



## EFFECT OF VISCOUS DISSIPATION ON POWER LAW - FLUID PAST A PERMEABLE STRETCHING SHEET IN A POROUS MEDIA

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**ABSTRACT:** The steady, two dimensional, laminar flow of a power - law fluid over a permeable a stretching sheet in a porous media with constant heat flux in the presence of MHD, viscous dissipation. Using the similarity transformations, the governing equations have been transformed into a system of ordinary differential equations. These differential equations are highly nonlinear which cannot be solved analytically. Therefore, bvp4c MATLAB solver has been used for solving it. Numerical results are obtained for the skin-friction coefficient and the local Nusselt number as well as the velocity and temperature profiles for different values of the governing parameters, namely permeability parameter, power-law index parameter, Eckert number, Prandtl number.

**Keywords:** power-law fluid, viscous dissipation, stretching sheet, porous media, constant heat flux.

### 1. INTRODUCTION

Since 1960, a considerable attention has been devoted to predict the drag force behavior and energy transport characteristics of the non-Newtonian fluid flows. The main reason for this is probably that fluids (such as molten plastics, pulps, slurries, emulsions), which do not obey the assumption of Newtonian fluids that the stress tensor is directly proportional to the deformation tensor, are found in various engineering applications. A variety of constitutive equations have been proposed to describe the flow and heat transfer non-Newtonian characteristics, among them the empirical Ostwald-de Waele model, which is known as the so-called power-law model, gained much acceptance. Schowalter [1] and Acrivos et al. [2] successfully applied the power-law model to the boundary layer problems. Kumari et al. [3] investigated the non-similar mixed convection flow of a non-Newtonian fluid past a vertical wedge. Nadeem and Akbar [4] investigated the peristaltic flow of Walter's B fluid in a uniform inclined tube. Mahmoud [5] investigated the effects of surface slip and heat generation (absorption) on the flow and heat transfer of a non-Newtonian power-law fluid on a continuously moving surface. Olanrewaju et al. [6] investigated the thermal and thermo diffusion on convection heat and mass transfer in a power law flow over a heated porous plate in the presence of magnetic field. Yacob and Ishak [7] investigated the laminar flow of a power-law fluid over a permeable shrinking sheet of constant surface temperature. Aziz et al. [8] conducted the study of forced convective boundary layer flow of power-law fluid along with heat transfer over a porous plate in a porous medium. Rashidi et al. [9] analyzed the convective flow of a third grade non-Newtonian fluid due to a linearly stretching sheet subject to a magnetic field.

Hady and Hassanien [10] studied the magneto hydrodynamic and constant suction/injection effects of axisymmetric stagnation point flow and mass transfer for power-law fluids. Jadhav and Waghmode [11] investigated the effect of suction is to decrease in temperature and the rate of heat

transfer, while reverse nature occurs for injection. Sahu and Mathu [12] concluded that the suction influence decreases the skin-friction. Olanrewaju and Makinde [13] studied the free convective heat and mass transfer fluid past a moving vertical plate in the presence of suction and

injection with thermal diffusion (Soret) and diffusion-thermo (Dufour) effects. Rosali et al. [14] studied the micropolar fluid flow towards a stretching/shrinking sheet in a porous medium with suction. Raju and Varma [15] investigated the unsteady MHD free convective of non-Newtonian fluid through porous medium bounded by an infinite porous plate in the presence of constant suction. Chinyoka and Makinde [16] analyzed the unsteady and porous media flow of reactive non-Newtonian fluids subjected to buoyancy and suction/injection.

The heat source/sink effects in thermal convection, are significant where there may exist a high temperature differences between the surface (e.g. space craft body) and the ambient fluid. Heat generation is also important in the context of exothermic or endothermic chemical reactions. Singh et al. [17] investigated the effect of volumetric heat generation/absorption on mixed convection stagnation point flow on an isothermal vertical plate in porous media. Das and his co-workers [18] analyzed the effect of mass transfer on MHD flow and heat transfer past a vertical porous plate through a porous medium under oscillatory suction and heat source. More recently, Das et al. [19] investigated the hydromagnetic convective flow past a vertical porous plate through a porous medium with suction and heat source. Khan [20] studied the effect of heat transfer on a viscoelastic fluid flow over a stretching sheet with heat source/sink, suction/blowing and radiation. Pal and Talukdar [21] studied the unsteady MHD heat and mass transfer along with heat source past a vertical permeable plate using a perturbation analysis, where the unsteadiness is caused by the time dependent surface temperature and concentration.

Viscous dissipation which, appears as a source term in the fluid flow generates appreciable temperature, gives the rate at which mechanical energy is converted into heat in a viscous

fluid per unit volume. This effect is of particular significant in natural convection in various devices that are subjected to large variation of gravitational force or that operate at high rotational speeds, pointed by Gebhart [22] in his study of viscous dissipation on natural convection in fluids. Lawal and Mujumdar [23] investigated the forced convection heat transfer to power-law fluids in arbitrary cross-sectional ducts with finite viscous dissipation. Kairi et al. [24] concluded that the heat transfer coefficient increases with increasing in the power law index  $n$  and viscosity parameter, while it decreases with the dissipation parameter. Boubaker et al. [25] investigated the effects of viscous dissipation on the thermal boundary layer of pseudoplastic power-law non-Newtonian fluids.

The present study investigates the steady, two dimensional, laminar flow of a power - law fluid over a stretching sheet with constant heat flux in the presence of MHD, viscous dissipation and porous medium. Using the similarity transformations, the governing equations have been transformed into a set of ordinary differential equations, which are nonlinear and cannot be solved analytically, therefore, byp4c MATLAB solver has been used for solving it. The results for velocity and temperature functions are carried out for the wide range of important parameters namely; namely, suction/injection parameter, power-law index parameter, convective parameter, Prandtl number and Eckert number. The skin friction, the couple wall stress and the rate of heat transfer have also been computed.

**2. MATHEMATICAL FORMULATION**

Consider a steady, two-dimensional laminar viscous flow of a power-law fluid over a permeable shrinking sheet of constant surface temperature  $T_w$  as shown in Fig. 1. The shrinking velocity is assumed to be of the form  $U_w(x) = ax^{1/3}$ , where  $a$  is a positive constant. A strong magnetic field  $B = (0, B_0, 0)$  is applied in the  $y$  direction.

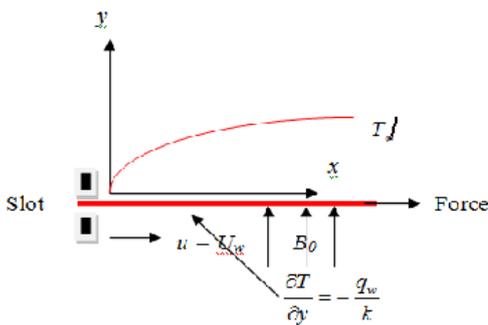


Figure 1: Flow geometry and coordinate system

The  $x$ -axis extends parallel, while the  $y$ -axis extends upwards, normal to the surface of the sheet. The boundary layer equations are :

Continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2.1}$$

Momentum equation

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{1}{\rho} \frac{\partial \tau}{\partial y} - \frac{\sigma B_0^2}{\rho} u - \frac{\nu}{k^*} u \tag{2.2}$$

Energy equation

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \frac{\nu}{c_p} \left( \frac{\partial u}{\partial y} \right)^{n+1} \tag{2.3}$$

The boundary conditions are

$$u = U_w(x), v = V_w(x), \frac{\partial T}{\partial y} = -\frac{q_w}{k} \quad \text{at } y = 0$$

$$u \rightarrow 0, T \rightarrow T_\infty \quad \text{as } y \rightarrow \infty$$

(2.4)

where  $u$  and  $v$  are the velocity components along the  $x$  and  $y$  directions, respectively,  $\tau_{xy}$  is the shear stress,  $h$  is the convective heat transfer coefficient,  $Q_0$  is the heat source/sink coefficient,  $k$  is the thermal conductivity of the fluid,  $c_p$  is the specific heat,  $\nu$  is the kinematic viscosity of the fluid,  $\rho$  is the fluid density and  $V_w(x)$  (which will be defined later) is the mass transfer velocity at the surface of the sheet.

The stress tensor is defined as (Andersson and Irgens [26]; Wilkinson [27]),

$$\tau_{ij} = 2K \left( 2D_{ij}D_{kl} \right)^{(n-1)/2} D_{ij} \tag{2.5}$$

Where

$$D_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \tag{2.6}$$

denotes the stretching tensor,  $K$  is called the consistency coefficient and  $n$  is the power-law index. The index  $n$  is non-dimensional and the dimension of  $K$  depends on the value of  $n$ . The two-parameter rheological (2.5) is known as the Ostwald-de-Waele model or, more commonly, the power-law model. The parameter  $n$  is an important index to subdivide fluids into pseudoplastic fluids ( $n < 1$ ) and dilatant fluids ( $n > 1$ ). For  $n = 1$ , the fluid is simply the Newtonian fluid. Therefore, the deviation of  $n$  from unity indicates the degree of deviation from Newtonian behavior (Wang [28]). With  $n \neq 1$ , the constitutive (2.5) represents shear-thinning ( $n < 1$ ) and shear-thickening ( $n > 1$ ) fluids. Using (2.5) and (2.6), the shear stress appearing in (2.2) can be written as:

$$\tau_{xy} = K \left( \frac{\partial u}{\partial y} \right)^n \tag{2.7}$$

Now the momentum (2.2) becomes:

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{K}{\rho} \frac{\partial}{\partial y} \left( \frac{\partial u}{\partial y} \right)^n - \frac{\sigma B_0^2}{\rho} u - \frac{\nu}{k^*} u \tag{2.8}$$

The continuity (2.1) is satisfied by introducing a stream function  $\psi$  such that

$$u = \frac{\partial \psi}{\partial y} \text{ and } v = -\frac{\partial \psi}{\partial x} \tag{2.9}$$

The momentum (2.8) and the energy (2.3) can be transformed into the corresponding ordinary differential equations by the following transformation (Xu & Liao [29]):

$$\eta = \frac{y}{x} (\text{Re}_x)^{1/(n+1)}, \psi = U_w x (\text{Re}_x)^{-1/(n+1)} f(\eta) \tag{2.10}$$

$$\theta(\eta) = \frac{k}{xq_w} (T - T_\infty) (\text{Re}_x)^{1/(n+1)}$$

Where  $\eta$  is the similarity variable,  $f(\eta)$  is the dimensionless stream function and  $\text{Re}_x = \rho x^n U_w^{2-n} / K$  is the local Reynolds number.

Thus the mass transfer velocity  $V_w(x)$  may be defined as:

$$V_w(x) = -\frac{2}{3} a \left( \frac{\rho a^{2-n}}{K} \right)^{1/(n+1)} x^{-1/3} f_w \tag{2.11}$$

where  $f_w$  is the suction/injection parameter with  $f_w > 0$  is for suction,  $f_w < 0$  is for injection and  $f_w = 0$  corresponds to an impermeable plate.

The transformed nonlinear ordinary differential equations are:

$$n(f''(\eta))^{n-1} f'''(\eta) + \frac{2}{3} f(\eta) f''(\eta) - \frac{1}{3} f'(\eta)^2 - (M + kp) f''(\eta) = 0 \tag{2.12}$$

$$\frac{1}{\text{Pr}} \theta''(\eta) + \frac{2}{3} f \theta' + \text{Ec} (f''(\eta))^{n+1} = 0 \tag{2.13}$$

The boundary conditions become

$$f(0) = 0, f'(0) = 1, \theta'(0) = -1$$

$$f'(\eta) \rightarrow 0, \theta(\eta) \rightarrow 0 \text{ as } \eta \rightarrow \infty \tag{2.14}$$

Here primes denote differentiation with respect to  $\eta$ .

$\text{Pr}$  is the Prandtl number,  $\text{Ec}$  is the Eckert number and  $\text{Bi}$  is the convective parameter defined respectively as

$$M = \frac{\sigma B_0^2}{\rho a}, kp = \frac{\nu}{k^* a}, \text{Pr} = \frac{a}{\alpha} \left( \frac{\rho a^{2-n}}{K} \right)^{2/(n+1)}, \text{Ec} = \frac{U_w^2}{c_p (T_w - T_\infty)} \tag{2.15}$$

The physical quantities of interest are the skin friction coefficient  $C_f$  and the local Nusselt number  $\text{Nu}_x$ , which are defined as

$$C_f = \frac{\tau_w}{\rho U_w^2 / 2}, \text{Nu}_x = \frac{xq_w}{k(T_w - T_\infty)} \tag{2.16}$$

where the surface shear stress  $\tau_w$  and the surface heat flux  $q_w$  are given by

$$\tau_w = \left[ K \left( \frac{\partial u}{\partial y} \right)^n \right]_{y=0}, q_w = -k \left( \frac{\partial T}{\partial y} \right)_{y=0} \tag{2.17}$$

Using the similarity variables (2.10), we obtain

$$\frac{1}{2} C_f \text{Re}_x^{1/(n+1)} = [f''(0)]^n, \frac{\text{Nu}_x}{\text{Re}_x^{1/(n+1)}} = 1/\theta(0) \tag{2.18}$$

### 3 SOLUTION OF THE PROBLEM

The set of equations (2.12) to (2.14) were reduced to a system of first-order differential equations and solved using a MATLAB boundary value problem solver called **bvp4c**. This program solves boundary value problems for ordinary differential equations of the form  $y' = f(x, y, p), a \leq x \leq b$ , by implementing a collocation method subject to general nonlinear, two-point boundary conditions  $(y(a), y(b), p)$ . Here  $p$  is a vector of unknown parameters. Boundary value problems (BVPs) arise in most diverse forms. Just about any BVP can be formulated for solution with **bvp4c**. The first step is to write the ODEs as a system of first order ordinary differential equations. The details of the solution method are presented in Shampine and Kierzenka [30].

### 4 RESULTS AND DISCUSSION

The main aim of the present investigated for the low and heat flux treatment to a power law fluid over a stretching sheet in a porous media with viscous dissipation. In order to get a clear insight of the physical problem, velocity distribution  $f(\eta)$  and temperature distribution  $\theta(\eta)$  are presented graphically for different values of magnetic parameter  $M$ , permeability parameter  $kp$ , Prandtl number  $Pr$ , Power law index  $n$ , Eckert number  $Ec$ . The effect of magnetic field parameter  $M$  on velocity distribution  $f(\eta)$  for shear thinning ( $n=0.85$ ), Newtonian ( $n=1.0$ ) and shear thickening ( $n=1.25$ ) fluids is presented in figure1. It is quite clear that velocity distribution  $f(\eta)$  as well as thickness of the boundary layer decelerates with the increment of the magnetic field strength parameter  $M$  due to the resistive force called Lorentz force generated by the magnetic field. We can also noticed that the effect magnetic parameter  $M$  on velocity profile  $f(\eta)$  becomes low dominating with increment of power law index  $n$ . Figure 3 show the influence of magnetic fields strength  $M$  on the temperature distribution  $\theta(\eta)$  for various values off power law index  $n=0.8, 1$  and  $1.25$  respectively. I can be seen this figure that temperature profile  $\theta(\eta)$  increase with an increase in the values of magnetic parameter  $M$  thus enhances boundary layer thickness. Also this figure the influence of the power law index  $n$  on the wall temperature is more significant when the surface is accelerated stretching the effect of  $kp$  on velocity profile  $f(\eta)$  is shown in figure 4 due to  $n=0.85, 1, 1.25$  respectively.

Increasing the porosity parameter  $kp$  reduce velocity profiles. As the porosity of the medium increases the value of  $kp$  decrease. For higher porosity of the medium i.e. for decreasing  $kp$  fluid gets more space to flow as a consequence its velocity increases. Thus an increase of porosity parameter  $kp$  leads to a decrease of the horizontal velocity profiles which leads to the enhanced deceleration of the flow and, hence, the velocity decreases. Figure 5 exhibit that  $\theta(\eta)$  in boundary layer increases with increasing permeability parameter  $kp$  in the case of  $n = 0.85, 1, 1.25$  respectively. The thermal boundary layer thickness becomes thinner with the decreasing permeability parameter  $kp$ . The effect of increasing values of  $kp$  opposes the flow in the boundary layer region which results in above heat transfer from the sheet to the fluid. This is because the presence of the porous medium is to increase the resistance to the fluid motion, this cause the fluid velocity to decrease (see fig 4) and due to which there is rise in the temperature in the boundary layer (see Fig 5).

Figure 6 shows influence of Prandtl number and Eckert number (viscous dissipation coefficient) on non-dimensional temperature profile  $\theta(\eta)$ . An increase in Prandtl number reduces the thermal boundary layer thickness. Prandtl number signifies the ratio of momentum diffusivity to thermal diffusivity. As compared to the case for no viscous dissipation, it is observed that the temperature increases

as  $Ec$  increases. The increase in the fluid temperature due to frictional heating is noticed to be more pronounced for higher values of  $Ec$  as expected. Table 1 represents the skin friction coefficient and local Nusselt number for different values of power – index  $n$ . It is noticed that both skin friction coefficient and local Nusselt number increases for  $n > 1$  whereas skin friction coefficient and local Nusselt number decreases for  $n < 1$ . Table 2 shows that the present results perfect agreement to the previously published data.

### 5 CONCLUSIONS

In the present paper, the steady, two dimensional, laminar flow of a power - law fluid over a stretching sheet in the presence of MHD, viscous dissipation and porous medium. The governing equations are approximated to a system of non-linear ordinary differential equations by similarity transformation. Numerical calculations are carried out for various values of the dimensionless parameters of the problem. It has been found that

- 1) Velocity of the fluid in the boundary region decreases with an increase in  $M$ . Opposite for skin friction coefficient. Similar results were found for porous media.
- 2) Velocity is higher for Pseudo-plastic fluids.
- 3) Viscous dissipation and heat generation is strongly influenced the thermal boundary layer.
- 4) Dilatant fluids are higher for temperature profile.

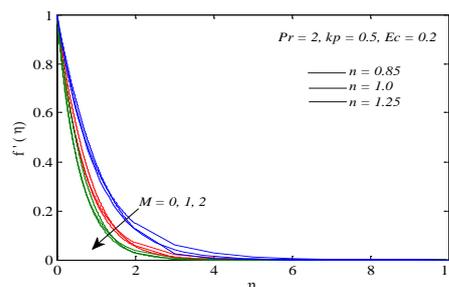


Fig.2 Velocity for different values of  $M$ .

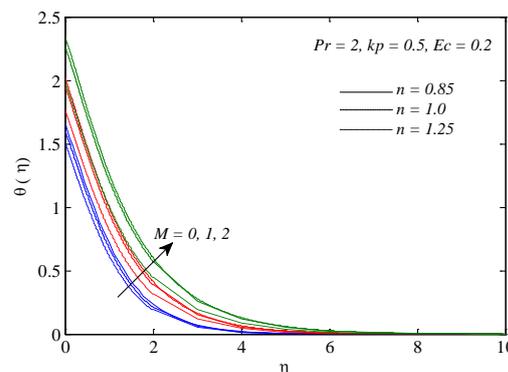


Fig.3 Temperature for different values of  $M$ .

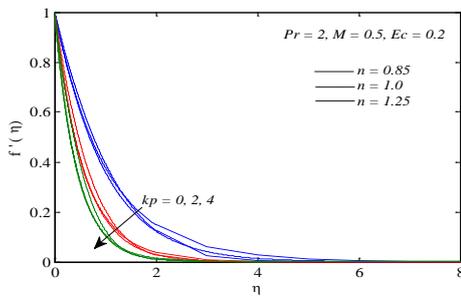


Fig.4 Velocity for different values of  $k_p$ .

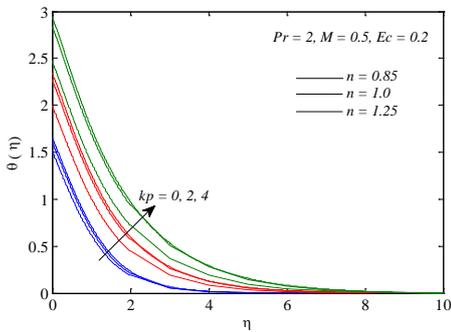


Fig.5 Temperature for different values of  $k_p$ .

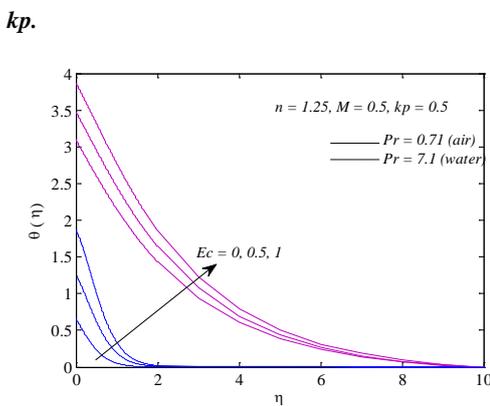


Fig.6 Temperature for different values of  $Ec$ .

Table 1 Computation for the values of  $[f''(0)]^n$  and  $1/\theta(0)$  for the values of  $n$  when  $Pr=0.71, Ec=0.2, M = 0.5, k_p = 0.5$ .

$n$	$[f''(0)]^n$	$1/\theta(0)$
0.6	-0.172879	0.313883
0.7	-0.533471	0.302661
0.9	-1.124511	0.291987
1.0	-1.227832	0.553545
1.1	-1.117591	0.293792
1.2	-0.949973	0.304635
1.3	-0.643089	0.324944
1.4	-0.272624	0.361681

Table 1. Comparison for the values of  $f''(0)$  when  $n = 1$ .

$f''(0)$			
Present study	Mutlag et al. [31]	Postelnicu and Pop [32]	Ishak et al. [33]
1.232588	1.232587	1.23259	1.2326

REFERENCES

- Schwalter, W. R., (1960), The application of boundary-layer theory to power-law pseudo plastic fluids: similar solutions, *AICHE Journal*, Vol. 6, pp. 24–28.
- Acrivios, A., Shah, M. J., and Petersen, E. E., (1960), Momentum and heat transfer in laminar Boundary-Layer flows of non-Newtonian fluids past external surfaces, *AICHE Journal*, Vol.6, pp. 312–317.
- Kumari, M., Takhar, H. S., and Nath G., (1995), Nonsimilar mixed convection flow of a non-Newtonian fluid past a vertical wedge, *Acta Mechanica*, Vol. 113, Issue 1, pp. 205-213.
- Nadeem S. and Noreen Sher Akbar N.S., (2010), Peristaltic flow of Walter’s B fluid in a uniform inclined tube, *Journal of Biorheology*, Vol.24, pp.22-28.
- Mahmoud M.A.A., (2011), Slip velocity effect on a non-Newtonian power-law fluid over a moving permeable surface with heat generation, *Mathematical and Computer Modelling*, Vol. 54, Pp. 1228–1237.
- Olanrewaju, P.O., Fenuga, O.J., Gbadayan, J.A., Okedayo, T.G., (2013), Dufour and Soret effects on convection heat and mass transfer in an electrical conducting power law flow over a heated porous plate, *International Journal for Computational Methods in Engineering Science and Mechanics*, Vol.14(1), pp.32-39.
- Yacob N. A. & Anuar A., (2014), Flow and Heat Transfer of a Power-Law Fluid over a Permeable Shrinking Sheet, *Sains Malaysiana*, Vol.43(3), pp.491–496.
- Aziz A, Ali Y, Aziz T, Siddique J.I., (2015), Heat Transfer Analysis for Stationary Boundary Layer Slip Flow of a Power-Law Fluid in a Darcy Porous Medium with Plate Suction/Injection, *PLoS ONE* 10(9): e0138855. doi:10.1371/journal.pone.0138855.
- Rashidi, M.M., Bagheri, S., Momoniat, E., Freidoonimehr, N., (2015), Entropy analysis of convective MHD flow of third grade non-Newtonian fluid over a stretching sheet, *Ain Shams Engineering Journal*, doi:10.1016/j.asej.2015.08.012
- Hady, F.M., and Hassanien, I.A., (1986), Magnetohydrodynamic and constant suction/injection effects of axisymmetric stagnation point flow and mass transfer for power-law fluids, *Indian J. Pure Appl. Math.*, Vol.17(1), pp.108-120.
- Jadhav, B. P., Waghmode B. B., (1990), Heat transfer to non-Newtonian power-law fluid past a continuously moving porous flat plate with heat flux, *Wärme - und Stoffübertragung*, Vol. 25, pp 377-380.
- Sahu, A.K., and Mathur, M.N., (1996), Free convection in boundary layer flows of power law fluids past a vertical flat plate with suction/injection, *Indian J. Pure Appl. Math.*, Vol.27(9), pp.931-941.
- Olanrewaju P.O., Makinde O. D., (2011), Effects of thermal diffusion and diffusion thermo on chemically reacting MHD boundary layer flow of heat and mass transfer past a moving vertical plate with suction/injection, *Arabian Journal for Science and Engineering*, Vol.36(8), pp.1607-1619.

14. Rosali H., Ishak A., Pop I., (2012), Micropolar fluid flow towards a stretching/shrinking sheet in a porous medium with suction, *International Communications in Heat and Mass Transfer*, Vol.39, pp.826-829.
15. Raju, M.C., and Varma, S.V.K., (2014), Soret effects due to natural convection in a non-Newtonian fluid flow in porous medium with heat and mass transfer, *Journal of Naval Architecture and Marine Engineering*, Vol. 11, pp.147-156.
16. Chinyoka T., Makinde D.O., (2015), Unsteady and porous media flow of reactive non-Newtonian fluids subjected to buoyancy and suction/injection, *International Journal of Numerical Methods for Heat & Fluid Flow*, Vol. 25(7), pp.1682 – 1704.
17. Singh, G., Sharma, P. R., and Chamkha, A. J., (2010), “Effect of Volumetric Heat Generation/Absorption on Mixed Convection Stagnation Point Flow on an Isothermal Vertical Plate in Porous Media”, *Int. J. Industrial Mathematics*, Vol.2(2), pp.59-71.
18. Das, S.S., Satapathy, A., Das J. K., and Panda, J. P., (2009). Mass transfer effects on MHD flow and heat transfer past a vertical porous plate through a porous medium under oscillatory suction and heat source, *Int. J. Heat Mass Transfer*, Vol. 52, pp.5962-5969.
19. Das S.S., Tripathy, U.K., and Das, J.K., (2010), Hydromagnetic convective flow past a vertical porous plate through a porous medium with suction and heat source, *International Journal of Energy and Environment*, Vol.1(3), pp.467-478.
20. Khan, K., (2006), Heat transfer in a viscoelastic fluid flow over a stretching surface with heat source/sink, suction/blowing and radiation, *Int. J. Heat Mass Trans.*, Vol.49, pp.628–639.
21. Pal and Talukdar, B., (2010), Perturbation analysis of unsteady magnetohydrodynamic convective heat and mass transfer in a boundary layer slip flow past a vertical permeable plate with thermal radiation and chemical reaction, *Commun. Nonlin. Sci. Numer. Simul.*, Vol.15, pp.1813–1830.
22. Gebhart, B., (1962), Effect of viscous dissipation in natural convection, *Journal of Fluid Mechanics*, vol. 14, pp. 225-235, 1962.
23. Lawal, A., and Mujumdar, A. S., (1992), The effects of viscous dissipation on heat transfer to power law fluids in arbitrary cross-sectional ducts, *Wärme - und Stoffübertragung*, Vol. 27, pp. 437-446.
24. Kairi, R.R., Murthy, P.V.S. N., and Ng, C. O., (2011), Effect of Viscous Dissipation on Natural Convection in a Non-Darcy Porous Medium Saturated with Non-Newtonian Fluid of Variable Viscosity, *The Open Transport Phenomena Journal*, Vol. 3, pp.1-8.
25. Boubaker K., Li B., Zheng L., Bhrawy A.H., and Zhang X., (2012), Effects of Viscous Dissipation on the Thermal Boundary Layer of Pseudoplastic Power -Law Non-Newtonian Fluids Using Discretization Method and the Boubaker Polynomials Expansion Scheme, *ISRN Thermodynamics*, Vol. 2012, pp.1-6.
26. Andersson, H.I., and Irgens, F., (1990), Film flow of power-law fluids, In *Encyclopedia of Fluid Mechanics*, edited by Cheremisinoff, N.P. Houston: Gulf Publishing. pp.617-648.
27. Wilkinson, W.L., (1960), *Non-Newtonian Fluids*, London: Pergamon Press.
28. Wang, T.Y., (1994), Similarity solution of laminar mixed convection heat transfer from a horizontal plate to power-law fluid, *Mingchi Institute of Technology Journal*, Vol. 26, pp.25-32.
29. Xu, H. and Liao, S.J., (2009), Laminar flow and heat transfer in the boundary-layer of non-Newtonian fluids over a stretching flat sheet, *Computer & Mathematics with Applications*, Vol. 57, pp.1425-1431.
30. Shampine, L. F. and Kierzenka, J., (2000), Solving boundary value problems for ordinary differential equations in MATLAB with *bvp4c*, *Tutorial Notes*.
31. Mutlag, A. A., Jashim-Uddin, Md., Ismail, A. I. Md., & Hamad, M. A. A., (2012), Scaling group transformation under the effect of thermal radiation heat transfer of a non-Newtonian power-law fluid over a vertical stretching sheet with momentum slip boundary condition. *Applied Mathematical Sciences*, Vol.6(121), pp.6035-6052.
32. Postelnicu, A., & Pop, I., (2011), Falkner-Skan boundary layer flow of a power-law fluid past a stretching wedge, *Applied Mathematics and Computation*, Vol.217(9), pp.4359- 4368.
33. Ishak, A., Nazar, R., & Pop, I. (2007). Falkner-Skan equation for flow past a moving wedge with suction or injection. *Journal of Applied Mathematics and Computing*, Vol.25(1-2), pp. 67-89.