



A Literature Review on Minimizing the Number of Phasor Measurement Units (PMUs) and their Optimal Placement in Power Systems

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Abstract: This paper compares 12 methods of optimization techniques for the optimal placement of phasor measurement units (PMUs) in power systems. These techniques are further explained for minimizing the no. of pmu's in the real time power system. The planned methodology result in a reduction in the number of PMUs even though the system topological observability is complete. First, the number of PMUs is minimized in such a way that the system topological observability is complete. The optimal placement is also done to maximize the measurements redundancy which allows a bus to be observed more than one time. All these methods are categorized under Mathematical Programming Methods, Heuristic Methods and Meta-Heuristic Methods have been discussed.

Keywords: phasor measurement units, observability, optimal PMU placement, Conventional optimization, heuristic optimization, meta-heuristic optimization, phasor measurement.

I. INTRODUCTION

Power utilities are facing growing number of threats of security of operation due to over stressed power network in modest power market state. The intrusive changes in electrical networks has complicated the power system planning, operation, and protection. Consequently, the existing methods of data acquisition and processing in the supervisory control and data acquisition (SCADA) system need more technical advancement. Hence, the attention in the use of phasor measurement technology to obtain a better estimate of the power system state is enlarged. Hence, the power utilities are tracking to change the SCADA-based monitoring system to phasor measurement unit (PMU)-dominated wide area measurement system (WAMS)[1].

A. About Phasor Measurement Units (PMUs)

A phasor measurement unit is a device which measures voltage and current phasors on an electricity grid. PMUs are provided with the Global Positioning System (GPS) to give the time synchronized (real time) measurement of voltage and current phasors [2]. The device gives the 48 samples of resulting measurements per second which depicts its accuracy. In the **Figure 1** we get overall idea of the sinusoidal waveform and the angles. The total error in the measurement of phasor (magnitude and angle) is less than 1 %. Individually magnitude and phasor contributes error $< 1\%$ and $< 0.573\sigma$ respectively. The results obtained by the device is known as a synchro phasor [3]. That's why the terms "PMU" and "synchro phasor" are sometimes used interchangeably though they are two separate technical terms.

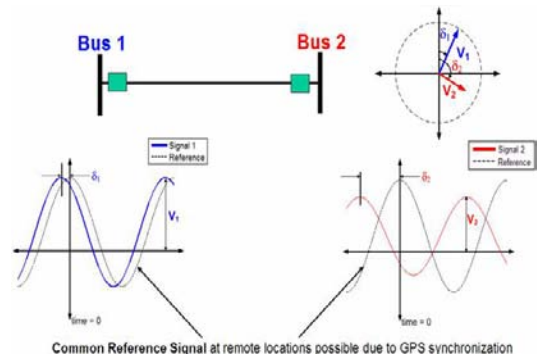


Figure 1 Phasor representation of sinusoidal waveform

B. Applications of PMU

- Wide-area situational awareness
- Voltage monitoring and trending
- State estimation
- Power oscillation monitoring

II. OPTIMAL PLACEMENT OF PMUS

Phasor measurement units (PMUs) provide time-synchronized (real-time) phasor measurements in power systems [4]. With the increasing demand of quality power, the use of phasor measurement units (PMUs) has increased a lot ever since it came into existence in 1980s. With this advanced meter, the operation, protection, monitoring and control of power system becomes accurate and easy. Using the PMUs data in state estimation (SE) equation make the SE algorithm linear which is easy to solve as compared to the nonlinear state estimation equations. Since it makes the SE algorithm linear, no iteration is required in getting the solution. Because of PMUs promising accuracy, its role is very crucial in SE algorithm. It is predicted that in the coming days SE technique will rely more on results of PMUs. However due to expensive nature of device (Rs. 27

lacs/PMU) they cannot be installed at all the buses. Therefore, a suitable technique is required to minimize the number of PMUs with complete observability of power system. A power system is said to be completely observable when the phasor voltage of all the buses in the system can be determined uniquely either directly or indirectly [5]. Therefore, observability study in the PMUs placement problem is important before the deployment of PMUs.

The block Diagram of PMU can be graphically observed in **Figure 2**. After assuring the complete system observability, it is necessary to find the optimal locations of the PMUs to maximize the measurement redundancy. The term "measurement redundancy" for a particular bus represents the number of times that bus is observed by PMUs. For example, if a bus observed by one PMU is make to observe by one more PMU, then the measurement redundancy of that bus will increase by one. The increase in value of measurement redundancy will ensure the observability of system in case of branch outage or PMU failure. Current energy management systems (EMSs) require accurate monitoring of power system state variables, such as the voltage phasors at all buses.

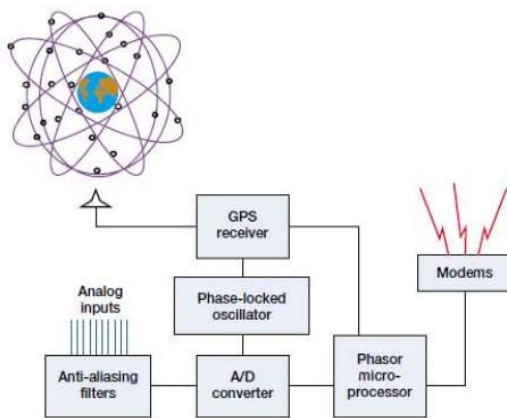


Figure 2. Block Diagram of PMU

After formation complete system observability, it is necessary to control the optimal places of the PMUs to maximize measurement redundancy [6]. A power system has measurement redundancy when its buses are observed by more than one PMU's or the number of observable buses is maximized. In other words, some of the PMUs can be detached from the measurement system while all of the buses remain observable.

II. FORMULATION OF OPTIMAL PMU LACEMENT PROBLEM

There are many topological and numerical methods for determining the optimal location of PMUs. But we have to aim for discovery out the one technique or the combination of techniques which will not only minimize the installation cost but will also provide full observability of all the buses using a minimum set of phasor measurements. Several conventional optimization techniques[7] have been proposed to solve the Optimal PMU Placement (OPP) problem, such as linear programming (LP), nonlinear programming (NLP), dynamic programming(DP), Greedy Algorithm (GA) or combinatorial optimization(CO).Graphical Representation of placing PMU in

a real power system is shown in **Figure 3**. To overcome the problems of conventional optimization techniques, such as difficulties in handling constraints, risk of trapping at local optima or numerical difficulties, advanced heuristic and modern metaheuristic optimization techniques have been proposed. An extensive range of such strategies can be cited from the OPP literature, like depth first search (DeFS), minimum spanning tree (MST), Tabu search (TS), simulated annealing (SA), genetic algorithms (GA), differential evolution (DE), immune algorithms (IA), particle swarm optimization (PSO), Binary Particle Swarm Optimization (BPSO) or ant colony optimization (ACO). These all techniques are divided in 3 different Methods I.e. Conventional Methods, Heuristic Methods and Metaheuristic Methods.

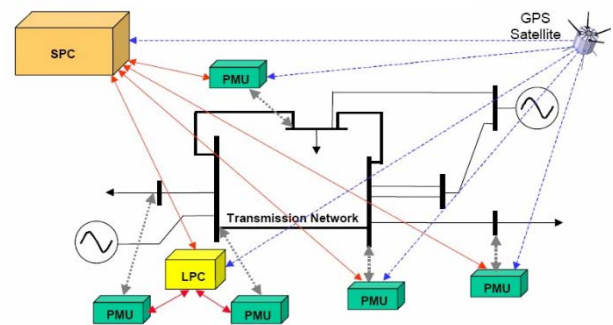


Figure 3 PMUs in Real Time Power Systems

IV. CONVENTIONAL METHODS

A. Integer Linear Programming (ILP)

A comprehensive formulation [8], considering situations with and without zero injections, shows that the problem of optimal PMU placement can be modelled linearly and solved by ILP for full and incomplete observability. A procedure for multistage PMU placement in a given time horizon, using an ILP framework, is presented in [9]. The zero injection constraints can be modelled as linear constraints. The OPP problem has multiple solutions and two indices are proposed to further rank these multiple solutions. The bus observability index (BOI) gives a measure of the number of PMUs observing a given bus and the system observability redundancy index (SORI) gives the sum of all BOI for the system.

In [10], a two level approach partitions the spanning tree of the network into two or more sub-networks using ILP. The ILP has been formulated based on eigenvectors of the adjacency matrix of the spanning tree. After decomposition, PMUs have been placed optimally in the subnetworks in order to minimize their installation cost.

B. Integer Quadratic Programming (IQP)

Quadratic programming (QP) concerns the optimization of a quadratic objective function, linearly constrained. In integer quadratic programming, all design variables take integer values. Method [11], is an integer quadratic optimization process that minimizes the number of PMUs needed to maintain complete network observability for normal operating conditions as well as for the outage of a transmission line or

PMU and maximizes the measurement redundancy at all system buses. It was applied on various IEEE test systems, considering the outage of a single transmission line or PMU. Another IQP approach [12], determines the solution of OPP problem, using the connectivity matrix to represent the network topology and formulate the optimization problem.

C. Greedy Algorithm

A combinatorial optimization algorithm that takes the best immediate, or local, solution while finding an answer, is called greedy algorithm. A virtual data elimination pre-processing method and a matrix reduction algorithm have been introduced to reduce the size of the placement model and the computational effort for the determination of the optimal placement set [13].

V. HEURISTIC METHODS

A. Depth First Search (DeFS)

An algorithm that marks all vertices in a directed graph in the order they are discovered and finished, while partitioning the graph into a forest, is called depth first search algorithm (DeFS). This method uses the Conditions 1 to 3 of Section II. It is based on the criterion of 'depth' and is non iterative. In [14]-[15], the OPP optimization problem is solved using PSAT, a MATLAB based toolbox, and DeFS method is compared with other methods. Another DeFS method is proposed in [16]. The DeFS algorithm is computationally faster, but the solution is not optimum, because the optimization criterion is stiff and unitary.

B. Minimum Spanning Tree (MST)

A modified depth first approach is the minimum spanning tree (MST) method. The MST algorithm improves the DeFS approach, which also has fast computing characteristics, and improves DeFS's complex and weak convergence. It changes optimization rules from "find a slip road linking the bus up to" to "search for the maximum coverage of the bus network". Many simulations have been performed with the IEEE-14 and IEEE-30 bus systems [17].

C. Index Method

The index method, uses an indices called connectivity index, to determine the number of favorable bus locations, depending on their connectivity with the rest of the system. Then, the selected locations are assigned as optimal locations for PMUs placements.

Since an HMD installed at a bus makes the entire buses incident on it to be observable, all such observable buses can be determined by defining the binary connectivity matrix as

$$A(i, i) = 1, \text{ for all buses}$$

$$A(i, j) = 1, \text{ if bus } i \text{ and bus } j \text{ are connected}$$

$$A(i, j) = 0, \text{ if bus } i \text{ and bus } j \text{ are not connected}$$

The index method starts with selecting the *terminal bus* in the system. The *terminal bus* is the bus which is connected to only a single bus of the entire system. A PMU installed at the terminal bus cannot observe more than two buses, the terminal

bus itself and the bus connected to that terminal bus. Thus, to observe any terminal bus, the PMU is placed at the bus connected to it.[18] After placing the PMU on the adjacent bus to the terminal bus, a unique bus having the highest *connectivity index*, if any, is found. The *connectivity index* of a bus is defined as the number of unobserved buses that can be observed by placing a PMU at that particular bus. For the *i*th bus, it will be given by the sum of all elements of the *i*th row of matrix A minus one.

VI. METAHEURISTICMETHODS

A. Simulated Annealing (SA)

Simulated Annealing (SA) is a technique that finds a good solution to an optimization problem, by trying random variations of the current solution. A worse variation is accepted as the new solution with a probability that decreases as the computation proceeds. *The slower the cooling schedule*, or rate of decrease, the more likely the algorithm is to find an optimal or near-optimal solution. The proposed SA method in [19] suggests a simple objective function that takes into account the distribution and installation cost of the measuring devices. The concept of depth of unobservability and how it affects the number of PMUs is presented in [20]. Test results have shown that this method guarantees a dispersed placement of PMUs around the system and ensures that the distance between unobserved and observed buses is not too great. SA Technique is utilized to solve the pragmatic communication constrained PMU placement problem. The SA algorithm is adopted in [21] to find the sensitivity constrained optimal PMU placement for system observability. A discrete objective function is minimized subject to the constraint that the system be topologically observable and PMUs be placed on buses with higher sensitivities. An observability topology analysis method is used to calculate parameter sensitivities of every bus in the system. The above method can be extended to consider the concept of unobservability depth [22]. Each control area includes one PMU and several RTUs. Voltage and current phasors are measured by the PMUs, while conventional measurements (power injections and flows) are measured by the RTUs. Pairs of power injection and flow measurements are placed to observe the raw data of boundary bus and tie line for data exchange in wide-area state estimator.

B. Genetic Algorithm (GA)

Genetic Algorithms (GA) are direct, parallel method for global search and optimization, which imitates the evolution of the living beings, described by Charles Darwin. GA are part of the group of Evolutionary Algorithms (EA)[23]. The evolutionary algorithms use the three main principles of the natural evolution: reproduction, natural selection and diversity of the species, maintained by the transformations of each generation with the previous. Genetic Algorithms works with a set of individuals, representing possible solutions of the task.. The best-suited entities create the next generation[24]. The large variety of problems in the engineering scope, as well as in other fields, requires the usage of algorithms from different type, with different characteristics and settings. Main ingredients of GA are Chromosome, Selection, Recombination (Crossover), Mutation.

C. Tabu Search (TS)

Tabu Search (TS) is a search technique used combinatorial for solving optimization problems by outlining and guiding the search. A novel topological method based on the supplement incidence matrix and TS algorithm, is proposed in [5]. The solution of the combinatorial OPP problem requires less computation and is highly robust. The method is faster and more convenient than conventional observability analysis methods using complicated matrix analysis, because it manipulates integer numbers. A TS method on meter placement to maximize topological observability is presented in [25].

D. Differential Evolution (DE)

Differential evolution (DE) is an optimization method that iteratively tries to increase a candidate solution with respect to a given measure of quality. The algorithm which is proposed in [26], is an integration of Pareto non-dominated sorting and differential evolution algorithm (NSDE). The schemes of PMU placement which are produced by the approach are flexible, differentiated, rational and practical. It has realistic enlightening significance for the decision-maker to make decision scientifically according to practical condition. It can realize global multi-objective optimization effortlessly and swiftly, hence can find a lot of Pareto optimal solutions and can attain accurate and complete Pareto front. Moreover, it is worth supplementary studying and researching on how to apply NSDE algorithm to PMU optimal placement problem with more objects.

E. Immune Algorithm (IA)

The immune algorithm (IA) is a search strategy based on meta heuristic technique i.e. genetic algorithm principles and inspired by protection mechanism of living organisms against bacteria and viruses. With reference in [27], the particular application of the immune genetic algorithm (IGA) method to the OPP problem is presented. Application of the native and previous knowledge associated with the considered problem is the main idea behind IGA. The prior knowledge of the OPP problem was inferred based on the topological observability analysis and was abstracted as some vaccines. The injection of these vaccines into the individuals of generations, revealed a remarkable increase in the convergence process.

F. Particle Swarm Optimization (PSO)

Particle Swarm Optimization (PSO) is an optimization method that offers a population-based search technique in which individuals, called particles, alter their positions with time. These particles fly around in a multidimensional search space. During this flight, every particle adjusts to its own position according to their own experience and the experience of neighboring particles, making use of that best position encountered by itself and its neighbors. The swarm (group of bees) direction of a particle is determined by the set of particles neighboring the particle and their history experience. In [28], a modified binary version of particle swarm algorithm (BPSO) is used, as an optimization tool to find the minimal number of PMUs for complete observability. By evolving a new rule based on analysis of zero-injections, an improved topological

observability assessment which is based upon topological analysis is implemented. A BPSO algorithm with the main objective of minimum PMU installation costs, is introduced in [29]. A number of issues may influence the cost, such as the communication situations at the located bus and the number of adjacent branches at the bus. These factors have been proved to be more qualified than conventional methods.

A modified algorithm was defined as BPSO algorithm which is used to obtain the minimal number of PMUs and their corresponding locations while sustaining associated constraints [30]. A similar procedure for the OPP problem using the BPSO algorithm is proposed in [31]. This optimization technique is not just for finding the minimal number of PMUs to be installed at the buses but also the maximum redundancy of that PMU which means the no. of times the buses are observed. A similar BPSO algorithm is suggested in [32]. Other kind of hybrid algorithm based on BPSO and immune mechanism is introduced in [33]. It delivers a speedy and general analyzing method of power network topology observation based on the properties of PMU and topological structure information of the power network. The feature of the projected algorithm is the mixture of the swiftness in BPSO and the diversity of antibodies in immune system, thus refining its ability of congregating in later evolution process.

G. Ant Colony Optimization (ACO)

The ant colony optimization (ACO) algorithm is a probabilistic technique for solving computational problems which can be reduced to finding good paths through graphs. A briefly described algorithm which constitute the proper mechanism is described in it [34]. The mechanism of adaptively adjusting the pheromone (chemical excreted by ants) trail persistence coefficient and stochastic perturbing is introduced to improve the algorithm on the ability to escape from immobility behavior and convergence speed.

VI. CONCLUSIONS

During the last 27 years, there came many optimization techniques, which have been developed to solve the problem of the optimal placement of PMUs. The proposed techniques can be classified into three main categories: conventional method, heuristic method and metaheuristic method. The literature reviews of all the different techniques presented in this paper will be useful for the researchers in order to choose wisely which method or methods to be used for their purpose and apply different methods for solving the stimulating OPP problem.

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