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INDEPENDENT ROMAN DOMINATION OF CORONA GRAPH $C_n \odot K_m$

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***ABSTRACT*;** Domination Theory is an important branch of Graph theory that has various applications in Communication Networks, Engineering and Social Sciences and in many branches of Science & Technology. An introduction and an extensive overview on domination in graphs and related topics is surveyed and studied in the two books by Haynes et al. [1, 2]. Nowadays the behavior of domination parameters in product graphs is an interesting topic of research in graph theory. In this paper we present some results on minimal independent Roman dominating functions of corona product graph of a cycle with a complete graph. Also independent Roman domination number is obtained.

Keywords; Domination, Communication Networks, dominating functions, complete graph.

INTRODUCTION

Domination in graphs has been studied extensively in recent years and it is an important branch of graph theory. Allan, R.B. and Laskar, R.[3], Cockayne, E.J. and Hedetniemi, S.T. [4] have studied various domination parameters of graphs. Recently dominating functions in domination theory have received much attention. The concept of Roman domination was introduced by Ian Steward [5] and further studied by some authors. A function $f : V(G) \rightarrow \{0, 1, 2\}$ is said to be a Roman dominating function (RDF) if every vertex 'u' for which f(u) = 0 is adjacent to at least one vertex 'v' for which f(v) = 2. The Roman domination number of a graph G is the minimum weight of an RDF on G and it is denoted by $\gamma_R(G)$. A function $f = (V_0, V_1, V_2)$ is called a $\gamma_R -$ function if it is an RDF and $f\big(V(G)\big) = \gamma_R(G)$.

Frucht and Harary [6] introduced a new product on two graphs G_1 and G_2, called corona product denoted by $G_1 \odot G_2$. The object is to construct a new and simple operation on two graphs G_1 and G_2 called their corona, with the property that the group of the new graph is in general isomorphic with the wreath product of the groups of G_1 and of G_2.
In this paper we study the concept of independent Roman dominating functions of corona product graph of a cycle with a complete graph and some results on minimal independent Roman dominating functions of this graph are obtained.

1. CORONA PRODUCT OF C_n AND K_m

The **corona product** of a cycle C_n with a complete graph K_m is a graph obtained by taking one copy of a $n-$ vertex graph C_n and n copies of K_m and then joining the i^{th} vertex of C_n to every vertex of i^{th} copy of K_m and this graph is denoted by $C_n \odot K_m$.

We present some properties of the corona product graph $C_n \odot K_m$ without proofs and the proofs can be found in Sivaparvathi.M.[7].

Theorem 2.2.1: The graph $G = C_n \odot K_m$ is a connected graph.

Theorem 2.2.2: The degree of a vertex v in G $= C_n \odot K_m$ is given by

$$d(v) = \begin{cases} m+2, & if\ v \in C_n, \\ m\ , & if\ v \in K_m. \end{cases}$$

Let us denote the vertices of C_n in $G = C_n \odot K_m$ as $u_1, u_2, \dots\dots, u_n$ and the vertices in the i^{th} copy of K_m in G as $v_{i1}, v_{i2}, \dots\dots\dots, v_{im}$, $1 \le i \le n$.

INDEPENDENT ROMAN DOMINATING FUNCTION OF A GRAPH

Cockayne et. al. [8] introduced the concept of Independent Roman domination in graphs. Independent domination in graphs was introduced by R.B.Allan & R.C.Laskar[3] in the year 1978. A dominating set I of a graph G(V, E) in which no two vertices are adjacent is called an independent domination set (IDS) of G. The induced subgraph <I> is a null graph if I is an IDS.

A Roman dominating function $f = (V_0, V_1, V_2)$ is called an independent Roman dominating function (IRDF) if the set $V_1 \cup V_2$ or V_2 is independent. The independent Roman domination number is the minimum weight of an independent Roman dominating function on G and it is denoted by $i_R(G)$ and $i_R(G) = |V_1| + 2|V_2|$.

Adabi et. al.[9] proved some properties, bounds and characterizations of independent Roman domination in graphs. Chellali et al.[10] proved some lower bounds, characterizations and comparisons of lower bounds on the Roman and independent Roman domination numbers of a graph.

Theorem 3.1 : A function $f : V \rightarrow \{0, 1, 2\}$ defined by

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$f(v) = \begin{cases} 2, & \text{if } v = u_i \in C_n,\ i \equiv 0 (\text{mod } 2) \text{ and} \\ & \text{for one vertex in the } i^{th} \text{ copy of } K_m \text{ where } i \equiv 1(\text{mod } 2), \\ 0, & \text{otherwise} \end{cases}$

is a Minimal Independent Roman Dominating Function of $= C_n \odot K_m$.

Proof: Let f be a function defined as in the hypothesis.

Let us denote the set of vertices of G whose functional value is 2 by V_2.

Now $V_1 = \varphi$, and $V_0 = V - V_2$.

Case 1: Suppose n is even

Sub case 1: Let $v = u_i \in C_n$ be such that $d(v) = m + 2$ in G.

If $2 \le i < n$ and $i \equiv 0(\text{mod } 2)$ then

$$\sum_{u \in N[v]} f(u) = u_{i-1} + u_i + u_{i+1} + [v_{i1} + v_{i2} + ------ + v_{im}] = 0 + 2 + 0 + [\underbrace{0 + 0 + ... + 0}_{m-times}] = 2.$$

If $i = n$ and $i \equiv 0(\text{mod } 2)$ then

$$\sum_{u \in N[v]} f(u) = u_i + [v_{i1} + v_{i2} + ------ + v_{im}] = 0 + [2 + \underbrace{0 + 0 + ... + 0}_{(m-1)-times}] = 2.$$

$$\sum_{u \in N[v]} f(u) = u_{n-1} + u_n + u_1 + [v_{n1} + v_{n2} + ------ + v_{nm}] = 0$$

If $i = 1$ then

$$\sum_{u \in N[v]} f(u) = u_n + u_1 + u_2 + [v_{11} + v_{12} + ------ + v_{1m}] = 2 +$$

If $1 < i < n$ and $i \equiv 1(\text{mod } 2)$ then

$$\sum_{u \in N[v]} f(u) = u_{i-1} + u_i + u_{i+1} + [v_{i1} + v_{i2} + ------ + v_{im}] = 2$$

Sub case 2: Let $= v_{ij} \in K_m$, $1 \le j \le m$ be such that $d(v) = m$ in G.

If $i \equiv 0(\text{mod } 2)$ then

$$\sum_{u \in N[v]} f(u) = u_i + [v_{i1} + v_{i2} + ------ + v_{im}] = 2 + [\underbrace{0 + 0 + ...}_{m-tim}$$

If $i \equiv 1(\text{mod } 2)$ then

Therefore for all possibilities, we get $\sum_{u \in N[v]} f(u) > 1, \ \forall \ v \in V$.

By the definition of the function, $V_2 = S_1 \cup S_2$, where $S_1 = \left\{u_{2k} : 1 \le k \le \frac{n}{2}\right\}$ and $S_2 = \left\{v_{(2k-1)j} : 1 \le k \le \frac{n}{2}\right\}$ for fixed j, $1 \le j \le m$.

$|S_1| = \frac{n}{2}$, $|S_2| = \frac{n}{2}$ and $S_1 \cap S_2 = \emptyset$.

Clearly the set V_2 dominates the set V_0.

Now a vertex u_{2k} in S_1 is a vertex of C_n which is not adjacent to any other vertex of S_1. So S_1 is an independent set.

Also the set S_2 is obtained by taking one vertex in the i^{th} copy of K_m where $i \equiv 1(\text{mod } 2)$, since by the definition of the corona product, the vertices in i^{th} copy of K_m are adjacent to the corresponding vertex of C_n but not adjacent to any other vertex in G. So S_2 is an independent set.

Hence $S_1 \cup S_2$ is independent. Therefore V_2 is independent.

This implies that f is an IRDF of G.

Now we check for the minimality of f.

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Define $g : V \rightarrow \{0,1,2\}$ by

$$g(v) = \begin{cases} 2, & \text{if } v = u_i \in C_n,\ i \equiv 0 \pmod 2, i \neq 2 \text{ and} \\ & \text{for one vertex in the } i^{th} \text{ copy of } K_m \text{ where } i \equiv 1 \pmod 2, \\ 1, & \text{if } v = u_2 \in C_n, \\ 0, & \text{otherwise.} \end{cases}$$

Sub case 3: Let $v = u_i \in C_n$ be such that $d(v) = m + 2$ in G.

If $i = 2$ then

$$\sum_{u \in N[v]} g(u) = u_1 + u_2 + u_3 + [v_{21} + v_{22} + ------ + v_{2m}] = 0+1+0+[\underbrace{0+0+........+0}_{m-times}] = 1.$$

If $2 \leq i < n$ and $i \equiv 0 \pmod 2$ then

$$\sum_{u \in N[v]} g(u) = u_{i-1} + u_i + u_{i+1} + [v_{i1} + v_{i2} + ------ + v_{im}] = 0+2+0+[\underbrace{0+0+........+0}_{m-times}] = 2.$$

If $i = n$ and $i \equiv 0 \pmod 2$ then

$$\sum_{u \in N[v]} g(u) = u_{n-1} + u_n + u_1 + [v_{n1} + v_{n2} + ------ + v_{nm}] = 0+2+0+[\underbrace{0+0+........+0}_{m-times}] = 2.$$

If $i = 1$ then

$$\sum_{u \in N[v]} g(u) = u_n + u_1 + u_2 + [v_{11} + v_{12} + ------ + v_{1m}] = 2+0+1+[2+\underbrace{0+0+........+0}_{(m-1)-times}] = 5.$$

If $i = 3$ then

$$\sum_{u \in N[v]} g(u) = u_2 + u_3 + u_4 + [v_{31} + v_{32} + ------ + v_{3m}] = 1+0+2+[2+\underbrace{0+0+........+0}_{(m-1)-times}] = 5.$$

If $1 < i < n$ and $i \equiv 1 \pmod 2$ then

$$\sum_{u \in N[v]} g(u) = u_{i-1} + u_i + u_{i+1} + [v_{i1} + v_{i2} + ------ + v_{im}] = 2+0+2+[2+\underbrace{0+0+........+0}_{(m-1)-times}] = 6.$$

Sub case 4: Let $v = v_{ij} \in K_m$ be such that $d(v) = m$ in G.

If $i = 2$ then

$$\sum_{u \in N[v]} g(u) = u_2 + [v_{21} + v_{22} + ------ + v_{2m}] = 1+[\underbrace{0+0+........+0}_{m-times}] = 1.$$

If $i \equiv 0 \pmod 2$ $i \neq 2$ then

$$\sum_{u \in N[v]} g(u) = u_i + [v_{i1} + v_{i2} + ------ + v_{im}] = 2+[\underbrace{0+0+........+0}_{m-times}] = 2.$$

If $i \equiv 1 \pmod 2$ then

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$$\sum_{u\in N[v]} g(u) = u_i + [v_{i1} + v_{i2} + ------ + v_{im}] = 0 + [2 + \underbrace{0 + 0 + ... + 0}_{(m-1)-times}] = 2.$$

This implies that $\sum_{u\in N[v]} g(u) \geq 1, \ \forall \ v \in V.$

i.e. g is a IDF. But g is not a IRDF, since the RDF definition fails in the 2^{nd} copy of K_m in G because all vertices $v_{2j}, 1 \leq j \leq m$ in this copy are assigned the value 0 and the corresponding vertex u_2 , where u_2 is assigned the value 1. Hence the vertices in the 2^{nd} copy of K_m in G are adjacent to the vertex u_2 for which $f(u_2) = 1$.

Therefore f is a MIRDF.

Case 2: Suppose n is odd.

In similar lines to sub case 1 & sub case 2 of Theorem 3.1 we have seen that

$\sum_{u\in N[v]} f(u) = 2,$ if $v = u_i \in C_n, 2 \leq i < n$ and $i \equiv 0(\text{mod } 2)$.

$\sum_{u\in N[v]} f(u) = 2,$ if $v = u_i \in C_n, \ i = n$ and $i \equiv 1(\text{mod } 2)$.

$\sum_{u\in N[v]} f(u) = 4,$ if $v = u_i \in C_n$ *and* $i = 1$.

$\sum_{u\in N[v]} f(u) = 6,$ if $v = u_i \in C_n, \ 1 < i < n$ and $i \equiv 1(\text{mod } 2)$.

$\sum_{u\in N[v]} f(u) = 2,$ if $v = v_{ij} \in K_m$ and $i \equiv 0(\text{mod } 2)$.

$\sum_{u\in N[v]} f(u) = 2,$ if $v = v_{ij} \in K_m$ and $i \equiv 1(\text{mod } 2)$,

Therefore for all possibilities, we get $\sum_{u\in N[v]} f(u) > 1, \ \forall \ v \in V.$

Let $S_1 = \left\{u_{2k} : 1 \leq k \leq \frac{n}{2}\right\}$ and

$S_2 = \left\{v_{(2k+1)j} : 1 \leq k \leq \frac{n}{2}\right\}$ for fixed j, $1 \leq j \leq m$.

$|S_1| = \frac{n}{2}$, $|S_2| = \frac{n}{2} + 1$ and $S_1 \cap S_2 = \emptyset$.

By the definition of the function, $V_2 = S_1 \cup S_2$.

Clearly the set V_2 dominates the set V_0.

Now a vertex u_{2k} in S_1 is a vertex of C_n which is not adjacent to any other vertex of S_1. So S_1 is an independent set.

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And the set S_2 is obtained by taking one vertex in the ith copy of K_m where $i \equiv 1 (\mathrm{mod}\ 2)$, since by the definition of the corona product, the vertices in ith copy of K_m are adjacent to the corresponding vertex of C_n but not adjacent to any other vertex in G. So S_2 is an independent set.

Hence $S_1 \cup S_2$ is independent. Therefore V_2 is independent.

This implies that f is an IRDF of G.

Now we check for the minimality of f.

Define $g : V \to \{0, 1, 2\}$ by

$$g(v) = \begin{cases} 2, & \text{if } v = u_i \in C_n,\ i \equiv 0 (\mathrm{mod}\ 2), i \neq 2 \text{ and} \\ & \text{for one vertex in the } i^{th} \text{ copy of } K_m \text{ where } i \equiv 1 (\mathrm{mod}\ 2), \\ 1, & \text{if } v = u_2 \in C_n, \\ 0, & \text{otherwise} \end{cases}$$

As in sub case 3 and 4, we have seen that

$\sum_{u \in N[v]} g(u) = 1,$ if $v = u_i \in C_n$ and $i = 2$.

$\sum_{u \in N[v]} g(u) = 2,$ if $v = u_i \in C_n$, $2 < i < n$ and $i \equiv 0 (\mathrm{mod}\ 2)$.

$\sum_{u \in N[v]} g(u) = 3,$ if $v = u_i \in C_n$, and $i = 1$.

$\sum_{u \in N[v]} g(u) = 5,$ if $v = u_i \in C_n$, and $i = 3$.

$\sum_{u \in N[v]} g(u) = 6,$ if $v = u_i \in C_n$, and $3 < i < n$ and $i \equiv 1 (\mathrm{mod}\ 2)$.

$\sum_{u \in N[v]} g(u) = 4,$ if $v = u_i \in C_n$, and $i = n$.

Also $\sum_{u \in N[v]} g(u) = 1,$ if $v = v_{ij} \in K_m$ and $i = 2$.

$\sum_{u \in N[v]} g(u) = 2,$ if $v = v_{ij} \in K_m$ and $i \equiv 0 (\mathrm{mod}\ 2)$.

$\sum_{u \in N[v]} g(u) = 2,$ if $v = v_{ij} \in K_m$ and $i \equiv 1 (\mathrm{mod}\ 2)$.

This implies that $\sum_{u \in N[v]} g(u) \geq 1, \ \forall \ v \in V$.

i.e. g is a IDF. But g is not a IRDF, since the RDF definition fails in the 2^{nd} copy of K_m in G because all vertices $v_{2j}, 1 \leq j \leq m$ in this copy are assigned the value 0 and the corresponding vertex u_2 of C_n, is

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assigned by the value 1. Hence the vertices in the 2^{nd} copy of K_m in G are adjacent to the vertex u_2 for which $f(u_2) = 1$.

Hence f is a MIRDF. ■

Theorem 3.2: A function $f : V \rightarrow \{0,1,2\}$ defined by

$$f(v) = \begin{cases} 2, & \text{if } v = u_i \in C_n,\ i \equiv 1(\text{mod } 2) \text{ and} \\ & \text{for one vertex in the } i^{th} \text{ copy of } K_m \text{ where } i \equiv 0(\text{mod } 2), \\ 0, & \text{otherwise} \end{cases}$$

is a Minimal Independent Roman Dominating Function of $= C_n \odot K_m$.

Proof: As similar lines of theorem 3.1, we have seen that f is a MIRDF.

Theorem 3.3 : A function $f : V \rightarrow \{0,1,2\}$ defined by

$$f(v) = \begin{cases} 2, & \text{for any one vertex } v = v_{i1} \text{ in each copy of } K_m \text{ in } G, \\ 0, & \text{otherwise.} \end{cases}$$

is a Minimal Independent Roman Dominating Function of $G = C_n \odot K_m$.

Proof: Siva Parvathi[7] proved that the function defined in the hypothesis is a MRDF of G.

Let S be the set of vertices of G whose functional value is 2 in G.

i.e., $S = \{v_{11}, v_{21}, \dots \dots .., v_{n1}\}$ and $|S| = n$.

Obviously the set S is obtained by taking one vertex from each of copy of K_m . As there are 'n' copies of K_m and by the definition of corona product, the vertices in the i^{th} copy of K_m are adjacent to the corresponding vertex of C_n but not adjacent to the vertices in other copies of K_m.

Therefore S is independent.

Now $V_1 = \varphi$, $V_2 = S$ and $V_0 = V - S$.

Clearly the set V_2 is an independent set.Therefore f is a MIRDF.

Theorem 3.4: The independent Roman domination number of a graph $G = C_n \odot K_m$ is 2n.

Proof: Let $f = (V_0, V_1, V_2)$ be a MIRDF for $G = C_n \odot K_m$ such that V_2 is independent.

By the definition of f, $V_2 = \{v_{11}, v_{21}, \dots \dots .., v_{n1}\}$ or $\{v_{11}, u_2, v_{31}, u_4, v_{51}, \dots \dots .., u_n\}$ or $\{u_1, v_{21}, u_3, v_{41}, \dots \dots .., v_{m1}\}$ if n is even.

If n is odd then $V_2 = \{v_{11}, v_{21}, \dots \dots .., v_{n1}\}$ or $\{v_{11}, u_2, v_{31}, u_4, v_{51}, \dots \dots .., v_{m1}\}$ or $\{u_1, v_{21}, u_3, v_{41}, \dots \dots .., u_n\}$.

Now $V_1 = \varphi, V_0 = V - V_2$ and $|V_2| = n$.

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Clearly V_2 is minimal and minimum independent Roman dominating set of G with cardinality n. By the definition, $i_R(G) = |V_1| + 2|V_2|$. Here $|V_1| = 0$. Hence $i_R(G) = 2n$.

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