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STRONG AND WEAK DOMINATION OF MIDDLE GRAPHS OF PATH AND CYCLE GRAPHS

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Abstract: Let G =(V,E) be a simple, finite, undirected and connected graph. Let $u, v \in V$ then strongly dominates *v* if (i) $uv \in E$ and (ii) $\deg(u) \geq \deg(v)$ .A non empty subset $S \subseteq V$ is a strong dominating set (sd-set) of G if every vertex in is strongly dominated by at least one vertex in S. Similarly, a weak dominating set (wd-set) is defined. The strong (weak)domination γ_s (γ_w) number of G is the minimum cardinality of a sd-set (wdset). The middle graph M(G) of a graph is obtained by subdividing each edge of G exactly once and joining all these newly added middle vertices to the adjacent edges of G. Let $\lceil x \rceil$ denote the greatest integer not greater than x and $\lfloor x \rfloor$ denote the least integer not less than x. In this paper, we investigate the strong and weak domination number of the middle graphs of the path P_n and the cycle C_n .

AMS Subject Classification: 05C09.

Keywords: Dominating set, Domination number, Strong dominating set, Weak dominating set, Strong domination number, Weak domination number, Middle graph of a graph G.

1.INTRODUCTION

Domination in graphs is one of the concepts in Graph theory which has attracted many researchers to work on it because of its many and varied applications in fields such as linear algebra and optimization, Design and analysis of communication networks, social sciences and military surveillance. We consider simple, finite, connected and undirected graph G= (V, E) of order n and size m . A non-empty subset $S \subseteq V$ of vertices in a graph G is called dominating set if every vertex in $V - S$ is adjacent to at least one vertex in S. The minimum cardinality of a minimal dominating set of G is called the domination number of G which is denoted by $\gamma(G)$ and the corresponding dominating set called the γ-set of G.

Sampathkumar and L.PushpaLatha introduced the concepts of strong and weak domination in graphs[8]. Routenbach[7] has derived a new bounds on γ_s(G).Strong and weak domination arise naturally in certain practical situations. For example, consider a network of roads, connecting a number of locations. In such a network, the degree of a vertex v is the number of roads meeting at v. Suppose $\deg(u) \geq \deg(v)$. Naturally, the traffic at u is heavier than that at v. If we consider the traffic between u and v, preference should be given to the vehicles going from u to v. Thus in some sense, u strongly dominates v and v weakly dominates u . Bounds on strong domination number are also reported by Rautenbach. V.Swaminathan and Thangaraju [9] have established the relation between strong domination and the maximum degree of the graph as well as weak domination and the minimum degree of the graph. In this paper, we investigate the strong and weak domination number of the middle graphs of the path P_n and the cycle C_n

2.Main Results

Strong and Weak Domination of the Middle Graph of Path P_n.

Definition 2.1

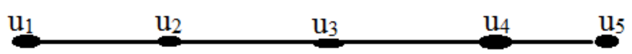
The middle graph of a connected graph G denoted by M(G) is the graph whose vertex set is V(G)UE(G) where two vertices are adjacent if

(i)They are adjacent edges of G (or)

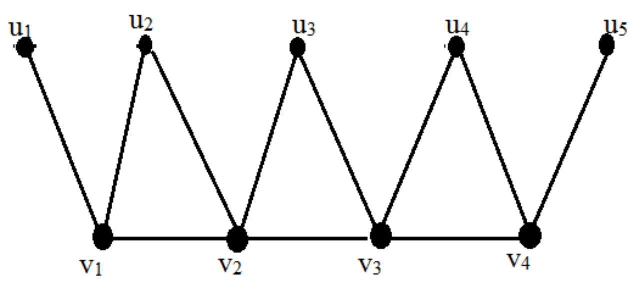
(ii) One is a vertex of G and the other is an edge incident with it.

Example 2.2

Path P_5 and the Middle Graph of P_5.



Path P_5



Middle graph of P_5 $M(P_5)$

In this section ,we obtain the strong and weak domination numbers of the middle graph of path P_n.

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Theorem2.3

For any positive integer n, the strong domination number of the middle graph of P_n is $\gamma_s(M(P_n)) = n - 1$

Proof:

Let $u_1, u_2, u_3, \ldots, u_n$ be the vertices of the path P_n and let $v_1, v_2, v_3, \ldots, v_{n-1}$ be the added vertices corresponding to the edges $e_1, e_2 e_3, \ldots e_{n-1}$ of P_n to obtain $M(P_n)$. Then $|V(M(P_n))| = 2n-1$ and $|V(M(P_n))| = 3n-1$.

Construct the set,

$$S = \begin{cases} \{ v_i : i \equiv 2 \ (mod3)\} \cup \{ v_{n-1} \}, & \text{if } n=3r-1, \text{ is an integer} \\ \{v_i : i \equiv 2 \ (mod3)\}, & \text{otherwise} \end{cases}$$

where $1 \le i \le n$ with $|S| = \left\lceil \frac{n-1}{2} \right\rceil$

Since each vertex $v \in S$ is adjacent to a vertex in $M(P_n)$. That is, u and v are two adjacent vertices ,in which v strongly dominates u such that $deg(v) \ge deg(u)$. Every vertex in V-S is strongly dominated by at least one vertex in S. Thus $\gamma_s(M(P_n)) \le n-1$. As $\Delta(M(P_n)) \ge 3$,we observe that $deg(v) \ge 3$ for all $v \in S$. In particular, every vertex in S lies on the path $(v_1, v_2, v_3, \ldots, v_{n-1})$ of $M(P_n)$. Now suppose that $\gamma_s(M(P_n)) < n-1$. Then there is a vertex $v_i \in M(P_n)$ such that $v_i \notin S$ and the degree of the vertex is one. This is the contradiction to our assumption. Therefore , every vertex V –S is strongly dominated by at least one vertex in S. We obtain, $\gamma_s(M(P_n)) = n - 1$.

Theorem2.4

For any positive integer n, the weak domination number of the middle graph of P_n is $\gamma_w(M(P_n)) = n$

Proof:

Let $u_1, u_2, u_3, \ldots, u_n$ be the vertices of path P_n and let $v_1, v_2, v_3, \ldots, v_{n-1}$ be the added vertices corresponding to the edges $e_1, e_2 e_3, \ldots e_{n-1}$ of P_n to obtain $M(P_n)$. Then $|V(M(P_n))| = 2n-1$ and $|E(M(P_n))| = 3n-1$.Given two adjacent vertices u and v, we say that u strongly dominates v if $deg(u) \ge deg(v)$. Similarly, we say that u weakly dominates v if $deg(u) \le deg(v)$. Therefore, S is a weak dominating set of M(G) if every vertex in V-S is weakly dominated by at least one vertex in S. Therefore , $\gamma_w(M(P_n)) = n$.

Observation 2.5 For any middle graph of P_n ,

$\gamma_s(M(P_n)) \le \gamma_w(M(P_n))$.

Theorem 2.6

For any path P_m ,

$$\gamma_s(M(P_m)) = \begin{cases} n-1 & \text{if } m=n, \ n \in N \\ n & \text{if } m=n+1, \ n \in N \\ n+1 & \text{if } m=n+2, \ n \in N \end{cases}$$

Proof:

Case (i):

Let $m=n$, $n \in N$ .Let$\{ u_1, u_2, u_3, \ldots, u_n \}$ be the vertices of the path P_n and $\{v_1, v_2, v_3, \ldots, v_{n-1} \}$ be the added vertices of $M(P_n)$ by subdividing each edge of G = P_n .

Since $\{ v_1, v_2, v_3, \ldots, v_{n-1} \}$is the strong dominating set of $M(P_n)$, we have , $\gamma_s(M(P_n)) = n-1, n \in N$.

Case (ii):

Let $m=n+1, n \in N$. Let$\{ u_1, u_2, u_3, \ldots, u_n ,u_{n+1}\}$ be the vertices of the path P_n and $\{ v_1, v_2, v_3, \ldots, v_n \}$be the added vertices of $M(P_{n+1})$ by subdividing each edge of G = P_{n+1}.

Since $\{ v_1, v_2, v_3, \ldots, v_n \}$ is the strong dominating set of $M(P_{n+1})$, we get $\gamma_s(M(P_{n+1})) = n$, $n \in N$.

Case (iii):

Let $m=n+ 2, n \in N$ .Let $\{ u_1, u_2, u_3, \ldots, u_n ,u_{n+1}, u_{n+2}\}$ be the vertices of P_{n+2} and $\{ v_1, v_2, v_3, \ldots, v_n ,v_{n+1}\}$ be the added vertices of $M(P_{n+2})$. Since $\{ v_1, v_2, v_3, \ldots, v_n ,v_{n+1}\}$ is the strong dominating set of $M(P_{n+2})$, we obtain $\gamma_s(M(P_{n+2})) = n+1$, $n \in N$.

Theorem2.7

For any path P_m ,

$$\gamma_w(M(P_m)) = \begin{cases} n & \text{if } m=n, \ n \in N \\ n+1 & \text{if } m=n+1, \ n \in N \\ n+2 & \text{if } m=n+2, \ n \in N \end{cases}$$

Proof:

Case(i):

Let $m=n$, $n \in N$ .Let $\{ u_1, u_2, u_3, \ldots, u_n \}$ be the vertices of the path P_n , and $\{ v_1, v_2, v_3, \ldots, v_{n-1} \}$ be the added vertices corresponding to the edges $\{e_1, e_2, e_3, \ldots, e_{n-1}\}$ of P_n to obtain $M(P_n)$. Since $\{ u_1, u_2, u_3, \ldots, u_n \}$ is a weak dominating set of $M(P_n)$,we get, $\gamma_w(M(P_n)) = n$, $n \in N$.

Case(ii):

Let $m=n+1, n \in N$. Let $\{ u_1, u_2, u_3, \ldots, u_n ,u_{n+1}\}$ be the vertices of the path P_{n+1} and let $\{ v_1, v_2, v_3, \ldots, v_{n-1} \}$ be the added vertices corresponding to the edges $\{e_1, e_2, e_3, \ldots, e_{n-1}\}$ of P_n and obtain $M(P_{n+1})$.Since $\{ u_1, u_2, u_3, \ldots, u_n ,u_{n+1}\}$ is the weak dominating set of $M(P_{n+1})$, we obtain, $\gamma_w(M(P_{n+1})) = n+1$, $n \in N$.

Case(iii):

Let $m=n+ 2, n \in N$ .Let $\{ u_1, u_2, u_3, \ldots, u_n ,u_{n+1}, u_{n+2}\}$ be the vertices of the path P_{n+2} and $\{ v_1, v_2, v_3, \ldots, v_{n-1} \}$be the added vertices of $M(P_{n+2})$. Since $\{ u_1, u_2, u_3, \ldots, u_n ,u_{n+1}, u_{n+2}\}$ is the weak dominating set of $M(P_{n+2})$, we get,

$\gamma_w(M(P_{n+2})) = n+2$, $n \in N$

Observation 2.8 For any path P_n $\gamma(M(P_n)) = n$.

3. Strong and Weak Domination of the Middle Graph of Cycle C_n.

In this section , we obtain the strong and weak domination numbers of the middle graph of the cycle C_n.

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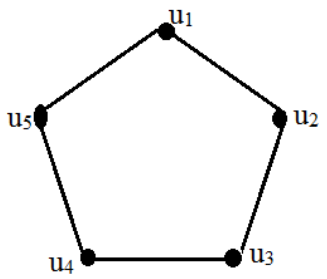
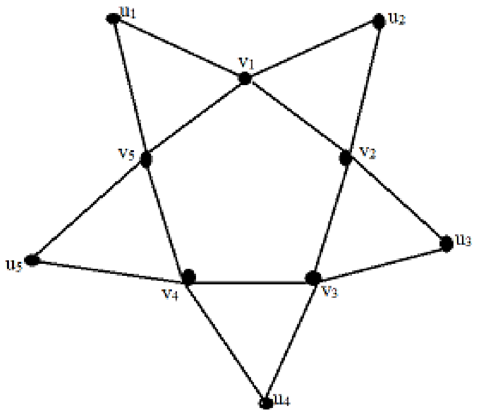
Example 3.1

Cycle C_5 and the Middle graph of C_5



Cycle graph C_5



Middle graph of C_5 $M(C_5)$

Theorem3.2

For any positive integer n, the strong domination number of the middle graph of C_n is $\gamma_s(M(C_n)) = n$.

Proof:

Let $u_1, u_2, u_3, \ldots, u_n$ be the vertices of the cycle C_n and let $v_1, v_2, v_3, \ldots, v_n$ be the added vertices corresponding to the edges $e, e_2, e_3, \ldots, e_n$ of C_n to obtain $M(C_n)$.Then $|V(M(C_n))| = 2n$ and $|E(M(C_n))| = 3n$.

Construct the set,

$S = \{ v_i : i \equiv 1 \ (mod)\}$ where $1 \le i \le n$ with $|S| = \left\lceil \frac{n}{3} \right\rceil$.

Since each vertex in $V(M(C_n))$ is adjacent to a vertex u in $M(C_n)$. That is, u and v are adjacent vertices. That v can be strongly dominated u such that $deg(v) \ge deg(u)$. Every vertex in V-S is strongly dominated by at least one vertex in S. Thus $\gamma_s(M(C_n)) \le n$. As $\Delta(M(C_n))=4$, we observe that $deg(v)=4$. In particular, every vertex of S lies on the cycle $v_1, v_2, v_3, \ldots, v_n$ of $M(C_n)$.Now, suppose that $\gamma_s(M(C_n)) < n$. Then there is a vertex $v_i \in M(C_n)$ such that $v_i \notin S$ and V-S is not strongly dominated. This is a contradiction to our assumption. Therefore, every vertex in V-S is strongly dominated by at least one vertex in S. Thus, $\gamma_s(M(C_n)) = n$.

Theorem3.3

For any positive integer n, the weak domination number of the middle graph of C_n is $\gamma_w(M(C_n)) = n$.

Proof:

Let $u_1, u_2, u_3, \ldots, u_n$ be the vertices of the cycle C_n and let $v_1, v_2, v_3, \ldots, v_n$ be the added vertices corresponding to the edges $e_1, e_2, e_3, \ldots, e_n$ of C_n to obtain $M(C_n)$. Then $|V(M(C_n))| = 2n$ and $|E(M(C_n))| = 3n$. Given two adjacent vertices u and v, we say that u strongly dominates v If $deg(u) \ge deg(v)$. Similarly, we say that u weakly dominates v if $deg(u) \le deg(v)$. Thus, S is a weak dominating set of $M(C_n)$ as every vertex in V-S is weakly dominated by at least one vertex in S. Therefore, $\gamma_w(M(C_n)) = n$.

Theorem 3.4

For any cycle C_m

$$\gamma_s(M(C_m)) = \begin{cases} n & \text{if } m=n,\ n \in N \\ n+1 & \text{if } m=n+1,\ n \in N \\ n+2 & \text{if } m=n+2,\ n \in N \end{cases}$$

Similar proof as that of theorem 3.2

Theorem 3.5

For any cycle C_m

$$\gamma_w(M(C_m)) = \begin{cases} n & \text{if } m=n,\ n \in N \\ n+1 & \text{if } m=n+1,\ n \in N \\ n+2 & \text{if } m=n+2,\ n \in N \end{cases}$$

Similar proof as that of theorem 3.3

Observation 3.6 For any middle graph of C_n,

$\gamma_s(M(C_m)) = \gamma_w(M(C_m))$.

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4.Conclusion

The strong (weak) domination number is the $\gamma_s(G)$ ($\gamma_w(G)$) minimum cardinality of a strong (weak) dominating set of G. In this paper, we have computed the strong (weak) domination number of middle graphs of the path P_n and the cycle C_n. More results on the domination parameters of middle graphs of many special classes of graphs are under investigation.

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