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PROPERTIES OF REGULAR SEMIGROUPS

Dr D.Mrudula devi
Professor in Mathematics,
H&BS Department,
, Aditya college of Engineering and Technology,
suram palem,E.G (dt) A.P .

DrG.Shobhalatha
Professor in Mathematics,
Department Of Mathematics,
S.K.University,
Ananthapur, A.P

Abstract: In this paper we proved that a regular semigroup (S,.) is μ -Inverse then it is E – Inverse. It is also proved that (S,.) is left regular semigroup it is GC- Commutative semigroup and left permutable. In the same way if (S,.) be a commutative left regular and left zero semigroup then S is H-commutative if it is regular. On the other hand (S,.) be completely regular semigroup then (S,.)is H-commutative if (s,.) is Externally Commutative left Zero Semigroup. It is also observe that a semigroup (S,.) with different properties satisfies some equivalent conditions of regular semigroup. The motivation to prove the theorems in this paper due to results J.M.Howie[2] and P.Srinivasulu Reddy, G.Shobhalatha[3].

***keywords*:** Regular semigroup , μ -Inverse , E – Inverse, GC- Commutative semigroup, H-commutative, Externally Commutative, left Zero Semigroup

INTRODUCTION

Various concepts of regularity on semigroup have been investigated by R.Croisot. His studieshave been presented in the book of A.H.Clifford and G.B.priston[1] as R.Croisots theory.One of the central places in this theory held by the left(right) regularity. One area of research in the field of semigroup theory in which there have been significant success in recent years has been the subject of completely regular semigroups. The aim of this chapteris to give a brief review of some of achievements in the theory of completely regular and regular semigroups.

Properties of Regular semigroups

In this paper we present preliminaries and basic concept of regular semigroups.

1.1. Definition: An element a of a semigroup (S,.) is left (right) regular if there exists an element x in S such that $xa^2 = a(a^2x = a)$.

1.2. Definition : An element a of a semigroup (S,.) is said to be regular if there exist x in S such that a x a = a**.**

1.3. Definition : A semigroup (S,.) is called regular if every element of S is regular

1.4 Definition : A semigroup (S,.) is said to be μ -inverse semigroup if baxc = bc and byac = bc, for all x, y, a, b, $c \in S$

1.5 Definition : A semigroup (S,.) is said to be GC-commutative semigroup if $x^2yx = xyx^2$ for all x, $y \in S$

1.6. Definition : A semigroup (S,.) is said to be left permutable if axb = xab for all a, b, $x \in S$

1.7 Definition : A semigroup (S,.) is said to be H-commutative if ab = bxa for all a, b, $x \in S$

1.8 Definition : A semigroup (S,.) is said to be R-commutative if ab = bax for all a, b, $x \in S$

1.9 Definition : A semigroup (S, .) is said to be weakly balance semigroup if ax = bx, ya = yb for all a, b, x, $y \in S$

1.10 Definition : A semigroup (S, .) is said to be externally commutative if axb = bxa, $\forall$ a, b, $x \in S$

1.11 Definition: A semigroup (S, .) is said to be completely regular if a = axa and ax = xa. for all a, b, $x \in S$

1.12 Theorem : If (S, .) is regular semigroup. If S is μ -inversive semigroup then it is E-inversive

Proof: Given that (S, .) is a regular semigroup.

Suppose (S, .) is μ -inversive semigroup

i.e baxc = bc and byac = bc $\forall$ a, b, x, $y \in S$

To prove that S is E-inversive semigroup.

Let $b \in S$ then $\exists\, c \in S$ such that

$$\begin{aligned}(bc)^2 &= (bc)(bc)\\ &= (baxc)(bc)\\ &= bax\,(cbc)\\ &= baxc\\ &= bc\end{aligned}$$

Or

$$\begin{aligned}(bc)^2 &= (bc)(bc)\\ &= (byac)(bc)\\ &= bya\,(cbc)\\ &= byac\\ &= bc\end{aligned}$$

$\Rightarrow$ bc is an E-inversive element in S.

Hence (S, .) is an E-inversive semigroup.

1.13 Theorem : If (S, .) is left(right) regular semigroup then it is GC-commutative semigroup.

Proof: Let (S, .) be a left regular semigroup.

i.e $x = yx^2$ or $y = xy^2$ for any x, $y \in S$

To prove that S is GC-commutative

Let
$$\begin{aligned}x^2yx &= x^2y(yx^2)\\ &= x(xy)yx^2\\ &= x(xy^2)x^2\\ &= xyx^2\end{aligned}$$

Similarly if (S, .) is right regular then $x = x^2y$ or $y = y^2x$

Let
$$\begin{aligned}xyx^2 &= x^2y(yx^2)\\ &= x^2y\,yx.x\\ &= x^2y^2x\,x\\ &= x^2yx\end{aligned}$$

$\therefore$ ((S, .) is GC-commutative.

1.14 Theorem : Every left regular semigroup is left permutable.

Pro **proof:** Let (S, .) be a left regular semigroup.

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To prove that S is left permutable.

i.e axb = xba for all $x,a,b \in S$

Conconsider axb $= (xa^2)xb$ $(xa^2 = a)$

$= x(a^2x)b$

$= xab$ $(a^2x = a)$

$\therefore$ (S, .) is left permutable

1.15. Theorem : Let (S, .) be a commutative left regular and left zero semigroup then S is H-commutative iff It is regular

Proproof : Given that (S, .) is H-commutative i.e ab = bxa for all a, b, $x \in S$

To prove that (S, .) is regular

Let ab = bxa

$\Rightarrow$ $aba = bxa^2$

$\Rightarrow$ = ba ((S, .) is left regular)

$\Rightarrow$ aba = a ((S, .) is left zero)

$\therefore$ (S, .) is regular

Conconversely Let (S, .) be a regular semigroup

To pprove that it is H-commutative

Now $a \in S \Rightarrow$ a is regular $\Rightarrow \exists\ x \in S$ such that axa = a

ab = (axa)b

= a(xa)b

= a(ax)b

= axb (ax = x)

= a(bx)

= bax

$\Rightarrow$ ab = bxa

Hence (S, .) is H-commutative.

1.16 Theorem : Every regular commutative left zero semigroup is R-commutative

Proof : Given that (S, .) is regular semigroup.

$a \in S$ is regular $\Rightarrow \exists\ x \in S$ such that axa = a

To prove that (S, .) is R-commutative.

i.e., ab = bax for all $a,b,x \in S$

Consider ab = (axa)b

= a(xa)b

= aaxb

$= a^2(xb)$

$= a^2(bx)$

$= (ba^2)x$

= (ba)ax

= bax

$\therefore$ (S, .) is R-commutative.

1.17 Theorem : Let (S, .) be a completely regular semigroup. If (S, .) is externally commutative left zero semigroup then it is H-commutative.

Proof : Let (S, .) be a completely regular semigroup.

To prove that (S, .) is H-commutative

Let ab = a(bxb) (b is regular $\Rightarrow \exists\ x \in S$ such that bxb = b)

= a(bx)b

= axb (bx = x)

= bxa ($\therefore$ (S, .) is externally commutative)

$\therefore$ ab = bxa

Hence (S, .) is H-commutative

1.18 Theorem : Let (S, .) be a weakly balance semigroup. If (S, .) is completely regular semigroup then it is μ-inverse.

Proof : Let (S, .) be weakly balance semigroup

$\Rightarrow$ ax = bx, ya = yb for all a, b, x, $y \in S$

Given that (S,.) is completely regular

i.e bxb = b, bx = xb for all x, y, $b \in S$

To Prove that (S,.) is μ-inverse

i.e., baxc = bc and byac = bc

Let baxc = b(bx)c ($\therefore$ ax = bx)

= b(xb)c (completely regular)

= (bxb)c = bc

byac = b(ya)cr

= b(yb)c

= (byb)c

$\therefore$ byac = bc

Hence (S,.) is μ-inverse semigroup.

1.19. Theorem : If (S, .) is GC-commutative semigroup. Then every (1, 2) regular semigroup is (2, 1) regular semigroup.

Proproof: Let (S, .) be GC-commutative semigroup.

i.e $a^2xa = axa^2$ for all a, $x \in S$

Let (S,.) be (1,2) regular semigroup. Then $axa^2 = a$ ---------(1)

but $axa^2 = a^2xa$ ----------(2)

from (1) and (2) we have $a^2xa = a$

(S, .) is (2,1) regular semigroup.

1.20. Theorem : Every regular semigroups is GC-commutative

Proof : Given that (S,.) be a regular semigroup

i.e xyx = x for all x, $y \in S$

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To prove that (S,.) is GC-commutative

Now let $x^2yx = x(xyx)$

$= x.x$

$= (xyx)\, x$

$x^2yx \quad = xyx^2$

Hence (S,.) is GC-commutative.

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