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IDEALS WITH SYMMETRIC REVERSE BI-DERIVATIONS ON PRIME AND SEMIPRIME RINGS

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Abstract: Let R be a 2 and 3-torsion free non-commutative prime ring and I be a nonzero ideal of R. Suppose there exist a symmetric reverse bi-derivations $D_1(.,.): R \times R \to R$ and $D_2(.,.): R \times R \to R$ such that $d_1(x)d_2(x) = 0$, for all $x \in I$, where d_1 and d_2 are the traces of D_1 and D_2 respectively. In this case either $D_1 = 0$ or $D_2 = 0$ and R be a 2-torsion free semiprime ring and I be a nonzero ideal of R. Let $D(.,.): R \times R \to R$ be a symmetric reverse bi-derivation such that $D(I, I) \subseteq I$. If d is a trace of D such that $D(d(x), x) = 0$, for all $x \in I$, then $D = 0$ on I.

Keywords: Prime ring, Semiprime ring, Symmetric mapping, Bi-additive mapping, Symmetric bi-additive mapping, Trace, Symmetric bi-derivation, Symmetric reverse bi-derivation.

I. INTRODUCTION

The concept of a symmetric bi-derivation was introduced by Gy.Maksa[2, 3]. It was shown in [3] and [6] the symmetric bi-derivations are related to general solution of some functional equations. Some results in symmetric bi-derivations in prime and Semiprime rings can be found in [4, 5, 7]. The notation of additive commuting mappings are closely connected with the notation of bi-derivations. Every commuting bi-additive mapping $f: R \to R$ givesrise to a bi-derivation on R. Asma Ali, V. De Filippis and Faiza Shujat [1] has studied some results concerning symmetric generalized bi-derivations of prime and semiprime rings. In this paper, we proved some results concerning on ideals with symmetric reverse bi-derivations on prime and Semiprime rings.

Throughout this paper R will be associative. We shall denote by $Z(R)$ the center of a ring R. Recall that a ring R is prime if $aRb = (0)$ implies that either $a = 0$ or $b = 0$ and it is a semiprime if $aRa = (0)$ implies $a = 0$.

We shall write $[x, y]$ for $xy - yx$ and use the identities $[xy, z] = [x, z]y + x[y, z], [x, yz] = [x, y]z + y[x, z]$. An additive map $d: R \to R$ is called derivation if $d(xy) = d(x)y + xd(y)$, for all $x, y \in R$. A mapping $B(.,.): R \times R \to R$ is said to be symmetric if $B(x, y) = B(y, x)$, for all $x, y \in R$. A mapping $f: R \to R$ defined by $f(x) = B(x, x)$, where $B(.,.): R \times R \to R$ is a symmetric mapping, is called a trace of B. It is obvious that, in case $B(.,.): R \times R \to R$ is symmetric mapping which is also bi-additive (i. e. additive in both arguments) the trace of B satisfies the relation $f(x + y) = f(x) + f(y) + 2B(x, y)$, for all $x, y \in R$. We shall use the fact that the trace of a symmetric bi-additive mapping is an even function. A symmetric bi-additive mapping $D(.,.): R \times R \to R$ is called a symmetric bi-derivation if $D(xy, z) = D(x, z)y + xD(y, z)$, for all $x, y, z \in R$. Obviously, in this case also the relation $D(x, yz) = D(x, y)z + yD(x, z)$, for all $x, y, z \in R$. A symmetric bi-additive mapping $D(.,.): R \times R \to R$ is called a symmetric reverse bi-derivation if $D(xy, z) = D(y, z)x + yD(x, z)$, for all $x, y, z \in R$. Obviously, in this case also the relation $D(x, yz) = D(x, z)y + zD(x, y)$, for all $x, y, z \in R$. A mapping $f: R \to R$ is said to be commuting on R if $[f(x), x] = 0$, for all $x \in R$. A mapping $f: R \to R$ is said to be centralizing on R if $[f(x), x] \in Z(R)$, for all $x \in R$. A ring R is said to be n-torsion free if whenever $na = 0$, with $a \in R$, then $a = 0$, where n is nonzero integer.

Lemma 1:[5, Lemma 1] Let $d: R \to R$ be a derivation, where R is a prime ring. Suppose that either (i) $ad(x) = 0$, for all $x \in R$ or (ii) $d(x)a = 0$, for all $x \in R$ holds. In both the cases we have $a = 0$ or $D = 0$.

Lemma 2: Let R be a 2-torsion free non-commutative prime ring and I be a nonzero ideal of R. If $D(.,.): R \times R \to R$ be a symmetric reverse bi-derivation and d be a trace of D such that $d(x) = 0$, for all $x \in I$ then $D = 0$.

Proof: We have $d(x) = 0$, for all $x \in I$.

(1)

We replace x by $x + y$ in (1), we get

$d(x + y) = 0$

$d(x) + d(y) + 2D(x, y) = 0$

By using (1) in the above equation we get $2D(x, y) = 0$

Since R is 2-torsion free, which implies that,

$D(x, y) = 0$, for all $x, y \in I$.

(2)

We replace y by yr in (2), we get

$D(x, yr) = 0$

$D(x, r)y + rD(x, y) = 0$

By using (2) in the above equation we get

$D(x, r)y = 0$, for all $x, y, \in I$ and $r \in R$.

(3)

We replace x by xs in (3), we get

$D(xs, r)y = 0$

$\big(D(s, r)x + sD(x, r)\big)y = 0$

$D(s, r)xy + sD(x, r)y = 0$

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By using (3) in the above equation we get
$D(s,r)xy = 0$, for all $x, y, \in I$ and $r \in R$.
$D(s,r)R[x,y] = 0$, for all $x, y, \in I$ and $r \in R$.
Since R is prime and non commutative ring, which implies $D(s,r) = 0$, for all $s, r \in R$.

Theorem 1: Let R be a 2 and 3-torsion free prime ring. Suppose there exist a symmetric reverse bi-derivations $D_1(.,.): R \times R \to R$ and $D_2(.,.): R \times R \to R$ such that $d_1(x)d_2(x) = 0$, for all $x \in R$, where d_1 and d_2 are the traces of D_1 and D_2 respectively. In this case either $D_1 = 0$ or $D_2 = 0$.

Proof: We have $d_1(x)d_2(x) = 0$, for all $x \in R$.

(4)

We replace x by $x + y$ in (4), we get
$d_1(x+y)d_2(x+y) = 0$
$(d_1(x) + d_1(y) + 2D_1(x,y))(d_2(x) + d_2(y) + 2D_2(x,y)) = 0$
$d_1(x)d_2(x) + d_1(x)d_2(y) + 2d_1(x)D_2(x,y) + d_1(y)d_2(x) + d_1(y)d_2(y) + 2d_1(y)D_2(x,y) + 2D_1(x,y)d_2(x) + 2D_1(x,y)d_2(y) + 4D_1(x,y)D_2(x,y) = 0$

By using (4) in the above equation we get
$d_1(x)d_2(y) + 2d_1(x)D_2(x,y) + d_1(y)d_2(x) + 2d_1(y)D_2(x,y) + 2D_1(x,y)d_2(x) + 2D_1(x,y)d_2(y) + 4D_1(x,y)D_2(x,y) = 0$
, for all $x, y \in R$. (5)

We replace x by $-x$ in (5), we get
$d_1(-x)d_2(y) + 2d_1(-x)D_2(-x,y) + d_1(y)d_2(-x) + 2d_1(y)D_2(-x,y) + 2D_1(-x,y)d_2(-x) + 2D_1(-x,y)d_2(y) + 4D_1(-x,y)D_2(-x,y) = 0$

$d_1(x)d_2(y) - 2d_1(x)D_2(x,y) + d_1(y)d_2(x) - 2d_1(y)D_2(x,y) - 2D_1(x,y)d_2(x) - 2D_1(x,y)d_2(y) + 4D_1(x,y)D_2(x,y) = 0$
, for all $x, y \in R$. (6)

By adding (5) and (6) we get
$d_1(x)d_2(y) + d_1(y)d_2(x) + 4D_1(x,y)D_2(x,y) = 0$, for all $x, y \in R$. (7)

By subtracting (6) from (5) we get
$d_1(x)D_2(x,y) + d_1(y)D_2(x,y) + D_1(x,y)d_2(x) + D_1(x,y)d_2(y) = 0$
, for all $x, y \in R$. (8)

We replace x by $2x$ in (8), we get
$d_1(2x)D_2(2x,y) + d_1(y)D_2(2x,y) + D_1(2x,y)d_2(2x) + D_1(2x,y)d_2(y) = 0$

$8d_1(x)D_2(x,y) + 2d_1(y)D_2(x,y) + 8D_1(x,y)d_2(x) + 2D_1(x,y)d_2(y) = 0$

$4d_1(x)D_2(x,y) + d_1(y)D_2(x,y) + 4D_1(x,y)d_2(x) + D_1(x,y)d_2(y) = 0$
, for all $x, y \in R$. (9)

By subtracting (8) from (9), we get
$3d_1(x)D_2(x,y) + 3D_1(x,y)d_2(x) = 0$
$d_1(x)D_2(x,y) + D_1(x,y)d_2(x) = 0$, for all $x, y \in R$.
(10)

$d_1(x)D_2(x,y) = -D_1(x,y)d_2(x)$
$D_1(x,z)d_2(x) = -d_1(x)D_2(x,z)$, for all $x, y, z \in R$. (11)

We replace y by zy in (10), we get
$d_1(x)D_2(x,zy) + D_1(x,zy)d_2(x) = 0$
$d_1(x)\big(D_2(x,y)z + yD_2(x,z)\big) + \big(D_1(x,y)z + yD_1(x,z)\big)d_2(x) = 0$

$d_1(x)D_2(x,y)z + d_1(x)yD_2(x,z) + D_1(x,y)z\, d_2(x) + yD_1(x,z)d_2(x) = 0$

By using (11) in the above equation we get
$-D_1(x,y)d_2(x)z + d_1(x)yD_2(x,z) + D_1(x,y)z\, d_2(x) - yd_1(x)D_2(x,z) = 0$

$D_1(x,y)z\, d_2(x) - D_1(x,y)d_2(x)z + d_1(x)yD_2(x,z) - yd_1(x)D_2(x,z) = 0$

$D_1(x,y)(z\, d_2(x) - d_2(x)z) + \big(d_1(x)y - yd_1(x)\big)D_2(x,z) = 0$
$D_1(x,y)[z, d_2(x)] + [d_1(x), y]D_2(x,z) = 0$, for all $x, y, z \in R$.
(12)

We replace in particular $z = d_2(x)$ in (12), we get
$D_1(x,y)[d_2(x), d_2(x)] + [d_1(x), y]D_2(x, d_2(x)) = 0$
$[d_1(x), y]D_2(x, d_2(x)) = 0$, for all $x, y \in R$.
(13)

Let us assume that D_1 and D_2 both different from zero. In this case there exist $a \in R$ such that $D_2(a, d_2(a)) \neq 0$; otherwise D_2 would be zero by theorem 4 in [4]. Since $D_2\big(a, d_2(a)\big) \neq 0$, it follows from (13) and Lemma 1 that $d_1(a) \in Z(R)$ (Note that $y \to [d_1(a), y]$ is an inner derivation). That is $[d_1(a), y] = 0$
$d_1(a)y - y\, d_1(a) = 0$
$d_1(a)y = y\, d_1(a)$, for all $y \in R$.
(14)

Now left multiplication of (4) by y gives us
$yd_1(a)d_2(a) = 0$
By using (14) in the above equation we get
$d_1(a)yd_2(a) = 0$, for all $y \in R$.
(15)

From (15) it follows that either $d_1(a) = 0$ or $d_2(a) = 0$ by the primeness of R. But $d_2(a)$ cannot be zero since $D_2\big(a, d_2(a)\big) \neq 0$; hence we have $d_1(a) = 0$.
Now we replace x by a in (10), we get
$d_1(a)D_2(a,y) + D_1(a,y)d_2(a) = 0$
$D_1(a,y)d_2(a) = 0$, for all $y \in R$.
(16)

From (16) and Lemma 1 we conclude that $D_1(a,y) = 0$, for all $y \in R$, since $d_2(a) \neq 0$ (recall that $y \to D_1(a,y)$ is a derivation).
Now we replace y by a in (7), we get
$d_1(x)d_2(a) + d_1(a)d_2(x) + 4D_1(x,a)D_2(x,a) = 0$
$d_1(x)d_2(a) = 0$, for all $x \in R$.
(17)

We replace x by $x + y$ in (17), we get
$d_1(x+y)d_2(a) = 0$
$d_1(x)d_2(a) + d_1(y)d_2(a) + 2D_1(x,y)d_2(a) = 0$
By using (17) in the above equation we get
$2D_1(x,y)d_2(a) = 0$
$D_1(x,y)d_2(a) = 0$, for all $x, y \in R$, which implies $D_1 = 0$ according to Lemma 1, since $d_2(a) \neq 0$. But $D_1 = 0$ is contrary to our assumption. This contradiction completes the proof.

Theorem 2: Let R be a 2 and 3-torsion free semiprime ring. Suppose there exist a symmetric reverse bi-derivation $D(.,.): R \times R \to R$ such that $d(x)d(x) = 0$, for all $x \in R$, where d is the trace of D. In this case $D = 0$.

Proof: We have $d(x)d(x) = 0$, for all $x \in R$.
(18)

We replace x by $x + y$ in (18), we get
$d(x+y)d(x+y) = 0$
$(d(x) + d(y) + 2D(x,y))(d(x) + d(y) + 2D(x,y)) = 0$
$d(x)d(x) + d(x)d(y) + 2d(x)D(x,y) + d(y)d(x) + d(y)d(y) + 2d(y)D(x,y) + 2D(x,y)d(x) + 2D(x,y)d(y) + 4D(x,y)D(x,y) = 0$

By using (18) in the above equation, we get
$d(x)d(y) + 2d(x)D(x,y) + d(y)d(x) + 2d(y)D(x,y) + 2D(x,y)d(x) + 2D(x,y)d(y) + 4D(x,y)D(x,y) = 0$
, for all $x, y \in R$.
(19)

We replace x by $-x$ in (19), we get
$d(-x)d(y) + 2d(-x)D(-x,y) + d(y)d(-x) + 2d(y)D(-x,y) + 2D(-x,y)d(-x) + 2D(-x,y)d(y) + 4D(-x,y)D(-x,y) = 0$

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$d(x)d(y) - 2d(x)D(x,y) + d(y)d(x) - 2d(y)D(x,y) - 2D(x,y)d(x) - 2D(x,y)d(y) + 4D(x,y)D(x,y) = 0$, for all $x,y \in R$. (20)

By adding (19) and (20), we get
$d(x)d(y) + d(y)d(x) + 4D(x,y)D(x,y) = 0$, for all $x,y \in R$. (21)

By subtracting (20) from (19), we get
$d(x)D(x,y) + d(y)D(x,y) + D(x,y)d(x) + D(x,y)d(y) = 0$, for all $x,y \in R$. (22)

We replace x by $2x$ in (22), we get
$d(2x)D(2x,y) + d(y)D(2x,y) + D(2x,y)d(2x) + D(2x,y)d(y) = 0$

$8d(x)D(x,y) + 2d(y)D(x,y) + 8D(x,y)d(x) + 2D(x,y)d(y) = 0$

$4d(x)D(x,y) + d(y)D(x,y) + 4D(x,y)d(x) + D(x,y)d(y) = 0$, for all $x,y \in R$. (23)

By subtracting (22) from (23), we get
$3d(x)D(x,y) + 3D(x,y)d(x) = 0$
$d(x)D(x,y) + D(x,y)d(x) = 0$, for all $x,y \in R$. (24)

$D(x,z)d(x) = -d(x)D(x,z)$ and $d(x)D(x,y) = -D(x,y)d(x)$, for all $x,y \in R$. (25)

We replace y by zy in (24), we get
$d(x)D(x,zy) + D(x,zy)d(x) = 0$
$d(x)\big(D(x,y)z + yD(x,z)\big) + \big(D(x,y)z + yD(x,z)\big)d(x) = 0$
$d(x)D(x,y)z + d(x)yD(x,z) + D(x,y)z\,d(x) + yD(x,z)d(x) = 0$

By using (25) in the above equation, we get
$-D(x,y)d(x)z + d(x)yD(x,z) + D(x,y)z\,d(x) - yd(x)D(x,z) = 0$

$D(x,y)z\,d(x) - D(x,y)d(x)z + d(x)yD(x,z) - yd(x)D(x,z) = 0$

$D(x,y)(z\,d(x) - d(x)z) + \big(d(x)y - yd(x)\big)D(x,z) = 0$
$D(x,y)[z, d(x)] + [d(x), y]D(x,z) = 0$, for all $x,y,z \in R$. (26)

We replace in particular $z = d(x)$ in (26), we get
$D(x,y)[d(x),d(x)] + [d(x),y]D(x,d(x)) = 0$
$[d(x),y]D(x,d(x)) = 0$, for all $x,y \in R$. (27)

Let us assume that D different from zero. In this case there exist $a \in R$ such that $D(a,d(a)) \neq 0$. Since $D\big(a,d(a)\big) \neq 0$, it follows from (27) and Lemma 1, $[d(a),y] = 0$
$d(a)y - y\,d(a) = 0$, for all $a,y \in R$. (28)

We replace y by xy in (24), we get
$d(x)D(x,xy) + D(x,xy)d(x) = 0$
$d(x)\big(D(x,y)x + yd(x)\big) + \big(D(x,y)x + yd(x)\big)d(x) = 0$
$d(x)D(x,y)x + d(x)yd(x) + D(x,y)xd(x) + yd(x)d(x) = 0$, for all $x,y \in R$.
By using (18) and (25) in the above equation, we get
$d(x)yd(x) + D(x,y)xd(x) - D(x,y)d(x)x = 0$
$d(x)yd(x) + D(x,y)(xd(x) - d(x)x) = 0$, for all $x,y \in R$.
By using (28) in the above equation, we get
$d(x)yd(x) = 0$, for all $x \in R$.
Since R is semiprime, which implies that $d(x) = 0$, for all $x \in R$. (29)

We replace x by $x+y$ in (29), we get
$d(x+y) = 0$
$d(x) + d(y) + 2D(x,y) = 0$, for all $x \in R$.
By using (29) in the above equation, we get
$D(x,y) = 0$, for all $x,y \in R$.

Theorem 3: Let R be a 2-torsion free non-commutative prime ring and I be a nonzero ideal of R. Suppose there exist a symmetric reverse bi-derivations $D_1(.,.): R \times R \to R$ and $D_2(.,.): R \times R \to R$ such that $d_1(x)d_2(x) = 0$, for all $x \in I$ holds, where d_1 and d_2 are the traces of D_1 and D_2 respectively. In this case either $D_1 = 0$ or $D_2 = 0$.

Proof: We have from (13) of Theorem 1,
$[d_1(x),y]D_2(x,d_2(x)) = 0$, for all $x,y \in I$.
We replace y by yz in above equation, we get
$[d_1(x),yz]D_2(x,d_2(x)) = 0$
$y[d_1(x),z]D_2(x,d_2(x)) + [d_1(x),y]zD_2(x,d_2(x)) = 0$
By using (13) in the above equation, we get
$[d_1(x),y]zD_2(x,d_2(x)) = 0$, for all $x,y,z \in I$. This implies that $[d_1(x),y]zRD_2(x,d_2(x)) = 0$, for all $x,y,z \in I$. Primeness of R yields that either $[d_1(x),y]z = 0$ or $D_2(x,d_2(x)) = 0$, for all $x,y,z \in I$. If $D_2(x,d_2(x)) = 0$, for all $x \in I$, then conclusion follows from by theorem 4 in [4]. Now consider the case when $[d_1(x),y]z = 0$, for all $x,y,z \in I$. Primeness of R yields that $[d_1(x),y] = 0$, for all $x,y \in I$. (30)

We replace x by $x+u$ in (30), we get
$[d_1(x+u),y] = 0$
$[d_1(x) + d_1(u) + 2D_1(x,u),y] = 0$
$[d_1(x),y] + [d_1(u),y] + [2D_1(x,u),y] = 0$
By using (30) in the above equation, we get
$2[D_1(x,u),y] = 0$, for all $x,y,u \in I$.
$[D_1(x,u),y] = 0$, for all $x,y,u \in I$. (31)

We replace x by xz in (31), we get
$[D_1(xz,u),y] = 0$
$[D_1(z,u)x + zD_1(x,u),y] = 0$
$[D_1(z,u)x,y] + [zD_1(x,u),y] = 0$
$[D_1(z,u),y]x + D_1(z,u)[x,y] + [z,y]D_1(x,u) + z[D_1(x,u),y] = 0$

By using (31) in the above equation, we get
$[D_1(z,u)[x,y] + [z,y]D_1(x,u) = 0$, for all $x,y,u,z \in I$. (32)

We replace y by x in (32), we get
$D_1(z,u)[x,x] + [z,x]D_1(x,u) = 0$
$[z,x]D_1(x,u) = 0$, for all $x,u,z \in I$. (33)

We replace z by zv in (33), we get
$[zv,x]D_1(x,u) = 0$
$[z,x]vD_1(x,u) + z[v,x]D_1(x,u) = 0$
By using (33) in the above equation, we get
$[z,x]vD_1(x,u) = 0$, for all $x,u,v \in I$. Since R is noncommutative prime ring, which implies that $D_1(x,u) = 0$, for all $x,u \in I$. Application of Lemma 2 gives that $D_2 = 0$.

Theorem 4: Let R be a 2-torsion free semiprime ring and I be a nonzero ideal of R. Let $D(.,.): R \times R \to R$ be a symmetric reverse bi-derivation such that $D(I,I) \subseteq I$. If d is a trace of D such that $D(d(x),x) = 0$, for all $x \in I$, then $D = 0$ on I.

Proof: We have $D(d(x),x) = 0$, for all $x \in I$. (34)

We replace x by $x+y$ in (34), we get
$D(d(x+y),x+y) = 0$
$D(d(x) + d(y) + 2D(x,y),x+y) = 0$
$D(d(x),x) + D(d(x),y) + D(d(y),x) + D(d(y),y) + 2D(D(x,y),x) + 2D(D(x,y),y) = 0$
By using (34) in the above equation, we get
$D(d(x),y) + D(d(y),x) + 2D(D(x,y),x) + 2D(D(x,y),y) = 0$, for all $x,y \in I$. (35)
We replace y by $-y$ in (35), we get
$D(d(x),-y) + D(d(-y),x) + 2D(D(x,-y),x) + 2D(D(x,-y),-y) = 0$

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$-D(d(x),y) + D(d(y),x) - 2D(D(x,y),x) + 2D(D(x,y),y) = 0$

, for all $x, y \in I$. (36)

By adding (35) and (36), we get

$2D(d(y),x) + 4D(D(x,y),y) = 0$

$D(d(y),x) + 2D(D(x,y),y) = 0$, for all $x, y \in I$. (37)

We replace x by zx in (37), we get

$D(d(y),zx) + 2D(D(zx,y),y) = 0$

$D(d(y),zx) + 2D(D(x,y)z + xD(z,y),y) = 0$

$D(d(y),zx) + 2D(D(x,y)z,y) + 2D(xD(z,y),y) = 0$

$D(d(y),x)z + xD(d(y),z) + 2D(z,y)D(x,y) + 2zD(D(x,y),y) + 2D(D(z,y),y)x + 2D(z,y)D(x,y) = 0$

$D(d(y),x)z + xD(d(y),z) + 2zD(D(x,y),y) + 2D(D(z,y),y)x + 4D(z,y)D(x,y) = 0$

, for all $x, y, z \in I$. (38)

We multiply (37) by z on left hand side, we get

$zD(d(y),x) + 2zD(D(x,y),y) = 0$, for all $x, y, z \in I$. (39)

Subtract (39) from (38), we get

$[D(d(y),x),z] + xD(d(y),z) + 2D(D(z,y),y)x + 4D(z,y)D(x,y) = 0$

, for all $x, y, z \in I$. (40)

We replace x by z in (37), we get

$D(d(y),z) + 2D(D(z,y),y) = 0$, for all $x, z \in I$. (41)

By multiplying (41) by $x$on right hand side, we get

$D(d(y),z)x + 2D(D(z,y),y)x = 0$, for all $x, y, z \in I$. (42)

Subtract (42) from (40), we get

$[D(d(y),x),z] + [x, D(d(y),z)] + 4D(z,y)D(x,y) = 0$

$[D(d(y),x),z] - [D(d(y),z),x] + 4D(z,y)D(x,y) = 0$, for all $x, y, z \in I$. (43)

We replace x by z and z by x in (43), we get

$[D(d(y),z),x] - [D(d(y),x),z] + 4D(x,y)D(z,y) = 0$, for all $x, y, z \in I$. (44)

By adding (43) and (44), we get

$4D(z,y)D(x,y) + 4D(x,y)D(z,y) = 0$

Since R is 2-torsion free, we have

$D(z,y)D(x,y) + D(x,y)D(z,y) = 0$, for all $x, y, z \in I$. (45)

We replace z by x and y by x in (45), we get

$D(x,x)D(x,x) + D(x,x)D(x,y) = 0$

$d(x)d(x) + d(x)d(x) = 0$

$2d(x)d(x) = 0$

$d(x)d(x) = 0$, for all $x \in I$.

Hence theorem 2 completes the proof.

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