Volume 8, No. 3, March – April 2017
International Journal of Advanced Research in Computer Science
RESEARCH PAPER
Available Online at www.ijarcs.info
ISSN No. 0976-5697

IFZ-Graph and Extracting Shortest Path by IFZ-Dijkstra's Algorithm

Siddhartha Sankar Biswas
Department of Computer Science and Engineering
Faculty of Engineering and Technology
Jamia Hamdard
New Delhi, India

Abstract: In this paper the author introduces the notion of IFZ-graph in Graph Theory. The classical Dijkstra’s algorithm to find the shortest path in graphs is not applicable to IFZ-graphs. Consequently the author proposes a new algorithm called by IFZ-Dijkstra's Algorithm to solve the Shortest Path Problem (SPP) in an IFZ-graph.

Keywords: Z-number, Z-distance, Z-graph, IFZ-number, IFZ-distance, IFZ-graph, IFZ-Dijkstra's

I. INTRODUCTION

Shortest Path problems (SPP) are among the fundamental problems studied by researchers in Computational Geometry, Graph Algorithms, Geographical Information Systems (GIS), Network Optimization etc. to list a few only out of many. In many situations, the network of a communication or transportation problem can not be modeled into a graph but into a multigraph [5-15] only. In such a case the standard algorithms of graph theory cannot be applied in SPP. Biswas et. el. [5-15] has done rigorous analysis of SPP in multigraphs. However, in our work here we consider only those networks which can be modeled into graphs.

The notion of classical graphs is extended to define fuzzy graphs [4, 16] for soft computing using graph theoretic algorithm some soft-computing set theory’s under . The Fuzzy Shortest Path Problem (FSPP) [18, 23, 25, 33-37, 39-48] is a generalization of the classical SPP for applications in ill-defined environment and has been found important to many applications such as Communication or Transportation Network, Computational Geometry, Graph Algorithms, Geographical Information Systems (GIS), Network Optimization, etc. In the classical shortest path problems, the cost of an arc of the corresponding network takes crisp numbers. But in the real-life situation the arc length may represent transportation time or cost which can be known only by an approximate amount due to impreciseness of information, and hence it can be considered a fuzzy number or an intuitionistic fuzzy number. The nodes are well precise, but the data about the weights or costs of the links are sometimes not available as classical crisp numbers, rather fuzzy numbers or intuitionistic fuzzy numbers or Z-numbers. Recently Zadeh has introduced a new direction in the theory of soft computing by defining the concept of Z-number [52-54]. Then Velammal and Shahila Bhanu in [49] defined Intuitionistic Z-number (IFZ-number) as a generalization of Z-number. In our work here we use the theory of IFZ-numbers.

Fuzzy shortest path problem (FSPP) in graphs was initiated by Dubois and Prade [28] and then by Klein [34]. The work of Dubois and Prade [28] was a break-through in Graph Theory, but that paper lacked practical interpretation as even if fuzzy shortest path is mathematically computed, but this need not be any of the path in the corresponding network. According to the approach proposed by Dubois and Prade [28], the shortest path length can be computed, but the corresponding path in the network may not exist in reality. This drawback of their solution method made sometimes infeasible for application in network problems. Klein [34] proposed a dynamic programming recursion-based fuzzy algorithm. Lin and Chen [36] found the fuzzy shortest path length in a network by means of a fuzzy linear programming approach. Okada and Soper [39-45] proposed a fuzzy algorithm, which was based on multiple-labelling methods to offer non-dominated paths to a decision maker. Chuang and Kung [20] proposed a fuzzy shortest path length procedure that can find a fuzzy shortest path length among all possible paths in a network. Yao and Lin [50] presented two new types of fuzzy shortest path network problems.

The main results obtained from their studies were that the shortest path in the fuzzy sense corresponds to the actual paths in the network, and the fuzzy shortest path problem is an extension of the crisp case. Nayeem and Pal [38] have proposed an algorithm based on the acceptability index introduced by Sengupta and Pal [46] which gives a single fuzzy shortest path or a guideline for choosing the best fuzzy shortest path according to the decision maker’s viewpoint. Thus, numerous papers have been published in different journals/books on the fuzzy shortest path problem (FSPP).

In this work the authors introduces IFZ-graph and solves the SPP in an IFZ-graph by developing a new algorithm called by IFZ-Dijkstra's Algorithm. It is claimed that the IFZ-Dijkstra's Algorithm can be applicable in much wider domains of optimization problem.

II. ABOUT Z-NUMBERS AND IFZ-NUMBERS

The Z-number is a new fuzzy-theory based notion introduced by Zadeh in [52-54]. Not much work reported so far in the literature about the theory of Z-numbers because of its recent birth. But because of its inbuilt strong modeling it will surely take a huge role to the scientists and engineers of several fields in the near future.

A. Z-number

Decisions are based on information. To be useful, information must be reliable. Basically, the concept of a Z-number relates to the issue of reliability of information.

A Z-number Z is an ordered pair, having two components and hence can be expressed using the expression Z = (A, B). The first component, A, is a restriction (generalized constraint) on the values which a real-valued uncertain variable, X, is allowed to take. The second component, B, is a measure of reliability (certainty) of the first component. Typically, A and B are described in a natural language.

Thus a Z-number can be expressed as an ordered pair of fuzzy numbers denoted as Z = (A, B). For simplicity, in our work here A and B are assumed as triangular fuzzy numbers. Clearly Z-numbers can be used to model uncertain information in real world. For example, in risk analysis, the loss of severity of the fifth component is very low, with a confidence of very likely, which can be written as a Z-number as follows Z = (very low; very likely). Example of another type of Z-numbers are :

Z1 = (about 45 minutes, very sure), Z2 = (about 30 minutes, sure).

A Z-number Z = (A, B) is called to be a null Z-number denoted by 0z, if both A and B are null fuzzy numbers.

B. *IFZ-number*

In [49], Velammal and Shahila Bhanu defined : IFZ-number as - An ordered pair (A,B) is said to be an intuitionistic Z-number (IFZ-number) if A is a fuzzy set defined on the real line and B is a intuitionistic fuzzy set defined on the interval [0,1].

C. *Z-valuation of IFZ-number*

The Let X be a real random variable and (A,B) be an IFZ-number. Let A(x) be the membership function of A. Let B(x) be the membership function of B, and b(x) be the non-membership function of B .Then (the ordered triple (X,A,B) is a Z-valuation which is equivalent to the following statement:

FEP (X is A) is B. More explicitly we may write:
Possibility (FEP(X is A) = u) = B(u)
Impossibility (FEP(X is A) = u) = b(u)

D. *Some Operation on IFZ-numbers*

Consider two IFZ-numbers Z1 and Z2 given by

Z1 = (A1, B1) and Z2 = (A2, B2), where Ai and Bi are respectively triangular fuzzy numbers and triangular intuitionistic fuzzy numbers.

There are a number of methods (viz. [1], [2], [21], [29]) for ranking triangular fuzzy numbers. Similarly, there are a number of methods for ranking triangular intuitionistic fuzzy numbers. We follow the Centroid Method because of its simplicity and appropriateness in the philosophy of soft computing.

We say that Z1 is strongly greater than Z2 if A1 > A2 and B1 > B2.

Otherwise, We say that Z1 is weakly greater than Z2 if A1 > A2.

Addition ($\oplus$) of two IFZ-numbers Z1 and Z2 yields another IFZ-number Z3 given by

$$Z3 = Z1 \oplus Z2 = (A1 + A2, B1 + B2).$$

As comment made by Zadeh, it is fact that computing with Z-numbers is an important generalization of computing with real numbers. In particular, the generality of Z-numbers opens the door to construction of better models of reality, especially in fields such as decision analysis, planning, risk assessment, economics and biomedicine. The IFZ-numbers being the generalization of Z-numbers will provide further amount of scope to the soft-computing scientists.

III. IFZ-GRAPH

In this section we introduce the notion of IFZ-graph. An IFZ-graph is basically a generalized concept of the Z-graph which is a generalized concept of fuzzy graph. Consider the following graph (see Figure 1) where at least one of the weights is IFZ-number. This type of graph is called an IFZ-weighted graph or IFZ-graph (in short).

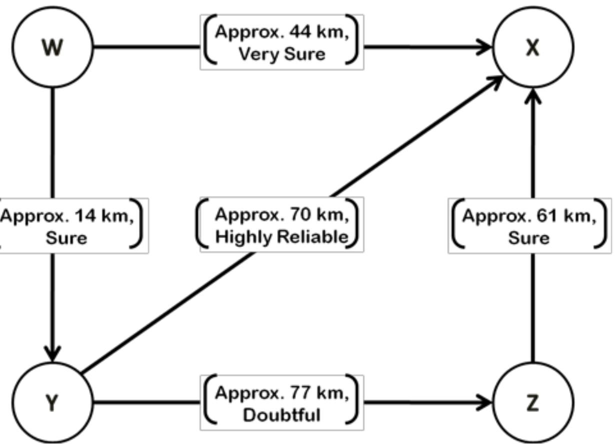

Figure 1. Example of a One-Column figure caption.

In this IFZ-graph in Figure 1, the edge WX has the FZ-weight the IFZ-number (Approx. 44 km, very sure), the edge WY has the IFZ-weight the FZ-number (Approx. 14 km, very sure), etc. The IFZ-number weight of an edge is also called IFZ-distance between the corresponding two nodes.

IV. IFZ-DIJKSTRA'S ALGORITHM FOR SPP IN AN IFZ-GRAPH

The Classical Dijkstra's algorithm [27] solves the single-source shortest path problems in a simple graph. In this section we develop a new algorithm called by IFZ-Dijkstra's Algorithm (of the style of the classical famous Dijkstra's algorithm) to solve a SPP in a IFZ-graph.

A. *Shortest path estimate of a vertex in a directed IFZ-graph*

Consider an IFZ-weighted directed graph G = (V, E). IFZ-shortest path estimate d[v] of any vertex v, where vertex v is one of the neighboring vertices of the currently traversed vertex u , is the IFZ-distance between the vertex v and vertex u , added with the shortest IFZ-distance between the starting vertex s and vertex u , where s, u, v ∈ V[G].

$$\therefore \ d[v] = (\text{shortest IFZ-distance between } s \text{ and } u) \oplus (\text{IFZ-weight of arc between } v \text{ and } u)$$

This is shown below in Figure 2.

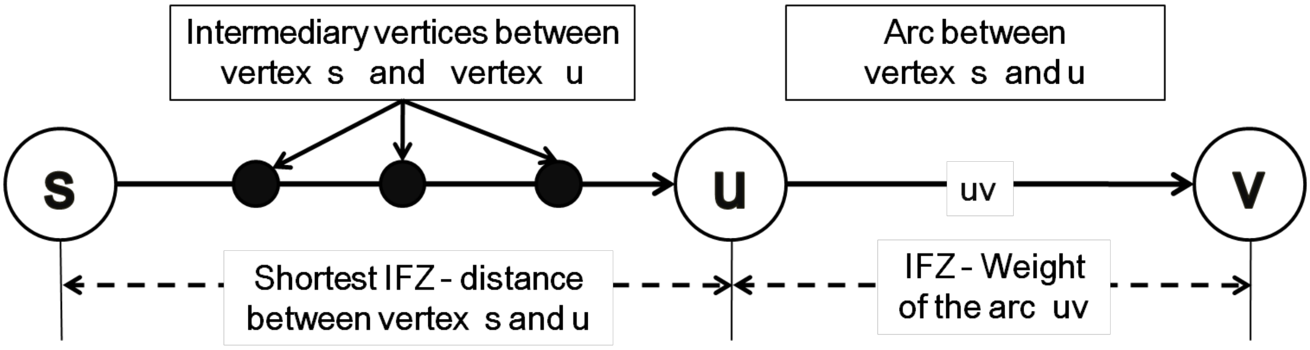

Figure 2. d[v] in an IFZ-graph

B. *Relaxation of an arc in our proposed IFZ-Dijkstra's Algorithm*

For the relaxation process of an arc to happen, we must first initialize the graph along with its starting vertex s and IFZ-shortest path estimate for each vertices of the graph G.

```
INITIALIZE-SINGLE-SOURCE(G, s)
FOR each vertex v ∈ V[G]
      d[v] = ∞
      v.π = NIL
d[s] = 0z
```

(Note : We store all predecessor nodes of u in the attribute u.π. Thus s.π is always Nil, because s is the source node).

Now on the basis of this initialization process, IFZ-Dijkstra's algorithm proceeds further and the process of relaxation of each arc begins. The sub-algorithm RELAX, plays the vital role to update d[v] i.e. the IFZ-shortest distance value between the starting vertex s and the vertex v *(which is neighbor of the current traversed vertex u, ∀ u, v ∈ V[G]).* The RELAX algorithm runs as shown below :

```
RELAX(u, v, w)
IF d[v] > d[u] ⊕ w(u,v)
   THEN d[v] ← d[u] ⊕ w(u, v)
              v.π ← u
```

where, w(u, v) is the IFZ-weight of the arc from vertex u and vertex v, and v.π denotes the parent node of a vertex v, ∀ v, v ∈ V[G]. This is shown below in Figure 3.

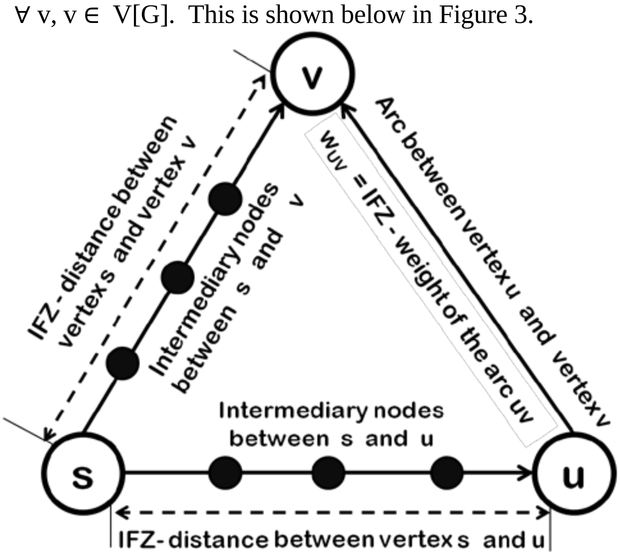

Figure 3. Diagram showing how RELAX algorithm works.

IFZ-Dijkstra's algorithm solves the single-source shortest-path on an IFZ-weighted directed IFZ-graph G = (V, E) for the case in which all edge IFZ-weights are non-negative. IFZ-Dijkstra's algorithm maintains a set S of vertices whose final IFZ-shortest path weights from the source vertex s has already been determined. The algorithm repeatedly selects the vertex u ∈ V – S with the minimum IFZ-shortest- path estimate, adds u to S, and relaxes all edges leaving u. Our proposed algorithm is as follows:

```
IFZ-DIJKSTRA (G, w, s)
INITIALIZE-SINGLE-SOURCE (G, s)
S ← ∅
Q ← V[G]
WHILE Q ≠ ∅
      DO u ← EXTRACT-MIN(Q)
            S ← S ∪ {u}
            FOR each vertex v ∈ Adj[u]
                  DO RELAX(u, v, w)
```

V. Conclusion

The notion of IFZ-graph is a generalization of Z-graph, where the notion of Z-graph is a generalization of fuzzy graph. There are many real life problems of networks in communication systems, transportation, circuit systems, etc. which cannot be modeled into traditional graphs but either into fuzzy graphs or into Z-graphs or into IFZ-graphs only. The classical Dijkstra's algorithm [27] which is for extracting the shortest path in graphs is not applicable to IFZ-graphs.

Consequently, in this work we have done relevant adjustment in the classical Dijkstra's algorithm to update it applicable to IFZ-graphs to find the IFZ-shortest path from a source vertex to a destination vertex. The modified algorithm is called by IFZ-Dijkstra's algorithm. The networks, where the classical Dijkstra's algorithm cannot be applied, can now be well dealt with the IFZ-Dijkstra's algorithm in many such cases.

VI. References

[1] Abbasbandy, Saeid, Ranking of fuzzy numbers, some recent and new formulas, IFSA-EUSFLAT, 2009, Page 642- 646.
[2] Abbasbandy, Saeid., Allahviranloo, T. and Saneifard, R., A Method for Ranking of Fuzzy Numbers using New Weighted Distance, Mathematical and Computational Applications, Vol. 16, No. 2, Page 359-369, 2011.
[3] Balakrishnan, V. K.; Graph Theory, McGraw-Hill; 1997.
[4] Bhutani,K.R. and Rosenfeld,A., Fuzzy End Nodes in Fuzzy Graphs, Information Sciences, Vol. 152 (2003) 323-326.

[5] Biswas, S. S., Alam, B. and Doja, M. N., A Generalized Real Time Multigraphs For Communication Networks : An Intuitionistic Fuzzy Theoretical Model, 17th International Conference on IFS, Sofia, Bulgaria, Proceedings published in Notes on Intuitionistic Fuzzy Sets (Bulgarian Journal) Vol.19 (3) 2013: pp 90-98, ISSN : 1310-4926

[6] Biswas, S. S., Alam, B. and Doja, M. N., GRT-Multigraphs For Communication Networks : A Fuzzy Theoretical Model, International Symposium on System Engineering and Computer Simulation (SECS-2013), Held in Danang, Vietnam. Published at Advanced in Computer Science and its Applications, Series Title : Lecture Notes in Electrical Engineering (Springer Berlin Heidelberg Publications) , Vol. 279 2014, Pages 633-641, Print ISBN : 978-3-642-41673-6 , Online ISBN : 978-3-642-41674-3, doi: 10.1007/978-3-642-41674-3_91.

[7] Biswas, S. S., Alam, B. and Doja, M. N., A Refinement of Dijkstra's Algorithm For Extraction of Shortest Paths in GRT-Multigraphs, Journal of Computer Science, Vol.10 (4) 2013: pp 593-603, ISSN 1549-3636, doi: 10.3844/jcssp.2014.593.603.

[8] Biswas, S. S., Alam, B. and Doja, M. N., Real Time Multigraphs For Communication Networks : An Intuitionistic Fuzzy Mathematical Model, Journal of Computer Science, Vol. 9 (7) 2013: pp 847-855, ISSN 1549-3636, doi: 10.3844/jcssp.2013.847.855.

[9] Biswas, S. S., Alam, B. and Doja, M. N., Intuitionistic Fuzzy Real Time Multigraphs For Communication Networks : A Theoretical Model, AASRI Conference on Parallel and Distributed Computing and Systems (DCS 2013), Held in Singapore, Published by AASRI Proceedings (Elsevier Publications), Vol.5, 2013, Pages 114–119, doi: 10.1016/j.aasri.2013.10.066.

[10] Biswas, S. S., Alam, B. and Doja, M. N., Real Time Graphs For Communication Networks : A Fuzzy Mathematical Model, Sadhana - Academy Proceedings in Engineering Sciences (Springer Publications) , ISSN (print version) : 0256-2499 ISSN(electronic version) : 0973-7677 , Journal no.: 12046.

[11] Biswas, S. S., Alam, B. and Doja, M. N., A Slight Adjustment of Dijkstra's Algorithm for Solving SPP in Real Time Environment, Third International Conference on Computational Intelligence and Information Technology – CIIT 2013, Held in Mumbai, India., Published at International Conference on ComNet CIIT & ITC 2013 Proceedings (Elsevier Publication), pp: 256-259 ISBN: 978-81-910691-6-3.

[12] Biswas, S. S., Alam, B. and Doja, M. N., An Algorithm For Extracting Intuitionistic Fuzzy Shortest Path in A Graph, Applied Computational Intelligence and Soft Computing , Vol.2(2) 2012 (Hindawi Publishing Corporation), Article ID 970197, ISSN: 1687-9724 e-ISSN:1687-9732, http://dx.doi.org/10.1155/2013/970197.

[13] Biswas, S. S., Alam, B. and Doja, M. N., Fuzzy Shortest Path in A Directed Multigraph, European Journal of Scientific Research, Vol.101 (3) 2013: pp 333-339, ISSN 1450-216X / 1450-202X.

[14] Biswas, S. S., Alam, B. and Doja, M. N., Generalization of Dijkstra's Algorithm For Extraction of Shortest Paths in Directed Multigraphs, Journal of Computer Science, Vol.9 (3) 2013: pp 377-382, ISSN 1549-3636, doi: 10.3844/jcssp.2013.377.382.

[15] Biswas, S. S., Alam, B. and Doja, M. N., A Theoretical Characterization of The Data Structure 'Multigraph' , Journal of Contemporary Applied Mathematics , Vol.2(2) 2012, pp 88-106 (Institute of Mathematics and Mechanics NAS of Azerbaijan) , ISSN: 2222-5498.

[16] Blue,M., Bush,B. and Puckett,J., Unified approach to fuzzy graph problems, Fuzzy Sets and Systems, Vol.125 (2002) 355–368.

[17] Bollobas, Bela. ; Modern Graph Theory, Springer; 2002.

[18] Boulmakoul, A., Generalized Path-Finding Algorithms on Semi rings and the Fuzzy Shortest Path Problem, Journal of Computational and Applied Mathematics, Vol. 162 (2004) 263-272.

[19] Chris Cornelis, Peter De Kesel, Etienne E. Kerre, Shortest Paths in Fuzzy Weighted Graphs, International Journal Of Intelligent Systems, Vol. 19 (2004) 1051–1068.

[20] Chuang T.N, Kung J.Y, The fuzzy shortest path length and the corresponding shortest path in a network, Computers and Operations Research Vol.32(2005), 1409–1428.

[21] Chu, T.C. and Tsao,C.T., Ranking fuzzy numbers with an area between the centroid point and original point, Comput. Math. Appl. Vol.43 (2002) 111–117.

[22] Chuang, T.N. and Kung,J.Y., The fuzzy shortest path length and the corresponding shortest path in a network, Computers and Operations Research 32 (2005) 1409–1428.

[23] Chuang, T.N. and Kung,J.Y., A new algorithm for the discrete fuzzy shortest path problem in a network, Appl. Math. Comput. 174 (2006) 1660–1668.

[24] Cormen, Thomas H.; Leiserson, Charles E.; Rivest, Ronald L.; Stein, Clifford, Introduction to Algorithms, McGraw-Hill. 2001

[25] Deng, Yong., Chen, Yuxin., Zhang, Yazuman., and Mahadevan, Sankaran., Fuzzy Dijkstra Algorithm for Shortest Path Problem under uncertain environment, Applied Soft Computing, Vol.12(3) 2012, pp. 1231-1237.

[26] Diestel, R., Graph Theory, Springer, 2000.

[27] Dijkstra, E. W., A note on two problems in connexion with graphs, Numerische Mathematik, Vol.1 (1959) 269–271.

[28] Dubois,D. and Prade,H., Fuzzy Sets and Systems: Theory and Applications, Academic Press, New York, USA, 1980.

[29] Dubois,D. and Prade,H., Ranking fuzzy numbers in the setting of possibility theory, Inform. Sci. 30 (1983) 183–224.

[30] Dubois,D. and Prade,H., Operations on fuzzy numbers, International Journal of Systems Science, vol.9, no.6,pp.613-626,1978.

[31] Dubois,D. and Prade,H., Additions of interactive fuzzy numbers, IEEE Transactions on Automatic Control, Vol. AC-26, No.4 1981, 926-936.

[32] Dubois, D. and Prade, H., "Fuzzy Numbers: An Overview", in: J. Bezdek (ed.) Analysis of Fuzzy Information, CRC Press, Boca Raton, 1988, Vol.2, pp. 3-39.

[33] Elizabeth, S. and Sujatha, L., Fuzzy Shortest Path Problem Based on Level -Triangular LR Fuzzy Numbers, Advances in Fuzzy Systems, Volume 2012 (2012), Article ID 646248, 6 pages doi: 10.1155/2012/646248

[34] Klein, C.M., Fuzzy shortest paths, Fuzzy Sets and Systems, 39 (1991), pp. 27–41.

[35] Li,Y., Gen,M. and Ida,K., Solving fuzzy shortest path problems by neural networks, Computers and Industrial Engineering 31 (1996)861–865.

[36] Lin,K. and Chen,M., The fuzzy shortest path problem and its most vital arcs, Fuzzy Sets and Systems, 58(3) (1994), 343–353.

[37] Mahdavi, I., Tajdin, A., Nourifar, R., Hasanzade, R. and Amiri, N.M., Designing a new algorithm for the fuzzy shortest path problem in a network. Proceeding of the 37th International Conference on Computers and Industrial Engineering, October 20-23, 2007, Alexandria, Egypt,2385-2392.

[38] Nayeem,S.M.A. and Pal, M., Shortest path problem on a network with imprecise edge weight, Fuzzy Optim. Decis. Making 4 (2005) 293–312.

[39] Okada,S. and Soper, T., A method for solving shortest path problem on the network with fuzzy arc lengths, Proc. 7th Internat. Fuzzy Systems Association World Congress, vol. 3, 1997, pp. 189–194.

[40] Okada,S. and Soper, T., A shortest path problem on a network with fuzzy arc lengths, Fuzzy Sets and Systems, 109(1)2000, pp. 129–140.

[41] Okada,S., Fuzzy shortest path problems incorporating interactivity among paths, Fuzzy Sets and Systems, Vol.142(3)2004, PP 335–357.

[42] Okada, S. and Gen, M., Fuzzy shortest path problem, Computers and Industrial Engineering 27 (1994) 465–468.

[43] Okada, S. and Gen, M., Order relation between intervals and its applications to shortest path problem, in: Proc. 15th Annual Conf. on Computers and Industrial Engineering, Vol. 25, 1993.

[44] Okada, S. and Gen, M., Fuzzy shortest path problem, in: Proc. 16th Ann. Conf. on Computers and Industrial Engineering, Vol. 27, 1994.

[45] Okada,S., Interactions among paths in fuzzy shortest path problems, Proceedings of the 9th International Fuzzy Systems Association World Congress (2001), 41-46.

[46] Sengupta, A and Pal, T. K., Solving the shortest path problem with intervals arcs, Fuzzy Optim. Decis. Making (5) (2006) 71–89.

[47] Sujatha,L. and Sattanathan,R., Fuzzy shortest path problem based on triangular LR type fuzzy number using acceptability index, International Journal of Engineering and Technology, Vol. 6, pp. 575–578, 2009.

[48] Sujatha,L. and Elizabeth,S., Fuzzy Shortest Path Problem Based on Similarity Degree, Applied Mathematical Sciences, Vol. 5, 2011, no. 66, 3263 – 3276.

[49] Velammal, G and Shahila Bhanu, M., Intuitionistic Z-Numbers, American International Journal of Research in Science, Technology, Engineering & Mathematics, Vol.9(1) (2015) pp 140-142.

[50] Yao, J.S. and Lin, F.T., Fuzzy shortest-path network problems with uncertain edge weights, Journal of Information Science and Engineering Vol.19 (2003) 321–351.

[51] Zadeh,L.A., Fuzzy sets, Information and Control, 8(1965) 338-353.

[52] Zadeh, L.A. (2011). A Note on Z-numbers, Information Sciences. Vol.181 pp 2923–2932.

[53] Zadeh, L.A., Z-Numbers — a New Direction in the Analysis of Uncertain and Complex Systems, 7th IEEE International Conference on Digital Ecosystems and Technologies, July 24, 2013, Menlo Park.

[54] Zadeh, L.A., The Concept of a Z-Number — a New Direction in Uncertain Computation, Lecture Note of Prof. Lotfi A. Zadeh in IRI 2011 on Aug 3, 2011, Las Vegas.

