



## Graphic Integer Sequence as Canopy of Graphs

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**Abstract:** Canopy is the organization or spatial arrangement. We are trying here to bring all the graph theoretic problems under it. Where our root is on the earth and head is towards infinity if any. The root of canopy may be zero (of course not well defined) but mind set dictates that the universe may be partially visual. Graphic integer sequence may play a vital role to cover all the related problems in graph theory.

**Keyword:** Canopy, Degree Sequence, Reconstruction, label, polynomial, relation, Cut set, circuit, tree and block.

### 1. INTRODUCTION

Any physical problem can be represented and manipulated in digital computer by codes (discrete, finite) [9,12]. Computer representation of a graph has different fascicles [12]. We are concentrating in a general number sequence usually denary system called degree sequence. Any of the sequence cannot be graphic sequence. They need some additional constraints [4]. After satisfying some constraints it become the degree sequence of a graph i.e. graphic sequence.

### 2. CANOPY(N)

When a new term or definition is introduced in graph theory like concept of quantum computing and later a prize winning terminology that may occupy a space in Wikipedia. As observed by Knuth and quoted as “The number of system of terminology presently used in graph theory is equal, to close approximation, to the number of graph theorists.(Richard P. Stanley(1986))[2]”. We introduce a “n” integer permutation and combination since that is wonderful, smell like an umbrella in a disturbed atmosphere, a place for small party in a resort or a pandle coverage for a ceremony. The reflection of a tree in the root and the top shows the scenario which is open to expand at any time. The canopy filters the sunlight, softens the rain, and blocks the wind so that younger, more tender life may thrive. In turn, the roots of the trees draw nutrients and moisture from the soil and organic material on the forest floor, conveying it up to the green canopy and keeping it lush and growing.

Similarly, interests and ideas which foster creativity keep people vital and growing. They provide sanctuary from the pressures of life, and allow people to discover who they truly are. Each person brings new perspective to ancient ideas,

stretching the boundaries much as the canopy stretches toward the sky.

The canopy is the topmost layer of life which provides shelter for all that resides beneath it.

**Definition1:-** Canopy (n) is the universal structure consisting of a set of points and their interconnection. It encompasses different sub areas.

For example in terms of ‘n’ points we can classify different graph theoretic structures following the basic graph theoretic

According to M.R. Garey / D.S. Johnson [3] most of the intractable problems can be reduced polynomial to graph problems. Degree sequence is a numerical sequence that represents many of the exemplary algorithmic paradigms.

In our realization on studying different problem, the degree sequence gives a focus on each of the problem. For example the tree of a graph is the skeleton of the canopy. Hamiltonian path is one of the covers (Like an umbrella). So we introduce one terminology in graph theory that is canopy. Our philosophy says that graphic integer sequence is suitable to identify as canopy of graph.

axioms.

Axioms:

i) A set of points  $n$ , finite or infinite.

ii) Sum of the interconnections among the points is even.

Canopy has the following structure which may not be finite. We initially give a graphical structure (like family tree) that summarises the present domain of graph theoretic cultural heritage.

We try to define properties of the graph theory as

Properties:-i) Adjacency ii) Incidence

iii) Degree Sequence

2.1. Million dollar question- Why another term in graph theory?

We find that scientists search for unified theory in any field for getting a glorified footage. We have met the axioms, theorem, conjecture, lemma in graph algorithms that need to be precisely collected and reformed always as a ready reference. For Example D. E. Knuth when writing his “Art of Computer Programming [2]” used graphs for his future plan or for reading the volumes. His infinite plan shows us a posturize view in volume 4. He should justify to the graph: that he described as the beginning. We think any mathematical term should study as canopy (n) before proceeding further with progenies.

The large number of graph algorithms can be solved in polynomial time if it is labeled graph. In other words we reduce the size of the problem humanologically. In our deigned figure of Canopy of the graph we tried to cover many of the well-known problems and ambitious that later a large number of problems should be included under this canopy. Our effort to bring all classes of graph under the same umbrella.

2.2. Classification:-

2.2.1. Classification for representation of graph.

Canopy is an ancestor with infinitely numerous siblings. Graph is one successor on which our attention is focused now. There are a large variety of parameters to define a graph. Some of them are independent to directly represent the graph and others are combinations of the parameters to represent the graph as unique or any graph. We should not deceive readers the in fact only integers can represent a graph with a variety combinatorial combination of the parameters. There are

- i) Relation and Functions [10]:-The first one is as relation and functions, where we can define the graph as “A graph G consists of a finite set of vertices and an irreflexive binary relation on vertices. The binary relation may be termed either as a collection of edges of ordered pairs or as a function from vertices to its power set”. The classical concept of domain and co-domain is also relevant in our general definition.
- ii) Level and adjacency information [10,11]:- When each vertex are assigned a unique name and the adjacency are mapped as: Adj: V -> p(V) i.e. a function from V to its power set.  
We can identify the unique graph using these techniques. A large number of graph theorist problem those are exponential in nature are polynomial solvable for their labeled representation. For example isomorphism checking, reconstruction etc.
- iii) Cut Sets [11]:- As we know from set theory that the union of all subset gives the original set. Here we take one special type of sub graph of connected graph whose removal from the original graph separates in two or more sub graphs. Another form of cut set is fundamental cut set [11]. Fundamental Cut Set determines the dimension of the particular graph.
- iv) Trees [11]:- A tree is a minimally connected sub graph. If we explore the all the trees of a graph, then their commutative sum can give the original graph. The reverse is also true. We have designed various algorithms [18, 20] in different approaches for the reverse circumstances.
- v) Circuit [1, 11]:- A walk is defined as a finite alternating sequence of vertices and edges beginning and ending with vertices. Occurrence of vertex may be more than once but edges only once. An open walk is path and a closed walk in which no vertex appears more than once is called a circuit [11].
- vi) Block [11]:-A connected non-trivial sub graph having no cut point is a block. A block is maximal with respect to his property. If we can get the all blocks of a graph, then we can construct the original graph. Reconstruction conjecture is one of the best examples for these techniques. We have also designed an algorithm for this purpose [15].
- vii) Polynomials[1,11,13,19]:- Given a non-negative integer d, a polynomial in ‘m’ of degree d is a function P(m) of the form  $P(m)=\sum_{i=0}^d a_i m^i$ ; Where the constraints  $a_0, a_1, \dots, a_d$  are the coefficient the polynomials and  $a_d \neq 0$ .

A graph can be expressed elegantly by means of polynomials. There are different types of polynomials those can expressed the different nature of the graph

different representation for the same graphs. Efficiency of graph algorithms depends upon the information structure for a problem specific graph [6]. Our primary objective is to explore unique representation of a graph that will serve this purpose. There is no standardization in this aspect [19]. The process is to start with a standard representation and improve upon it depending upon the experience gained in attempts to design an algorithm with the graph as input. The experience allows us to recognize the occurrences of simple notation like symmetry or useful sequence of operations that may lead us quickly towards solutions.

and can be applied to the different graph theoretic problems. Such as Characteristic polynomial [11,20] for the isomorphism testing of a graph, Chromatic polynomial [11,5,20] for graph coloring, rank polynomial [11,5,20] for reconstruction of trees and circuits and matching polynomials[13,20] are for reconstruction conjecture etc.. These all polynomials are reconstructible. Each representation of the polynomials can represent the unique some specific graphs. Sum of two or more polynomials can represent a unique graph. The polynomials information and degree sequence together can represent the unique graph or up to isomorphism. It can also overlap with the level and adjacency to represent the original graph.

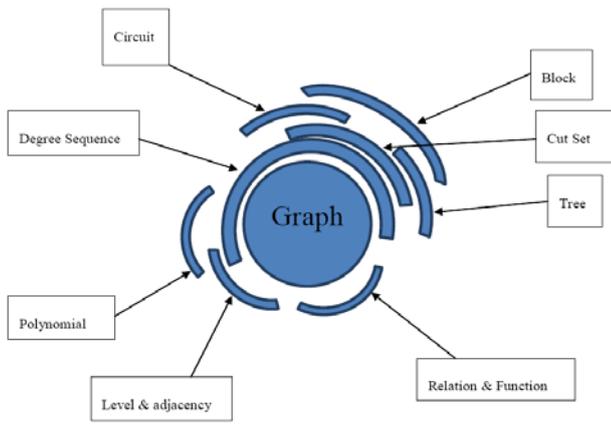
viii) Degree Sequence[1,2,11]:- A sequence  $d_1, d_2, \dots, d_n$  of non negative integers is called graphical if there's at least one graph on vertices  $\{1, 2, \dots, n\}$  such that vertex K has degree  $d_k$  [2,11].

Degree sequence is an inherent characteristic of any graph. From a non negative integer sequence we can recognize it as graphic sequence or degree sequence after some constraints satisfaction. We can draw a random graph [2] as well as we can identify different characteristics of the graph [7, 8, 13, 15,16,17,18]. If the labeling are added and perform we can draw the graph uniquely.

This parameter is much more essential among the others to represent the graph with the help of combinations of other parameters such as cut set, tree, circuit, block and polynomials as shown in the figure1. Degree sequence is acting as pandel coverage of graph or canopy.

ix) Etc.- For future inclusion of the different representation which can itself or power set of some extra information can represent a graph.

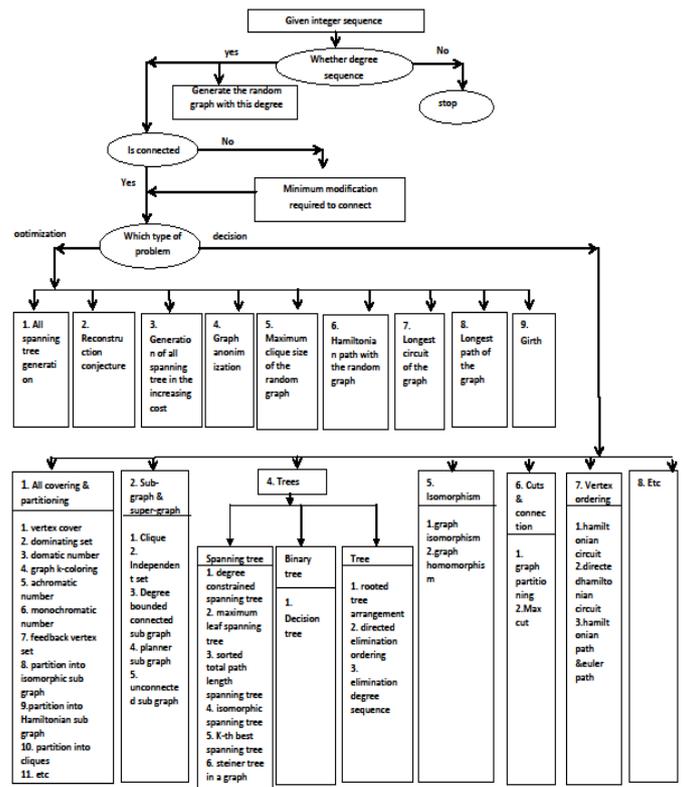
The combinations those are identified in the Fig1 can uniquely identify the graph is as i) Relation and function ii) Level and adjacency iii) Level and degree sequence iv) level, polynomials and degree sequence v) Degree sequence and circuit vi) Degree sequence, cut set and block vii) degree sequence and tree viii) degree sequence, cut set and tree. Ix) Degree sequence, block and tree x) degree sequence, block and circuit xi) degree sequence, cut set and circuit etc.



**Fig1. Representation of Graph**

**2.2.2. Classification by problem type:-**

At first of our designated diagram, we started with a finite integer sequence. After it satisfied with different constraints it recognized as the degree sequence. If it disagrees, it does not become a degree sequence. When it established as the degree sequence we tried to generate a random graph according to a well-known algorithm [2]. Then we look up to the checking that the random graph generated for that degree sequence is connected or not [14,15]? If not connected then need to minimum modification this random graph to make this graph as connected. Here we can check for the degree sequence can represent an Euler’s graph or trail [7] and whether it can represent an planer graph? If so then generate the random graph for the specified problem. Then we come to the one of the point where we have to decide the nature of the problem i.e., Decision and optimization. If it belongs to the first one then it covers the maximum of the NP-complete problems [3] in seven categories as shown in the figure. The categories are as 1) All covering and partitioning 2) Sub graph and Super Graph 3) Trees 4) Isomorphism 5) Cut’s and Connections 6) Vertex Ordering and 7) Etc. for future inclusion. Each of these categories consist a large number of problems. The third category of this division contains also three sub division. They are i) Tree ii) Spanning tree iii) Binary tree.



**Fig. Canopy of the graph**

**3. CONCLUSION**

Majority of graph algorithms mapped to integer sequence. Our coinage of the term graphic integer sequence is more appropriate because it is generalization and gives a global view of graphs used in general for illustrations and very much cohesion with the discipline of computing. We typically try to find patterns or arrangements that are the best possible ways to satisfy certain constraints. The number of such problems is vast. And the art of writing such programs is especially important and appealing because a single good idea can save years or even centuries of computer time.

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