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A NOTE ON FUZZY VERTEX GRACEFUL LABELING ON SOME SPECIAL GRAPHS

K.Ameenal Bibi, M.Devi
PG and Research Department of Mathematics
D.K.M College for Women(Autonomous)
Vellore,Tamilnadu, India.

Abstract: A labeling on a Graph G which can be gracefully numbered is called a graceful labeling. If a fuzzy graph G admits a graceful labeling and if all the vertex labelings are distinct then G is called a fuzzy vertex graceful graph. In this paper, we discussed fuzzy vertex graceful labeling on Helm graph, Closed Helm graph, Bistar graph, $B^2(m,n)$ graph and Star friendship graph SF(n,m).

Keywords: Fuzzy labeling, Fuzzy graceful labeling, Fuzzy Helm graph, Fuzzy Bistar graph, Fuzzy SF(n,m) graph.

1.INTRODUCTION

In 20th century, remarkable development had happened in mathematical modeling for uncertainty which was introduced by Lofti.A.Zadeh in 1965[13] called fuzzy sets. The applications of fuzzy sets in the field of cluster analysis, neutral networks etc were discussed by zimermann [12]. Zadeh [13] had developed the fuzzy relations on fuzzy sets which had better feature in making fuzzy graph model. In 1975, Rosenfeld [9] discussed the concept of fuzzy graphs whose basic idea was introduced by Kauffmann [3] in 1973. The fuzzy relations between fuzzy sets were also considered by Rosenfeld and he developed the structure of fuzzy graphs, obtaining analogs of several graph theoretical concepts. Bhattacharya gave some remarks on fuzzy graphs.

Many results on fuzzy graphs were proved by J.N.Moderson, K.R.Bhutani, A.Rosenfield, S.Mathew and M.Sunitha [5,2,9,4]. A.Nagoorgani, D.Rajalakshmi(a) Subhasini, introduced fuzzy graph labeling and studied such as graceful labeling, vertex graceful labeling in some special graphs, etc. S.Vimala and R.Jebesty Shajila has studied the edge vertex graceful labeling [11].

Already we published papers on fuzzy vertex graceful labeling on Double Fan graph and Double wheel graph [1]. This note is a further contribution on fuzzy graceful labeling. In this paper we discussed fuzzy vertex graceful labeling on some more special graphs.

In this paper, the basic definitions were followed as in [5].

2. PRELIMINARIES AND OBSERVATIONS:

Definition 2.1[5]

Let U and V be two non-empty sets. Then ρ is said to be a fuzzy relation from U into V if ρ is a fuzzy set of UxV.

Definition 2.2[5]

A fuzzy graph $G:(\sigma,\mu)$ is a pair of functions $\sigma:V\to[0,1]$ and , where for all u, v $\in$ V, we have $\mu(u,v)<\sigma(u)\wedge\sigma(v)$.

Definition 2.3[5]

A labeling of a graph is an assignment of values to the vertices and edges of a graph.

Definition 2.4[5]

A graph G= (σ,μ) admits fuzzy labeling if the mapping $\sigma:V\to[0,1]$ and $\mu:V\times V\to[0,1]$ are bijection such that the membership value of edges and vertices are distinct and $\mu(u,v)<\sigma(u)\wedge\sigma(v)$ for all u,v$\in$V.

Definition 2.5[5]

Let G=(p,q) be a simple, finite, connected, undirected graph. A Graceful labeling of a graph G with q edges is an injection f:V(G) $\to$ {0,1,2,…q} such that when each edge xy$\in$E(G) is assigned with the label $|f(x)-f(y)|$, and all of the edge labels are distinct.

Definition 2.6[11]

A fuzzy labeling in which all the vertex values are distinct is called graceful fuzzy labeling. A fuzzy graph which admits fuzzy labeling is called fuzzy graceful graph.

3.MAIN RESULTS:

Definition 3.1[11]

The Helm graph H_n is a graph obtained from a wheel graph by attaching a pendent edges at each vertex of the n-cycle. In a fuzzy labeling on Helm graph, if all edge vertex values are distinct then it is called fuzzy graceful labeling Helm graph.

Theorem 3.2

For n≥3, the Helm graph H_n admits fuzzy edge-vertex graceful labeling.

Proof:

A Helm graph H_n is a graph with n vertices only if n≥3. Here, v is the central vertex and u_i denotes the vertices in the outer cycle and the pendent vertices.

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A Helm graph is a Fuzzy Helm graph $(\widetilde{H_n})$ consists of two vertex sets U and V with $|U| > 1$ and $|V| = 1$ such that $\mu(v, u_i) > 0 \mu(u_i, u_{n+1}) > 0$ and $\mu(u_{n+1}, u_{n+2}) = 0$ for $1 \leq i \leq n$.

If the fuzzy Helm graph is vertex graceful, then

$\mu(u_i, u_{i+1}) = i \times 0.02$ for $1 \leq i \leq n$.

$\mu(u_i, u_{n+i}) = (n+1) \times 0.02$ for $1 \leq i \leq n$.

$\mu(v, u_{i+i}) = \mu(u_i, u_{i+i}) + \mu(v, u_i)$ for $1 \leq i \leq n$.

and also, $\sigma(u_i) = \sigma(v) - \mu(v, u_i)$

$\sigma(u_{i+n}) = \sigma(u_i) - \mu(u_i, u_{n+i})$ for $1 \leq i \leq n$.

Example 3.3

Consider the fuzzy Helm graph $(\widetilde{H_4})$. Here n=4

When $\sigma(v)$ starts from $\frac{n+2}{10}$

Here, $\sigma: v \rightarrow [0,1]$ and $\sigma: u_i \rightarrow [0,1]$

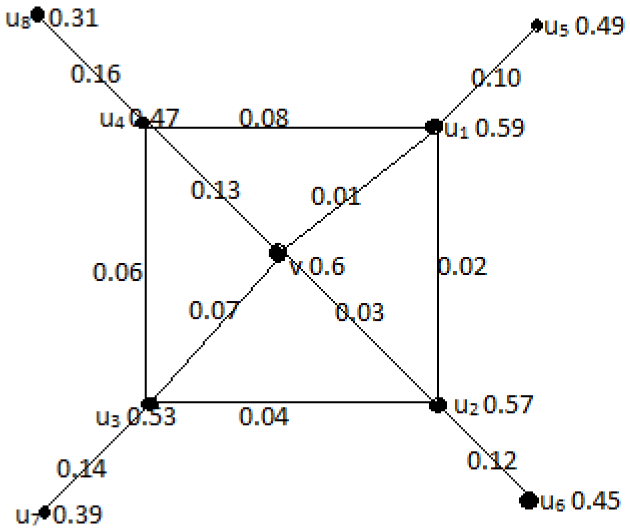

Figure (1) Fuzzy Helm graph $(\widetilde{H_4})$

In this figure, for the outer cycle,

$$\mu(u_1, u_2) = 1 \times 0.02 = 0.02$$

$$\mu(u_2, u_3) = 2 \times 0.02 = 0.04$$

$$\mu(u_3, u_4) = 3 \times 0.02 = 0.06$$

$$\mu(u_4, u_1) = 4 \times 0.02 = 0.08$$

For the pendent vertices,

$$\mu(u_1, u_5) = 5 \times 0.02 = 0.10$$

$$\mu(u_2, u_6) = 6 \times 0.02 = 0.12$$

$$\mu(u_3, u_7) = 7 \times 0.02 = 0.14$$

$$\mu(u_4, u_8) = 8 \times 0.02 = 0.16$$

Also, Let us take

$\mu(v, u_1) = 0.01$, then

$\mu(v, u_2) = 0.03 = \mu(v, u_1) + \mu(u_1, u_2)$

$\mu(v, u_3) = 0.07 = \mu(v, u_2) + \mu(u_2, u_3)$

$\mu(v, u_4) = 0.13 = \mu(v, u_3) + \mu(u_3, u_4)$

By using the above membership functions, we have defined the membership values of the vertices.

For the outer cycle,

$$\sigma(u_1) = 0.59 = \sigma(v) - \mu(v, u_1)$$

$$\sigma(u_2) = 0.57 = \sigma(v) - \mu(v, u_2)$$

$$\sigma(u_3) = 0.53 = \sigma(v) - \mu(v, u_3)$$

$$\sigma(u_4) = 0.47 = \sigma(v) - \mu(v, u_4)$$

For the pendent vertices,

$$\sigma(u_5) = 0.49 = \sigma(u_1) - \mu(u_1, u_5)$$

$$\sigma(u_6) = 0.45 = \sigma(u_2) - \mu(u_2, u_6)$$

$$\sigma(u_7) = 0.39 = \sigma(u_3) - \mu(u_3, u_7)$$

$$\sigma(u_8) = 0.31 = \sigma(u_4) - \mu(u_4, u_8)$$

Since all the membership values of the edges are greater than zero and

$$\mu(u, v) \leq \sigma(u) \wedge \sigma(v)$$

In the similar way, for any value of $\sigma: v \rightarrow [0,1]$, the labeling of all vertices in the outer cycle and the pendent vertices u_i 's are distinct when $1 \leq i \leq n$.

Therefore, this Helm graph admits fuzzy edge-vertex graceful labeling.

Definition 3.4

The closed Helm graph CH_n is the graph obtained from a Helm graph H_n by joining each pendent vertex to form a cycle. It contains three types of vertices, an apex of degree n, n vertices of degree 4 and n vertices of degree 3.

If the closed Helm graph admits fuzzy labeling then it is called as a fuzzy closed Helm graph

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and it is denoted by $\widetilde{CH}_n$ and if all the vertex values are distinct then $\widetilde{CH}_n$ admits fuzzy vertex graceful labeling.

Proposition 3.5

The closed Helm graph CH_n admits fuzzy graceful labeling in $n \geq 3$.

Proof

The Fuzzy membership functions defined in fuzzy Helm graph is the same as the membership function of fuzzy closed Helm graph. In addition for the outer cycle

$$\mu(u_{n+i}, u_{n+i+1}) = \sigma(u_{n+1}) - \sigma(u_{n+i+1}), \quad n \geq 3, \; 1 \leq i \leq n.$$

Definition 3.6

The Bistar graph $B_{m,n}$ is the graph obtained from K_2 by joining m pendent edges to one end and n pendent edges to the other end of K_2.

$B_{m,n}$ admits fuzzy labeling if all the vertex(edge) values are distinct. Then the graph $\widetilde{B_{m,n}}$ fuzzy vertex(edge) graceful labeling .

Theorem 3.7

All fuzzy Bistar graph $\widetilde{B_{m,n}}$ admits graceful labeling.

Proof:

Let $\widetilde{B_{m,n}}$ be a fuzzy Bistar graph consists of the vertex sets $\{u, v, u_i, v_j \,/ 1 \leq i \leq m\,,\; 1 \leq j \leq n\}$ with $|u_i| > 1$, $|v_j| > 1$ where u_i *and* v_j are the pendent vertices such that $\mu(v, u) > 0$, $\mu(u, u_m) > 0$, $\mu(v, v_n) > 0$ and

$$\mu(u_i, u_{i+1}) = 0 \qquad for\ 1 \leq i \leq m$$

$$\mu(v_j, v_{j+1}) = 0 \qquad for\ 1 \leq j \leq n.$$

Let $\sigma(v)\epsilon[0,1]$ and $\sigma(u)\epsilon[0,1]$ then

$$\mu(u, u_i) = \frac{i}{100} \quad for\ \ 1 \leq i \leq m$$

$$\mu(v, v_j) = \frac{i}{100} \quad for\ \ i = m+1, m+2, \ldots . and\ 1 \leq j \leq n$$

$$\sigma(u_i) = \sigma(u) - \mu(u, u_i) \quad for\ 1 \leq i \leq m$$

$$\sigma(v_j) = \sigma(u) - \mu(v, v_j) \quad for\ 1 \leq j \leq n$$

and

$$\mu(uv) = |\sigma(u) - \sigma(v)|.$$

If the number of pendent vertices is m+n then the number of edges is m+n+1.

If the number of pendent vertices is m+n then $\sigma(u)$ starts from $\frac{m+n+1}{100}$ and $\sigma(v)$ starts from $\frac{2(m+n+1)}{100}$.

Example 3.8

Edge-vertex graceful labeling of fuzzy Bistar graph $\widetilde{B_{4,5}}$

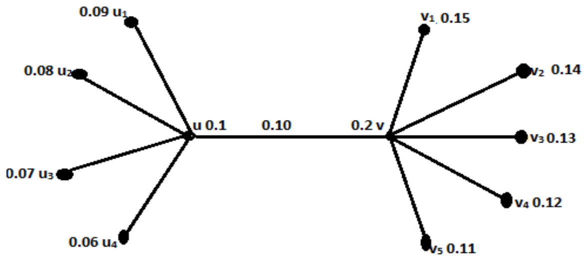

Figure (2) Fuzzy Bistar graph $\widetilde{B_{4,5}}$ (m<n)

In this figure, m=4, n=5

In this case m+n=9.

Therefore $\sigma(u)$ starts from $\frac{1}{10}$ and $\sigma(v)$ starts from $\frac{2}{10}$

$$\mu(u, u_1) = \frac{1}{100} = 0.01$$

$$\mu(u, u_2) = 0.02 \quad ,$$

$$\mu(u, u_3) = 0.03, \qquad \mu(u, u_4) = 0.04$$

$$\mu(v, v_1) = \frac{5}{100} = 0.05, \qquad \mu(v, v_2) = 0.06,$$

$$\mu(v, v_3) = 0.07, \quad \mu(v, v_4) = 0.08$$

Then

$$\sigma(u_1) = \sigma(u) - \mu(u, u_1) = \frac{1}{10} - 0.01 = 0.09$$

Similarly

$$\sigma(u_2) = 0.08, \quad \sigma(u_3) = 0.07, \quad \sigma(u_4) = 0.06$$

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$\sigma(v_1) = \sigma(v) - \mu(v, v_1) = \frac{2}{10} - 0.05 = 0.15$

$\sigma(v_2) = 0.14, \quad \sigma(v_3) = 0.13,$

$\sigma(v_4) = 0.12, \quad \sigma(v_5) = 0.11$

and $\mu(uv) = |\sigma(u) - \sigma(v)| = 0.10$

Since the membership functions of edges are all distinct in $\mu(u, u_i)$ and $\mu(v, v_j)$ as well as $\mu(u, u_i) < \sigma(u) \wedge \sigma(u_i),\ i = 1\ to\ m$ and $\mu(v, v_j) < \sigma(v) \wedge \sigma(v_j),\quad j = 1\ to\ n$

Since all the membership values of the vertices are distinct, this Bistar graph admits fuzzy edge-vertex graceful labeling.

llustration 3.9

Edge-vertex graceful labeling of fuzzy Bistar graph $\widetilde{B_{5,4}}$

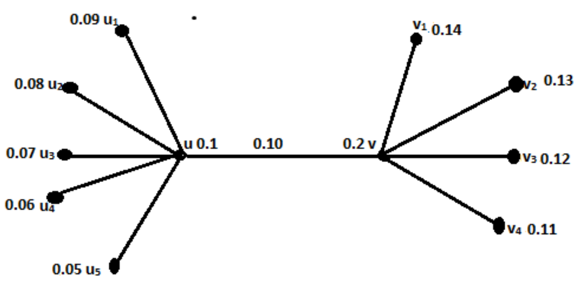

Figure (3) Fuzzy Bistar graph $\widetilde{B_{5,4}}$ (m>n)

Illustration 3.10

Edge-vertex graceful labeling of fuzzy Bistar graph $\widetilde{B_{5,5}}$

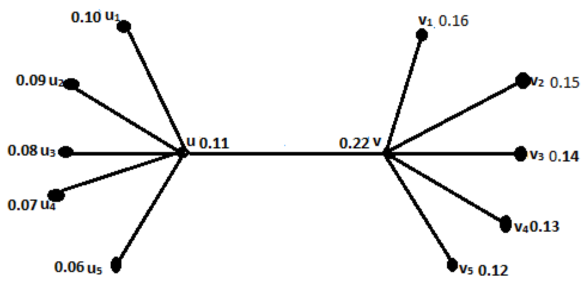

Figure (4) Fuzzy Bistar graph $\widetilde{B_{5,5}}$ (m=n)

Note 3.11

The membership functions defined in illustrations 3.8,3.9 and 3.10 represent Bistar graphs which exist if $m < n, m > n\ and\ m = n$ respectively.

Definition 3.12

The Bistar graph $B^2_{m,n}$ is the graph obtained from K_2 by joining m pendent edges to one end and n pendent edges to the other end of K_2 and also joining all the vertices between u, v_n and v, u_m. In $B^2_{m,n}$, note that each u_i and v_j(i=1 to m, j=1 to n) are adjacent to both *u* and *v*. Moreover *u* and *v* are adjacent vertices.

$B^2_{m,n}$ admits fuzzy labeling if all the vertex values are distinct. Then the fuzzy graph $\widetilde{B^2_{m,n}}$ admits vertex graceful labeling.

Proposition 3.13

The graph $\widetilde{B^2_{m,n}}$ admits fuzzy vertex graceful labeling.

Proof:

Let $\widetilde{B^2_{m,n}}$ be a fuzzy Bistar graph consists of the vertex sets $\{u, v, u_i, v_j\ /1 \le i \le m, 1 \le j \le n\}$ with $|u_i| > 1, |v_j| > 1$ such that $\mu(v, u) > 0$, $\mu(u, u_m) > 0, \mu(v, v_n) > 0$ and $\mu(u, v_n) > 0\ and\ \mu(v, u_m) > 0$.

Let $\sigma(v) \epsilon [0,1]$ and $\sigma(u) \epsilon [0,1]$ then the fuzzy membership functions are defined as follows:

$\mu(u, u_i) = \frac{i}{100} \quad for\ 1 \le i \le m$

$\mu(v, v_j) = \frac{m+j}{100} \quad for\ 1 \le j \le n$

$\mu(v, u_i) = \frac{m+n+i}{100} \quad for\ 1 \le i \le m$

$\mu(u, v_j) = \frac{2m+n+j}{100} \quad for\ 1 \le j \le n$

$\sigma(u_i) = \sigma(u) - \mu(u, u_i)\ for\ 1 \le i \le m$

$\sigma(v_j) = \sigma(u) - \mu(v, v_j) \quad for\ 1 \le j \le n$

and

$\mu(uv) = |\sigma(u) - \sigma(v)|$.

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Here, $\sigma(u)$ starts from $\frac{3(m+n)}{100}$ and $\sigma(v)$ starts from $\frac{4(m+n)}{100}$. Then $\widetilde{B^2_{m,n}}$ will be the fuzzy graceful Bistar graph.

Illustration 3.14

Vertex Graceful labeling of fuzzy Bistar graph $\widetilde{B^2_{4,5}}$

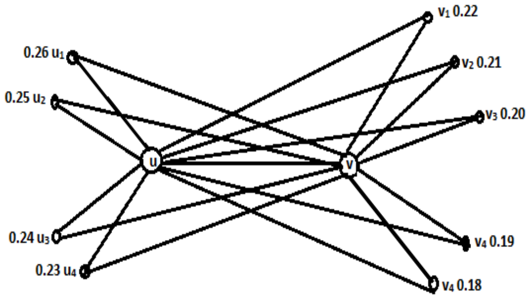

Figure (5) Fuzzy Bistar graph $\widetilde{B^2_{4,5}}$

In the above graph, all the membership values of the edges are greater than zero and

$\mu(u,u_i) < \sigma(u) \wedge \sigma(u_i), \quad i = 1 \; to \; m$

$\mu(v,v_j) < \sigma(v) \wedge \sigma(v_j), \quad j = 1 \; to \; n.$

Since all the membership values of the vertices are distinct, this fuzzy Bistar graph $\widetilde{B^2_{4,5}}$ admits fuzzy vertex graceful labeling.

Definition 3.15

A SF(n,m) is a star friendship graph consists of a cycle C_n $(n \geq 3)$ and n sets of m independent vertices where each set joins each of the vertices of C_n.

The graph SF(n,m) admits fuzzy labeling if all the vertex(edge) values are distinct. Then the fuzzy graph $\widetilde{SF(n,m)}$ admits vertex(edge) graceful labeling.

Theorem 3.16

The graph $\widetilde{SF(3,m)}$ admits fuzzy edge-vertex graceful labeling if $m \geq 2$.

Proof:

Let u_i, v_j, w_k be the vertices on the cycle $\widetilde{SF(3,m)}$ and u,v,w be the vertices joining u_i, v_j, w_k. That is, the vertex set of $\widetilde{SF(3,m)}$ is the set

$\{u_i/1 \leq i \leq m\} \cup \{v_j/1 \leq j \leq m\} \cup \{w_k/1 \leq k \leq m\} \cup \{u,v,w\}$
.

The edge set is the set

$\{u,u_i/1 \leq i \leq m\} \cup \{v,v_j/1 \leq j \leq m\} \cup \{w,w_k/1 \leq k \leq m\} \cup \{uv,vw,wu\}$
.

Here, the number of edges is 3(1+m).

In this graph,

$\mu(u,u_i) > 0, \mu(v,v_j) > 0, \mu(w,w_k) > 0$ for $1 \leq i,j,k \leq m$

and $(u_i,u_{i+1}) = 0, \mu(v_j,v_{j+1}) = 0,$ $\mu(w_k,w_{k+1}) = 0$ for $1 \leq i,j,k \leq m.$

Let $\sigma(u), \sigma(v), \sigma(w) \epsilon [0,1]$, then the fuzzy membership values are defined as follows:

$$\left.\begin{aligned} &\mu(u,u_i) = i \times 0.01 \\ &\mu(v,v_j) = (m+j) \times 0.01 \\ &\mu(w,w_k) = \{2m + (n-1) + k\} \times 0.01 \\ &\text{for } 1 \leq i,j,k \leq m \; and \; n = 3 \end{aligned}\right\}$$

$\mu(uv) = |\sigma(u) - \sigma(v)|$

$\mu(vw) = |\sigma(v) - \sigma(w)|$

$\mu(wu) = |\sigma(w) - \sigma(u)|$

and let us take,

$\sigma(u)$=m+n+(i-1) for i=1,m=2 and i=2,m=3 and i=3,m=4 and so on.

$\sigma(v)$= $2\sigma(u)$ and $\sigma(w)$=3(m+n)+3i-2, i=1,2,3…

Then $\sigma(u_1)$= $\sigma(u)$- $\mu(u,u_1)$

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$\sigma(u_2) = \sigma(u_2) - \mu(u, u_2)$ and so on.

Illustration 3.17

Edge-vertex labeling on Fuzzy Star Friendship graph $\widetilde{SF(3,4)}$

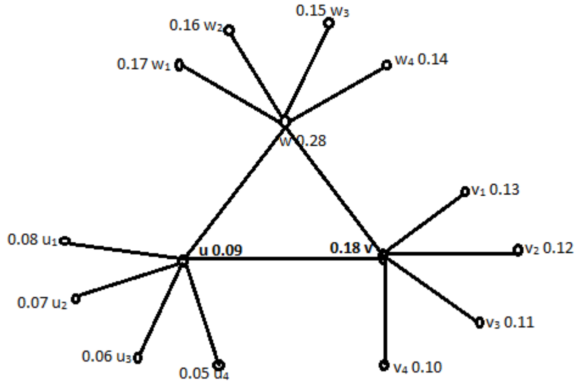

Figure (6) Fuzzy SF(n,m) graph $\widetilde{SF(3,4)}$

In the above graph, all the membership values of the edges are distinct and greater than zero and

$\mu(u, u_i) < \sigma(u) \wedge \sigma(u_i)$,

$\mu(v, v_j) < \sigma(v) \wedge \sigma(v_j)$ and

$\mu(w, w_k) < \sigma(w) \wedge \sigma(w_k)$

Since all the membership values of the vertices are distinct, this graph $\widetilde{SF(3,4)}$ admits edge-vertex graceful labeling.

Note: 3.18

(i) The above result will be true for all $n \geq 3$.

(ii) If n is unknown, then the number of edges is n(1+m).

4. CONCLUSION

In this paper, the concept of fuzzy edge-vertex graceful labeling on fuzzy Helm graph, fuzzy Bistar graph and fuzzy star friendship graph have been discussed. The fuzzy vertex graceful labeling on fuzzy Closed Helm graph and $\widetilde{B^2_{m,n}}$ have also been investigated. The extension of this study on some more special classes of graphs is under progress.

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