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Solution of Game Theory Problems by New Approach

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Abstract: In this paper, an alternative approach to the Simplex method for game theory problem is suggested. Here we proposed a new approach based on the iterative procedure for the solution of a game theory problem by alternative simplex method. The method sometimes involves less or at the most an equal number of iteration as compared to computational procedure for solving NLPP. We observed that the rule of selecting pivot vector at initial stage and thereby for some NLPP it takes more number of iteration to achieve optimality. Here at the initial step we choose the pivot vector on the basis of new rules described below. This powerful technique is better understood by resolving a cycling problem.

Keywords: Optimum solution, New method, Game theory Problem , operations research, no saddle point

INTRODUCTION

Game theory attempts to study decision-making in the situations where two or more intelligent and the rational opponents are involved under conditions of conflict and cooperation. The approach of the game theory is to seek to determine a rival's most profitable counter-strategy to one's own 'best' moves and to formulate the appropriate defensive measures.

Game theory is the formal study of conflict and cooperation. Game theoretic concepts apply whenever the actions of several agents are interdependent. These agents may be individuals, groups, firms, or any combination of these. The concepts of game theory provide a language to formulate structure, analyze, and understand strategic scenarios.

In practical life, it is required to take decision in a competing situation when there are two or more opposite parties with conflicting interests and the outcome is controlled by the decision of the all parties concerned. Such problems occur frequently in the economics, Business, Administration Sociology, Political Science and Military training. It is in this context that the game theory was developed in the twentieth century. However the mathematician treatment of the Game Theory was made available only in 1944, when John-Von-Newmann and the Oscar Morgenstrem [15] published their article 'Theory of the Game and Economics behaviour. The Von-Newmann's approach to solve the Game theory problems was based on the maximum losses. Most of the problems can be handled by this principle.

In 1994, B¨orgers' [1] discussed the theory of Weak Dominance and Approximate Common Knowledge. Brown [2] studied Iterative Solution of Games by Fictitious Play in Activity Analysis of Production and Allocation. Dantzig [3] discussed Maximization of linear function of variables subject to linear inequalities. Fudenberg and Levine [4] studied The Theory of Learning in Games. **Gass [5]**discussed Linear Programming. Ghadle, Pawar and Khobragade [6] find the Solution of Linear Programming Problem by New Approach. Khobragade and Khot [7] discussed Alternative Approach to the Simplex Method and Lokhande, Khobragade, Khot [8] studied Simplex Method: An Alternative Approach. O'Neill **[9]** discussed Non metric Test of the Minimax Theory of Two-person Zero-sum Games.

Rasmussen [10] studied Games and information: an introduction to game theory. Sharma [11] has written the book on Operation Research. Stinchcombe [12] discussed General Normal Form Games. Tang [13] studied Anticipatory Learning in Two-person Games: Some Experimental Results. Vaidya, Khobragade [14] found the Solution of Game Problems Using New Approach. Weibull [16] studied Evolutionary Game Theory.

In this paper, an attempt has been made to solve the game theory problems by KKL method.

ALGORITHM OF KKL METHOD

Step 1. For the $m \times n$ rectangular game when either m or n or both are greater than equal to three, new linear programming approach is as follows:

Let the two person zero sum game be defined as follows:

Player A has m course of action $(A_1, A_2, ..., A_m)$ and player B has n course of the action $(B_1, B_2, ..., B_n)$. The pay-off to the player A if he selects strategy A_i and player B select B_j is a_{ij}. Mixed strategy for player A is defined by the probabilities $p_1, ..., p_m$, where $p_1 + ... + p_m = 1$ and mixed strategy for player B is defined by $q_1, ..., q_n$ where $q_1 + ... + q_n = 1$.

Let the game can be defined as a linear programming problem as given below:

Player A

Minimize $z=\frac{1}{v}$ or $y_1+y_2+..........y_n$

Subject to the constraints:

$$a_{11}y_1+a_{12}y_2 ....+a_{1n}y_n \geq 1$$
$$a_{21}y_1+a_{22}y_2 ....+a_{2n}y_n \geq 1$$
$$..$$
$$..$$
$$a_{m1}y_1+a_{m2}y_2 ....+a_{mn}y_m \geq 1$$

Player B

Maximize $z=\frac{1}{v}$ or $x_1+x_2+..........x_n$

Subject to the constraints:

$$a_{11}x_1+a_{12}x_2 ....+a_{1n}x_n \leq 1$$
$$a_{21}x_1+a_{22}x_2 ....+a_{2n}x_n \leq 1$$
$$..$$
$$..$$
$$a_{m1}x_1+a_{m2}x_2 ....+a_{mn}x_m \leq 1$$

The steps for the computation of the optimal solution are as follows:

Step 2: Formulate the linear programming model of the real world problem that is obtained a mathematical representation of the problems objective function and constraints as stated below.

Maximize $M=c_1x_1+c_2x_2+..........+c_nx_n$

Subject to constraints:

$$a_{11}x_1+a_{12}x_2+..........+a_{1n}x_n \leq b_1$$
$$a_{21}x_1+a_{22}x_2+..........+a_{2n}x_n \leq b_2$$
$$..$$
$$..$$
$$a_{m1}x_1+a_{m2}x_2+..........+a_{mn}x_n \leq b_m$$
$$x_1, x_2, x_3,..........., x_n \geq 0$$

If the objective function is minimized, then convert it into a problem of maximizing by using the rule

Minimum M= - (Maximum (-M))

All $bi's$, $i=1,2,......,m$ must be non negative. If any one of bi is negative, multiply corresponding inequality by (-1), So as to get all $bi's$, $i=1,2,......,m$ non-negative.

Step 3: Convert all inequations of the constraints into the equations by introducing slack variables in the left hand side of constraints and assign a zero coefficient to the corresponding variable in the objective function. Thus we can reformulate the problem in terms of equation as follows:

Maximize

$$M=c_1x_1+c_2x_2+..........+c_nx_n+0p_1+0p_2+.......+0p_m$$

Subject to constraints:

$$a_{11}x_1+a_{12}x_2+..........+a_{1n}x_n+p_1=b_1$$
$$a_{21}x_1+a_{22}x_2+..........+a_{2n}x_n+p_2=b_2$$
$$..$$
$$...$$
$$a_{m1}x_1+a_{m2}x_2+..........+a_{mn}x_n+p_m=b_m$$

where $x_1, x_2, x_3,..........., x_n \geq 0$ and

$$p_1, p_2, p_3,..........., p_m \geq 0$$

Step 4: An initial basic feasible solution is obtained by setting $x_1=x_2=x_3=...=x_n=0$. Thus we get $p_1=b_1$, $p_2=b_2$,…, $p_m=b_m$.

Step 5: For computational, efficiency and simplicity, the initial basic feasible solution, the constraint of the standard linear programming problem as well as the function can be displayed in a tabular form,

Reducing The Game Problem To A L.P.P.

It is somewhat more difficult to solve a game problem with an $m \times n$ payoff matrix having neither a saddle point nor any dominant column or row.

Further, in order to avoid any graphical simplification, we consider the general case when neither m nor n is 2.

The problem is to determine m probabilities p_i for player A, say, with which he must mix his m pure strategies to get his mixed strategy, n probabilities q_j for player B, say, with which he should mix his n moves to get his mixed strategy; and the expected optimum value v of the game.

Consider an $m \times n$ rectangular payoff matrix (a_{ij}) for player A.

Let $S_m=\begin{bmatrix} A_1 & \cdots & A_m \\ p_1 & \cdots & p_m \end{bmatrix}$ and $S_n=\begin{bmatrix} B_1 & \cdots & B_n \\ q_1 & \cdots & q_n \end{bmatrix}$

where $\sum_{i=1}^{m} p_i = \sum_{j=1}^{n} q_j = 1$, be the mixed strategies for the two players respectively.

Player A select p_i that will maximize his minimum expected payoff in a column, white player B selects the q_j that will minimize his maximum expected loss in a row of the payoff matrix (a_{ij}).

Now, the expected gains $g_j (j=1\,2\ldots n)$ of player A against B's moves are given by

$$g_1=a_{11}p_1+a_{21}p_2+\ldots+a_{m1}p_m$$
$$g_2=a_{12}p_1+a_{22}p_2+\ldots+a_{m2}p_m$$
$$\vdots$$
$$g_n=a_{1n}p_1+a_{2n}p_2+\ldots+a_{mn}p_m$$

and the expected losses $l_i \; (i=1\,2\ldots m)$ of player B against A's moves are given by

$$l_1=a_{11}q_1+a_{12}q_2+\ldots+a_{1n}q_n$$
$$l_2=a_{21}q_1+a_{22}q_2+\ldots+a_{2n}q_n$$
$$\vdots$$
$$l_m+a_{m1}q_1+a_{m2}q_2+\ldots+a_{mn}q_n.$$

Thus, mathematically, minimax maximin principle suggests that player A should select $p_i(p_i \geq 0, \sum_{i=1}^{m} p_i = 1)$ that will yield $\max_i [\min_j (g_j)]$ for $j=1\,2\ldots n$ and the player B should select $q_j(q_j \geq 0, \sum_{j=1}^{n} q_j = 1)$ that will yield $\min_j [\max_i (l_i)]$ for $i=1\,2\ldots m$.

Let $u = \min_j (g_j)$ and $v = \max_i (l_i)$,

then the problem for player A is to

Maximize u

subject to the constraints :

$$g_1 = \sum_{i=1}^{m} a_{i1} p_i \ge u \qquad \sum_{i=1}^{m} p_i = 1,$$

$$g_2 = \sum_{i=1}^{m} a_{i2} p_i \ge u, \quad p_i \ge 0 \text{ for all } i.$$

$$\vdots$$

$$g_n = \sum_{i=1}^{m} a_{in} p_i \ge u$$

and the problem for player B is to

Minimize v

subject to the constraints :

$$l_1 = \sum_{j=1}^{n} a_{1j} q_j \le v$$

$$l_2 = \sum_{j=1}^{n} a_{2j} q_j \le v, \quad \sum_{j=1}^{n} q_j = 1$$

$$\vdots$$

$$l_m = \sum_{j=1}^{n} a_{mj} q_j \le v, \qquad q_j \ge 0 \text{ for all } j.$$

The above LPP formulation can be simplified by assuming that u and v both are positive. For, every element of (a_{ij}) can be made strictly greater than zero by adding some constant to all the entries of (a_{ij}).

After the optimum solution is obtained, the true value of the game is obtained by subtracting that constant. Thus assuming that $u > 0$, $v > 0$, we introduce the new variables

$$p_i' = \frac{p_i}{u} \quad i = 1\,2 \ldots m \text{ and } q_j' = \frac{q_j}{v}, \quad j = 1\,2 \ldots n$$

so that the two problem become :

Problem of Player A

Maximize u = Minimize $\frac{1}{u} = \sum_{i=1}^{m} \frac{p_i}{u} = \sum_{i=1}^{m} p_i'$

i.e. Minimize $p_0 = p_1' + p_2' + \ldots + p_m'$

subject to the constraints :

$$a_{11} p_1' + a_{21} p_2' + \ldots + a_{m1} p_m' \ge 1$$
$$a_{12} p_1' + a_{22} p_2' + \ldots + a_{m2} p_m' \ge 1$$
$$\vdots$$
$$a_{1n} p_1' + a_{2n} p_2' + \ldots + a_{mn} p_m' \ge 1$$
$$p_i' \ge 0, \quad i = 1\,2 \ldots m$$

Problem of Player B

Minimize v = maximize $\frac{1}{v} = \sum_{j=1}^{n} \frac{q_j}{v} = \sum_{j=1}^{n} q_j'$

i.e. Maximize $q_0 = q_1' + q_2' + \ldots + q_n'$

Subject to the constraints :

$$a_{11} q_1' + a_{12} q_2' + \ldots + a_{1n} q_n' \le 1$$
$$a_{21} q_1' + a_{22} q_2' + \ldots + a_{2n} q_n' \le 1$$
$$\vdots$$
$$a_{m1} q_1' + a_{m2} q_2' + \ldots + a_{mn} q_n' \le 1$$
$$q_j' \ge 0, \quad j = 1\,2 \ldots n.$$

After the optimum solution is obtained using the new simplex method, the original optimum values can be determined.

Notice that B's problem is actually the dual of A's problem. Thus if one problem is solved, that will automatically yield the solution to the other.

Example1: Solve the following 3×3 game by linear programming:

	Player B		
Player A	1	–1	–1
	–1	–1	3
	–1	2	–1

Solution: The given payoff matrix does not possess a saddle point. Since the maximin value is (–1), it is possible that the value of the game may be non – positive. Thus a constant $C \ge 1$ is added to all the elements of the payoff matrix.

Let $C = 2$, the payoff matrix then becomes

	Player B		
Player A	3	1	1
	1	1	5
	1	4	1

The problem of player A is to determine p_1, p_2 and p_3 so as to

Minimize $p_0 = \frac{1}{u} = p_1' + p_2' + p_3'$

Subject to the constraints :

$$3p_1' + p_2' + p_3' \ge 1$$
$$p_1' + p_2' + 4p_3' \ge 1$$
$$p_1' + 5p_2' + p_3' \ge 1,$$
$$p_1', p_2', p_3' \ge 0$$

where $p_i' = \frac{p_i}{u}$; $u =$ minimum expected gain of A.

The problem of player B is to determine q_1, q_2, q_3 so as to

Maximize $q_0 = \frac{1}{v} = q_1' + q_2' + q_3'$

subject to the constraints :

$$3q_1' + q_2' + q_3' \le 1$$
$$q_1' + q_2' + 5q_3' \le 1$$
$$q_1' + 4q_2' + q_3' \le 1,$$
$$q_1', q_2', q_3' \ge 0.$$

where $q_j' = \frac{q_j}{v}$; $v =$ maximum expected loss of B.

Let us solve B's problem by simplex method. Introducing the slack variable q_4', q_5', q_6' respectively in the constraints of the problem, one obtains the following simplex tables :

Table I. **Initial Simplex Table**

		C =	1	1	1	0	0	0	
C_B	y_B	x_B	y_1	y_2	y_3	y_4	y_5	y_6	Ratio
0	y_4	1	3	1	1	1	0	0	1/1
0	y_5	1	1	1	**5**	0	1	0	1/5
0	y_6	1	1	4	1	0	0	1	1/1
		0	–1	–1	–1	0	0	0	$Z_j - C_j$
		Ψj	4	6	**7**				
					↑		↓		

Here max [($Z_j - C_j$) + Ψj] is the entering vector, where $\Psi_j = \sum a_{ij}$.

First Iteration : Introduce y_3 and leave y_5 from the basis.

Table II.

		C =	1	1	1	0	0	0	
C_B	y_B	x_B	y_1	y_2	y_3	y_4	y_5	y_6	Ratio
0	y_4	4/5	4/5	4/5	0	1	–1/5	0	4/4
1	y_3	1/5	1/5	1/5	1	0	1/5	0	1/1
0	y_6	4/5	4/5	**19/5**	0	0	–1/5	1	4/19
		1/5	–4/5	–4/5	0	0	1/5	0	$Z_j - C_j$
		Ψj	9/5	24/5			1/5		

Table III. **Second Iteration:**

		C =	1	1	1	0	0	0	
C_B	y_B	x_B	y_1	y_2	y_3	y_4	y_5	y_6	Ratio
0	y_4	12/19	**50/19**	0	0	1	-3/19	-4/19	12/50
1	y_3	3/19	3/19	0	1	0	4/19	-1/19	3/3
1	y_2	4/19	4/19	1	0	0	-1/19	5/19	4/4
		7/19	-12/19	0	0	0	3/19	4/19	$Z_j - C_j$
		Ψj	57/19				0	0	
			↑			↓			

Table IV. Third Iteration :

		C =	1	1	1	0	0	0	
C_B	y_B	x_B	y_1	y_2	y_3	y_4	y_5	y_6	
1	y_1	6/25	1	0	0	19/25	–3/50	–2/25	
1	y_3	3/25	0	0	1	–3/50	11/50	–1/25	
1	y_2	4/25	0	1	0	–2/25	–1/25	7/25	
		13/25	0	0	0	6/25	3/25	4/25	$Z_j - C_j$

Since all $Z_j - C_j \geq 0$, the optimum solution has been attained. Thus, for the original problem, the expected value of the game is given by

$$v^* = \frac{1}{q_0} - C \;= \frac{25}{13} - 2 \;= \frac{-1}{13}$$

and the optimum mixed strategy for B is given by

$$q_1^* = \frac{q_1'}{q_0} = \frac{6}{25} \times \frac{25}{13} = \frac{6}{13},$$

$$q_2^* = \frac{q_2'}{q_0} = \frac{4}{25} \times \frac{25}{13} = \frac{4}{13},$$

$$q_3^* = \frac{q_3'}{q_0} = \frac{3}{25} \times \frac{25}{13} = \frac{3}{13}.$$

The optimum strategies for A are obtained from the dual solution to the above problem.

The optimum values for p_1', p_2' and p_3' , where $p_i' = \frac{p_i}{u}$ $(i = 1, 2 \dots 3)$ are read off from the net evaluation row of the above optimum simplex table under y_4 y_5 and y_6, because A's problem is the dual of B's problem.

Thus $p_1' = \frac{6}{25}$, $p_2' = \frac{3}{25}$, $p_3' = \frac{4}{25}$, $p_0 = q_0 = \frac{13}{25}$.

Hence the optimum mixed strategy for A is given by

$$p_1^* = \frac{p_1'}{p_0} = \left(\frac{2}{25}\right)\left(\frac{25}{13}\right) = \frac{6}{13},$$

$$p_2^* = \frac{p_2'}{p_0} = \left(\frac{3}{25}\right)\left(\frac{25}{13}\right) = \frac{3}{13},$$

$$p_3^* = \frac{p_3'}{p_0} = \left(\frac{4}{25}\right)\left(\frac{25}{13}\right) = \frac{4}{13}$$

Hence the optimum solution to the original game problem is

$$S_A = \begin{bmatrix} A_1 & A_2 & A_3 \\ 6/13 & 3/13 & 4/13 \end{bmatrix},$$

$$S_B = \begin{bmatrix} B_1 & B_2 & B_3 \\ 6/13 & 4/13 & 3/13 \end{bmatrix},$$

$$v^* = -\frac{1}{13}.$$

Example 2: Solve the following 3×3 game by linear programming:

	Player B		
Player A	8	9	3
	2	5	6
	4	1	7
	1	4	1

Solution The problem of player A is to determine p_1, p_2 and p_3 so as to

Minimize $p_0 = \frac{1}{u} = p_1' + p_2' + p_3'$

Subject to the constraints:

$$8p_1' + 2p_2' + 4p_3' \geq 1$$
$$9p_1' + 5p_2' + p_3' \geq 1$$
$$3p_1' + 6p_2' + 7p_3' \geq 1,$$
$$p_1',\ p_2',\ p_3' \geq 0$$

Where $p_i' = \frac{p_i}{u}$; $u =$ minimum expected gain of A.

The problem of player B is to determine q_1, q_2, q_3 so as to

Maximize $q_0 = \frac{1}{v} = q_1' + q_2' + q_3'$

subject to the constraints :

$$8q_1' + 9q_2' + 3q_3' \leq 1$$
$$2q_1' + 5q_2' + 6q_3' \leq 1$$
$$4q_1' + q_2' + 7q_3' \leq 1,$$
$$q_1', q_2', q_3' \geq 0.$$

where $q_j' = \frac{q_j}{v}$; $v =$ maximum expected loss of B.

Let us solve B's problem by simplex method. Introducing the slack variable q_4', q_5', q_6' respectively in the constraints of the problem, one obtains the following simplex tables :

Initial Simplex Table

		C =	1	1	1	0	0	0	
C_B	y_B	x_B	y_1	y_2	y_3	y_4	y_5	y_6	Ratio
0	y_4	1	8	9	3	1	0	0	1/3
0	y_5	1	2	5	**6**	0	1	0	1/6
0	y_6	1	4	1	7	0	0	1	1/7
		0	–1	–1	–1	0	0	0	$Z_j - C_j$
		Ψj	14	15	16				
					↑			↓	

First Iteration : Introduce y_3 and leave y_6 from the basis.

		C =	1	1	1	0	0	0	
C_B	y_B	x_B	y_1	y_2	y_3	y_4	y_5	y_6	Ratio
0	y_4	4/7	44/7	60/7	0	1	0	-3/7	1/15
0	y_5	1/7	-10/7	**29/7**	0	0	1	-6/7	1/29
1	y_3	1/7	4/7	1/7	1	0	0	1/7	1
		1/7	-17/7	22/7	-1	-1	-1	-6/7	$Z_j - C_j$
		Ψj							
					↑		↓		

Second Iteration : Introduce y_2 and leave y_4 from the basis

		C =	1	1	1	0	0	0	
C_B	y_B	x_B	y_1	y_2	y_3	y_4	y_5	y_6	Ratio
0	y_4	8/29	268/29	0	0	1	-60/29	39/29	12/50
1	y_2	1/29	-10/29	1	0	0	7/29	-6/29	3/3
1	y_3	4/29	63/29	0	1	0	-1/29	5/29	4/4

		13/29	321/29	0	0	0	66/2919	68/2919	$Z_j - C_j$
			↑			↓			

Third Iteration : Introduce y_1 and leave y_4 from the basis

		C =	1	1	1	0	0	0	
C_B	y_B	x_B	y_1	y_2	y_3	y_4	y_5	y_6	
1	y_1	2/67	1	0	0	29/68	-15/67	39/268	
1	y_2	3/67	0	0	1	5/134	11/67	-21/134	
1	y_3	8/67	0	1	0	-9/134	7/67	11/134	
		13/67	0	0	0	21/268	12/268	19/268	$Z_j - C_j$

Since all $Z_j - C_j \geq 0$, the optimum solution has been attained. Thus, for the problem, the expected value of the game is given by

$$v^* = \frac{1}{q_0} = 13/67$$

and the optimum mixed strategy for B is given by

$$q_1^* = \frac{q_1'}{q_0} = \frac{2}{67} \times \frac{67}{13} = \frac{2}{13},$$

$$q_2^* = \frac{q_2'}{q_0} = \frac{3}{67} \times \frac{67}{13} = \frac{3}{13},$$

$$q_3^* = \frac{q_3'}{q_0} = \frac{8}{67} \times \frac{67}{13} = \frac{8}{13}.$$

The optimum strategies for A are obtained from the dual solution to the above problem.

The optimum values for p_1', p_2' and p_3', where $p_i' = \frac{p_i}{u}$ $(i = 1, 2 \dots 3)$ are read off from the net evaluation row of the above optimum simplex table under y_4 y_5 and y_6, because A's problem is the dual of B's problem.

Thus $p_1' = 21/268$, $p_2' = 12/268$, $p_3' = 19/268$,

Hence the optimum mixed strategy for A is given by

$$p_1^* = \frac{p_1'}{p_0} = \left(\frac{21}{268}\right)\left(\frac{67}{13}\right) = \frac{21}{52},$$

$$p_2^* = \frac{p_2'}{p_0} = \left(\frac{12}{268}\right)\left(\frac{67}{13}\right) = \frac{12}{52},$$

$$p_3^* = \frac{p_3'}{p_0} = \left(\frac{19}{268}\right)\left(\frac{67}{13}\right) = \frac{19}{52}$$

Hence the optimum solution to the original game problem is

$$S_A = \begin{bmatrix} A_1 & A_2 & A_3 \\ 21/52 & 12/52 & 19/52 \end{bmatrix}, \quad S_B = \begin{bmatrix} B_1 & B_2 & B_3 \\ 2/13 & 3/13 & 8/13 \end{bmatrix},$$

$$v^* = \frac{67}{13}.$$

Example3:. Solve the following 2×3 game by linear programming:

	Player B		
Player A	3	–1	4
	6	7	-2

Solution : Since two of the entries in the pay-off matrix are negative a constant $C \geq 1$ is added to all the elements of the payoff matrix.

Let 3, the payoff matrix then becomes

	Player B		
Player A	6	2	7
	9	10	1

The problem of player A is to determine p_1, p_2 and p_3 so as to

Minimize $p_0 = \frac{1}{u} = p_1' + p_2'$

Subject to the constraints :

$$6p_1' + 9p_{23}' \geq 1$$

$$2p_1' + 10p_{23}' \geq 1$$

$$7p_1' + p_2' \geq 1,$$

$$p_1', p_2' \geq 0$$

Where $p_i' = \frac{p_i}{u}$; $u =$ minimum expected gain of A.

The problem of player B is to determine q_1, q_2, q_3 so as to

Maximize $q_0 = \frac{1}{v} = q_1' + q_2' + q_3'$

subject to the constraints :

$$6q_1' + 2q_2' + 7q_3' \leq 1$$

$$9q_1' + 10q_2' + q_3' \leq 1;$$

$$q_1', q_2', q_3' \geq 0.$$

where $q_j' = \frac{q_j}{v}$; $v =$ maximum expected loss of B.

Let us solve B's problem by simplex method. Introducing the slack variable q_4', q_5', q_6' respectively in the constraints of the problem, one obtains the following simplex tables :

Initial Simplex Table

		C =	1	1	1	0	0	
C_B	y_B	x_B	y_1	y_2	y_3	y_4	y_5	Ratio
0	y_4	1	6	2	7	1	0	1/6

0	y_5	1	**9**	10	1	0	1	1/9
		0	–1	–1	–1	0	0	$Z_j - C_j$
		Ψj	15	12	8			
					↑			

First Iteration : Introduce y_1 and leave y_5 from the basis.

		C =	1	1	1	0	0	
C_B	y_B	x_B	y_1	y_2	y_3	y_4	y_5	Ratio
0	y_4	1/3	0	-14/3	**19/3**	1	-2/3	1/19
1	y_1	1/9	1	10/9	1/9	0	1/9	1
		1/9		1/9	-8/9		1/9	$Z_j - C_j$
		Ψj	0	-2	39/9	0	-5/9	
					↑		↓	

Second Iteration : Introduce y_1 and leave y_5 from the basis

		C =	1	1	1	0	0	
C_B	y_B	x_B	y_1	y_2	y_3	y_4	y_5	Ratio
1	y_3	1/19	0	-14/19	1	3/19	-2/19	-14
1	y_1	2/19	1	**68/57**	0	-1/57	7/57	3/34
			0	11/57	0	-1/57	7/57	$Z_j - C_j$
		Ψj	2	26/57		8/57	1/57	

Third Iteration : Introduce y_2 and leave y_1 from the basis

		C =	1	1	1	0	0	
C_B	y_B	x_B	y_1	y_2	y_3	y_4	y_5	Ratio
1	y_3	2/17	42/68	0	1	5/34	-1/34	-14
1	y_2	3/34	57/68	1	0	-1/68	7/68	3/34
		7/34	29/68	0	0	9/68	5/68	$Z_j - C_j$

Thus, for the original problem, the expected value of the game is given by

$$v^* = \frac{1}{q_0} - C \quad = \frac{34}{7} - 3 \quad = \frac{13}{7}$$

and the optimum mixed strategy for B is given by

$$q_1^* = \frac{q_1'}{q_0} = \frac{9}{68},$$

$$q_2^* = \frac{q_2'}{q_0} = \frac{5}{68},$$

Hence the optimum solution to the original game problem is

$$S_A = \begin{bmatrix} A_1 & A_2 \\ 9/14 & 5/14 \end{bmatrix}, \; S_B = \begin{bmatrix} B_1 & B_2 & B_3 \\ 0 & 3/7 & 4/7 \end{bmatrix}, \; v^* = \frac{7}{34}.$$

CONCLUSION

We observed that the solution of Game Theory problem has been obtained by our technique very easily and requires less or at the most equal number of iterations than traditional simplex method. This technique is very useful to apply on numerical problems, reduces the labour work, gives more accuracy and improved optimum solution. Therefore this method is more powerful in solving Game Theory problems as compare to traditional simplex method.

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