *X* is D_β - T_1.

Theorem 3.13. The following statements are equivalent for a topological space (X,τ);

(i) *X* is D_β - T_2.

(ii) Let $x \in X$, for each $y \in X$ with $y \neq x$, there exists a D_β-open set *G* containing x such that $y \notin Cl_{D_\beta}(G)$.

(iii) For each $x \in X$, $\bigcap \{ Cl_{D_\beta}(G) : G$ is D_β-open in *X* and $x \in G \} = \{x\}$

Proof. (i)) $\Rightarrow$ (ii): Let *X* be any D_β - T_2 space and let $x, y \in X$, then for each $x \neq y$, there exists two disjoint D_β-open sets *G* and *H* such that $x \in G, x \notin H$ and $y \in H$, $y \notin G$. Since *H* is D_β-open, therefore $X \setminus H$ is D_β-closed and $G \subseteq (X \setminus H)$. Thus we have, $C\ell_{D_\beta}(G) \subseteq C\ell_{D_\beta}(X \setminus H) = (X \setminus H)$. Since $y \notin (X \setminus H)$, therefore $y \notin C\ell_{D_\beta}(G)$.

(ii) $\Rightarrow$ (iii) Let $x \in X$, then for each $y \neq x$ in *X*, there exists a D_β-open set *G* such that $x \in G$ and $y \notin C\ell_{D_\beta}(G)$. Therefore $y \notin \bigcap\{C\ell_{D_\beta}(G) : G$ is D_β-open in *X* and $x \in G\}$ and hence $\bigcap\{C\ell_{D_\beta}(G) : G$ is D_β-open in *X* and $x \in G\} = \{x\}$.

(iii) $\Rightarrow$ (i) : Let $x, y \in X$ such that $x \neq y$, then by assumption $y \notin \bigcap\{ C\ell_{D_\beta}(G) : G$ is D_β-open in *X* and $x \in G\} = \{x\}$. This implies that $y \notin Cl_{D_\beta}(G)$ where $G \in D_\beta O(X)$ and $x \in G$. This shows that $y \in (X \setminus C\ell_{D_\beta}(G))$. Since $C\ell_{D_\beta}(G)$ is D_β-closed, then $X \setminus C\ell_{D_\beta} G) = H$ (say) is D_β-open in *X* and $y \in H$.

Therefore $G \cap H = G \cap (X \setminus C\ell_{D_\beta}(G)) = \phi$.

Hence *X* is D_β - T_2.

4. D_β -CLOSED GRAPH AND STRONGLY D_β -CLOSED GRAPH

In this section we introduce D_β-closed graph and strongly D_β -closed graph and investigate some of their basic properties.

Definition 4.1. A function $f:(X,\tau)\to(Y,\sigma)$ has D_β-closed graph if for each $(x,y)\in(X\times Y)\setminus G(f)$, there exist $U\in D_\beta O(X,x)$ and $V\in\beta O(Y,y)$ such that $(U\times C\ell_\beta(V))\setminus G(f)=\phi$.

Theorem 4.2. A closed graph is always a D_β -closed graph.

Proof. The proof follows from the fact that every open set is β-open and D_β-open.

Remark. The converse of above theorem is not true in general.

Example 4.3. Let $X=\{a,b,c,d\}$ and $\tau=\{X,\phi,\{a,b\},\{b,c,d\},\{b\}\}$, then (X,τ) be a topological space. $C(X)=\{X,\phi,\{c,d\},\{a\},\{a,c,d\}\}$. Let $Y=\{1,2,3,4\}$, $\sigma=\{Y,\phi,\{1,2\},\{1,2,4\}\}$, then (Y,σ) be a topological space. $C(Y)=\{Y,\phi,\{3,4\},\{3\}\}$,

$D_\beta C(X)=\{X,\phi,\{c,d\},\{a\},\{a,c,d\},\{d\},\{c\},\{a,d\},$
$\{a,b,c\},\{a,b,d\},\{b,c\},\{b,d\},\{a,b\}.\{b\}\}$
,
$D_\beta O(X)=\{X,\phi,\{a,b\},\{b,c,d\},\{b\},\{a,b,c\},\{a,b,d\},$
$\{b,d\},\{b,c\},\{d\},\{c\},\{a,d\},\{a,c\},\{c,d\},\{a,c,d\}\}$

Let $f:(X,\tau)\to(Y,\sigma)$ be a function defined by f(a) = 2, f(b) = 3, f(c) = 4 and f(d) = 1. Let $\{c,d\}\in D_\beta O(X)$ and $\{2\}\in\beta O(Y)$, then $(\{c,d\}\times\{2\})\cap(\{c,4\},\{d,1\})=$ $\{\{c,2\},\{d,2\}\}\cap\{\{c,4\},\{d,1\}\}=\phi$ Therefore the graph G(f) is D_β-closed graph, but it is not closed graph, since the $\{c,d\}$ is not open in *X*.

Theorem 4.4. The function $f:(X,\tau)\to(Y,\sigma)$ has a D_β-closed graph if and only if for each $(x,y)\in X\times Y$ such that $f(x)\neq Y$, then there exits a D_β-open set *U* and a β-open set *V* containing x and y , respectively such that $f(U)\cap C\ell_\beta(V)=\phi$.

Proof. *Necessity* - Suppose for each $(x,y)\in X\times Y$ with $f(x)\neq y$. Since G(f) is D_β-closed graph, then there exits a D_β-open set *U* and a β-open set *V* containing x and y, respectively such that $(U\times C\ell_\beta(V))\cap G(f)=\phi$. Hence for each $f(x)\in f(U)$ and $y\in C\ell_\beta(V))$ with $f(x)\neq y$, we get $f(U)\cap C\ell_\beta(V)=\phi$.

Sufficiency - Let $(x,y)\notin G(f)$, then $f(x)\neq y$ and therefore there exist a D_β-open set *U* and an β-open set *V* containing x and y, respectively such that $f(U)\cap C\ell_\beta(V)=\phi$. This implies for each $x\in U$ and for each $y\in C\ell_\beta(V)$, $f(x)\neq y$. This proves that $(U\ x\ C\ell_\beta(V))\cap G(f)=\phi$. Hence f has a D_β-closed graph.

Theorem 4.5. If $f:(X,\tau)\to(Y,\sigma)$ is a D_β-continuous function from a space *X* into a Hausdorff space *Y*, then *f* has a D_β-closed graph in $X\times Y$.

Proof. Let $(x,y)\notin G(f)$, then $f(x)\neq y$. Since *Y* is Hausdorff space, there exist two disjoint open sets *P* and *Q* such that $f(x)\in Q$ and $y\in P$. Since f is D_β- continuous, therefore by Theorem 3.9 of [16], there exists a D_β-open set *U* in *X* such that $x\in U$ and $f(U)\subseteq Q$. Therefore $f(U)\subseteq Y\setminus C\ell(P)$. This implies that $f(U)\bigcap C\ell(P)=\phi$. Since every open set is β -open, therefore $C\ell_\beta(P)\subseteq C\ell(P)$. Hence we get $f(U)\cap C\ell_\beta(V)=\phi$, which implies that f has a D_β -closed graph.

Corollary 4.6. If the function $f:(X,\tau)\to(Y,\sigma)$ has a D_β-closed graph, then for each $(x,y)\in X\times Y$ such that $f(x)\neq y$, then there exits two D_β-open sets *U* and *V* containing x and y, respectively such that $(U\times C\ell_{D_\beta}(V))\bigcap G(f)=\phi$.

Proof. Suppose f has a D_β-closed graph, then for each $(x,y)\in X\times Y$ such that $f(x)\neq y$, then there exits D_β -open sets *U* and β-open set *V* containing x and y, respectively such that $(U\times C\ell_\beta(V))\cap G(f)=\phi$. Since every $\beta-$open set is $D_\beta-$open and therefore $C\ell_{D_\beta}(V)\subseteq C\ell_\beta(V)$, we have $(U\times C\ell_{D_\beta}(V))\cap G(f)=\phi$.

Corollary 4.7. The function $f:(X,\tau)\to(Y,\sigma)$ has a D_β-closed graph, then for each $(x,y)\in X\times Y$ such that $f(x)\neq y$, there exits two D_β -open sets *U* and *V* containing x and y, respectively such that $f(U)\cap C\ell_{D_\beta}(V)=\phi$.

Proof. Suppose the function $f:(X,\tau)\to(Y,\sigma)$ has a D_β -closed graph. By Theorem 4.4 for each $(x,y)\in X\times Y$ such that $f(x)\neq y$, there exits a D_β-open sets *U* and a β-open set *V* containing x and y, respectively such that $f(\mathrm{U}) \cap \mathrm{C}\ell_{\beta}(\mathrm{V}) = \phi$. Since every β -closed set is D_{β} -closed. i.e. $\mathrm{C}\ell_{\mathrm{D}_{\beta}}(\mathrm{V}) \subseteq \mathrm{C}\ell_{\beta}(\mathrm{V})$. Hence we get $f(\mathrm{U}) \cap \mathrm{C}\ell_{\mathrm{D}_{\beta}}(\mathrm{V}) = \phi$.

Theorem 4.8. If $f:(X, \tau) \to (Y, \sigma)$ is a surjective function and has a D_{β}-closed graph from a space *X* onto a space *Y*, then *Y* is $\beta - \mathrm{T}_2$.

Proof. Let y_1 and y_2 be any two distinct points in *Y*. Since f is subjective, then there exists a point $\mathrm{x}_1 \in \mathrm{X}$ such that $\mathrm{f}(\mathrm{x}_1) = \mathrm{y}_1 \neq \mathrm{y}_2$. Thus $(\mathrm{x}_1, \mathrm{y}_2) \notin \mathrm{G}(f)$. Since f has a D_{β}-closed graph, then by Theorem 4.4, there exists a D_{β}-open set *U* and a β-open set *V* containing x_1 and y_2, respectively such that $f(\mathrm{U}) \cap \mathrm{C}\ell_{\beta}(\mathrm{V}) = \phi$. $\mathrm{x}_1 \in \mathrm{U}$ implies $f(\mathrm{x}_1) = \mathrm{y}_1 \in f(\mathrm{U})$. Thus $\mathrm{x}_1 \notin \mathrm{C}\ell_{\beta}(\mathrm{V})$, then there exists a β -open set $\mathrm{Y} \setminus \mathrm{C}\ell_{\mathrm{D}_{\beta}}(\mathrm{V})$ such that $f(\mathrm{x}_1) = \mathrm{y}_1 \in \mathrm{Y} \setminus \mathrm{C}\ell_{\mathrm{D}_{\beta}}(\mathrm{V})$. Thus $\mathrm{V} \cap \mathrm{Y} \setminus \mathrm{C}\ell_{\beta}(\mathrm{V}) = \phi$. Hence *Y* is $\beta - \mathrm{T}_2$.

Corollary 4.9. If $f:(X, \tau) \to (Y, \sigma)$ is a surjective function and has a D_{β}-closed graph from a space *X* onto a space *Y*, then *Y* is $\mathrm{D}_{\beta} - \mathrm{T}_2$.

Proof. It follows from the Theorem 4.5 and the Corollary 4.7.

Definition 4.10. A function $f:(X, \tau) \to (Y, \sigma)$ has strongly D_{β} -closed graph if for each $(\mathrm{x}, \mathrm{y}) \in (\mathrm{X} \times \mathrm{Y}) \setminus \mathrm{G}(f)$,
there exist $\mathrm{U} \in \mathrm{D}_{\beta}\mathrm{O}(\mathrm{X}, \mathrm{x})$ and $\mathrm{V} \in \mathrm{O}(\mathrm{Y}, \mathrm{y})$ such that $(\mathrm{U} \ \mathrm{x}\ \mathrm{Cl}(\mathrm{V})) \setminus \mathrm{G}(f) = \phi$.

Remark. Every strongly D_{α}-closed graph is strongly D_{β}-closed. But the converse is not true. Since it is shown in the Example 3.4 that the set $\mathrm{A} = \{\mathrm{a}, \mathrm{d}\}$ is D_{β}-closed but it is not D_{α}-closed.

Theorem 4.11. The function $f:(X, \tau) \to (Y, \sigma)$ has strongly D_{β} -closed graph if and only if for each $(\mathrm{x}, \mathrm{y}) \in (\mathrm{X} \times \mathrm{Y})$ such that $f(\mathrm{x}) \neq \mathrm{y}$, then there exits a D_{β}-open set *U* and an open set *V* containing x and y, respectively such that $f(\mathrm{U}) \cap \mathrm{C}\ell(\mathrm{V}) = \phi$.

Proof. Its proof is similar to the Theorem 4.4.

Corollary 4.12. If the function $f:(X, \tau) \to (Y, \sigma)$ has a strongly D_{β} -closed graph, then for each $(\mathrm{x}, \mathrm{y}) \in \mathrm{X} \times \mathrm{Y}$ such that $f(\mathrm{x}) \neq \mathrm{y}$, then there exits a D_{β}-open sets *U* and a β-open set *V* containing x and y, respectively such that $(\mathrm{U} \times \mathrm{C}\ell_{\beta}(\mathrm{V})) \cap \mathrm{G}(f) = \phi$

Proof. Its proof is similar to the Corollary 4.6.

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