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Synchronization of Complex Networks with Multi-Links and Nodes with Different Structures

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Abstract: Recently, works which research on stability and adaptive synchronization of complex networks with multi-links appear a lot but a few on considering nodes with different structures. Based on previous studies, this paper considers adaptive synchronization of complex networks with multi-links and nonlinear coupling which have nodes with different structures and vary with time delay. Synchronous controller is also designed based on Lyapunov theory. The effectiveness and feasibility are proved by numerical simulation of this paper.

Key words: complex networks with multi-links; time-varying delay; synchronization; Lyapunov theory

I. INTRODUCTION

The research of complex networks can date back to the 20th century, 60 years and the ER random graph model [1] which proposed by Erdos and Renyi is the representative work. After nearly 40 years, the ER model has been the basic model. With the Small-World network model [2] proposed by Watts and Strogatz in 1998 and the Scale-Free network model [3] proposed by Barabási and Albert in 1999, the research which is on model, properties and process on the network has rapid development [4].And lots of reality system can describe by complex networks, such as Internet, WWW, Cellular network, Metabolic network and so on [5-7]. Also many synchronization methods have been used in complex networks, such as fully synchronized [8], Hysteresis synchronous [9], Projective synchronization [10], and Generalized synchronization [11].

In the process of the practical application of chaos synchronization, network structure is only partially known or completely unknown. So it is difficult to obtain accurate estimates of coupling strength. Thereby, a complex network with adaptive method has important practical significance. A large number of studies have discussed the adaptive synchronization of network [10-12].

Because of the limited transmission speed or traffic jam, during transmission information will have some time delay. In the biological and physical networks, time delay is very common. In a complex network model, considering the time delay is more realistic. Extensive literature has discussed the case of delay is constant [12-14]. For more general network stability of varying delay (time-varying delay), the research is less [15—17].

Recently, Gao yang has studied the stability and adaptive synchronization of complex traffic network system with multi-links [18-19]. Then based on the literature, Bian Qiuxiang published the synchronization standards of network which is aim to communications network with nonlinear coupling and weighted graph[20]. However, a common feature of these documents is that all nodes in the network have the same structure. In another word, chaotic system in the network nodes are the same and the difference is only the initial conditions of systems. In practice, the structures of the node of complex network are often different to each other. Therefore, it is necessary to study the chaotic synchronization of complex networks with different structure nodes [21].

Based on the results of these studies, the paper uses thought of network split [18] to apply the model in literature [18] to do research on network model of international relations with multi-links. What's more, the paper considers nodes with different structures and variable delay which based on literature [18-20]. Then it will strive to meet the actual situation as much as possible. Assuming the 244 countries and regions of the world are the nodes and the distance between each country is a side, then a basic network is formed. Generally, financial and trade, cultural exchange, military cooperation and so on between countries will occur. Based on the need, the paper will add the edge. Then it forms a new financial and trade network, cultural exchange network, military cooperation network and so on in the original node. It re-uses weighted graph show the total value of finance and trade, the frequency of cultural exchange, the extent of military cooperation and so on. As the exchanges and cooperation between nations will have time difference, it only consider the delay between different networks. Studying the stability, synchronization and community structure of international network has practical significance. For example, EU can analyze the stability of its own structure to prevent the next "dramatic changes in Eastern Europe" events. Of course, such data related to state secrets and we can't get it. But for the school community, company and agencies in society, we can build the network model with similar structure to analyze the cooperation among publicity department, organization department, liaison and logistics department in a team.

II. MODEL AND ASSUMPTIONS

Assuming that the time-delay among financial and trade network, cultural exchange network, military cooperation network are different, that is supposing that there are m types links with different nature. the links which are the actual distance between nations and its N nodes build a basic network. And we mark the delayed coupling $\tau_0(t)=0$, the remaining links that relative to the reference network have the same time delay $\tau_i(t)$ and N nodes form the i-th sub-networks. Therefore, the whole network can be divided into m different sub-networks. In addition, supposing that time delay between each node is the same as $\tau(t)$, then state equation of the whole controlled dynamic network is

$$\dot{x}_i = f_i(x_i(t), x_i(t-\tau(t))) + \sigma_0 \sum_{j=1}^{N} g_{(0)ij} h_0(x_j(t-\tau_0(t)))$$

$$+\sigma_1 \sum_{j=1}^{N} g_{(1)ij} h_1(x_j(t-\tau_1(t))) + \cdots$$

$$+\sigma_{m-1} \sum_{j=1}^{N} g_{(m-1)ij} h_{m-1}(x_j(t-\tau_{m-1}(t))) + U_i ,$$

$$i=1,2,\cdots,N \quad (1)$$

While $x_i=(x_{i1},x_{i2},\cdots,x_{in})^T \in R^n$, $i=1,2,\cdots,N$ is the state variables of node i . $f_i: R^n \to R^n$ Is continuously differentiable function, $\tau(t)>0$ is the node variable delay time, $\tau_l(t)>0$, $l=0,1,\cdots,m-1$ is the coupled variable delay time. Constant $\sigma_l>0$, $l=0,1,\cdots,m-1$ is the coupling strength of the l -th sub-network. $h_l: R^n \to R^n$, $l=0,1,\cdots,m-1$ is continuously differentiable function, and it is also the internal coupling function between state variables of each node. Coupling matrix $G_{(l)}=(g_{(l)ij})_{N\times N}$, $l=0,1,\cdots,m-1$ represent the topology of the l -th sub-network. While the definition of $g_{(l)ij}$ is as follows: each link of the original network defines a constant called weight. $g_{(l)ij}=g_{(l)ji} (i\neq j)$ is the sum of the weight of the node i and node j in l -th sub-network. If there is no connection between two nodes, we will get $g_{(l)ij}=g_{(l)ji}=0 (i\neq j)$ .diagonal of matrix is defined

as $g_{(l)ii} = -\sum_{\substack{j=1\\ j\neq i}}^{N} g_{(l)ij} = -\sum_{\substack{j=1\\ j\neq i}}^{N} g_{(l)ji}$, $i=1,2,\cdots,N$,

$l=0,1,\cdots,m-1$. It satisfies article dissipative coupling $\sum_{j=1}^{N} g_{(l)ij}=0$. $U_i=U_i(x_j(t),x_j(t-\tau_l)), i,j=1,2,\cdots,N$, $l=0,1,\cdots,m-1$ controls input.

Definition 1[18] Make $x_i(t;t_0,X_0)$, $i=1,2,\cdots,N$ be the solution of (1), $X_0=(x_1^0,x_2^0,\cdots,x_N^0)$ .If $f_i:\Omega\times R^+ \to R^n$ and $h_l:\Omega\times\cdots\times\Omega \to R^n$, $i=1,2,\cdots,N$, $l=0,1,\cdots,m-1$ ar e continuously differentialbe, while $\Omega \subseteq R^n$. If existing a nonempty subset $\Lambda \subseteq \Omega$ and $x_i^0 \in \Lambda$, $i = 1, 2, \cdots, N$, then to all $t \geq t_0$ and $1 \leq i \leq N$, $x_i(t; t_0, X_0) \in \Omega$ established. Moreover $\lim_{t\to\infty} \| x_i(t; t_0, X_0) - s_i(t; t_0, x_0) \| = 0$, $i = 1, 2, \cdots, N$,

While $s_i(t; t_0, x_0)$ is a solution of system equations $\dot{x}_i = f_i(x_i(t), x_i(t - \tau(t)))$ and $x_0 \in \Omega$. Norm $\|\cdot\|$ of vector x is defined as $\|x\| = (x^T x)^{1/2}$. It can be seen that the system described by (1) will reach synchronization. What's more, $\Lambda \times \cdots \times \Lambda$ is called synchronization field of dynamic network system that is described in(1). Sign $s_i(t; t_0, x_0)$ is $s(t)$, then $\dot{s}(t) = f_i(s(t), s(t - \tau(t)))$, while $s(t)$ can be acted a balance point, a cycle track, a non-periodic orbits or a chaotic orbit of a phase space.

Error variable of network system is defined as $e_i(t) = x_i(t) - s(t)$ $e_i(t - \tau_l(t)) = x_i(t - \tau_l(t)) - s(t - \tau_l(t))$, $1 \leq i \leq N, 0 \leq l \leq m-1$ The following is aim to design controller U_i and make dynamic network (2)tend to synchronization, that is $\lim_{t\to\infty} \|e_i(t)\| = 0, 1 \leq i \leq N$. then the error system is:

$$\dot{e}_i = f_i(x_i(t), x_i(t - \tau(t))) - f_i(s(t), s(t - \tau(t)))$$
$$+\sigma_0 \sum_{j=1}^{N} g_{(0)ij} H_0(e_j(t - \tau_0(t)))$$
$$+\sigma_1 \sum_{j=1}^{N} g_{(1)ij} H_1(e_j(t - \tau_1(t))) + \cdots$$
$$+\sigma_{m-1} \sum_{j=1}^{N} g_{(m-1)ij} H_{m-1}(e_j(t - \tau_{m-1}(t))) + U_i \quad , \quad i = 1, 2, \cdots, N$$

(2)

While

$$H_l(e_j(t - \tau_l(t))) = h_l(x_j(t - \tau_l(t))) - h_l(s(t - \tau_l(t))) ,$$
$0 \leq l \leq m-1$.

Assumption 1 Supposing that nonnegative constants α_i and satisfy $\forall\ t \in R^+$, then we will get

$$\| f_i(x_i(t), x_i(t - \tau(t))) - f_i(s(t), s(t - \tau(t))) \| \leq \alpha_i (\| x_i(t) - s(t) \| + \| x_i(t - \tau(t)) - s(t - \tau(t)) \|)$$

$i = 1, 2, \cdots, N$

Assumption 2 Supposing that nonnegative constants L_l is exist and satisfy $\forall\ t \in R^+$, then it will exist the following Lipschitz condition

$$\| h_l(x_j(t)) - h_l(s(t)) \| \leq L_l \, \| x_j(t) - s(t) \|$$

$0 \leq l \leq m-1, 1 \leq j \leq N$

Assumption 3 Delay function $\tau(t)$ and $\tau_l(t)$ are differential function and respectively satisfy

$0 \leq \tau(t) \leq \tau$, $0 \leq \dot{\tau}(t) \leq \delta < 1$

$0 \leq \tau_l(t) \leq \tau_l$, $0 \leq \dot{\tau}_l(t) \leq \delta_l < 1$, $0 \leq l \leq m-1$

Lemma 1 $\forall\ x, y \in R^n$, the following inequality is established:

$$x^T y \leq |x^T y| \leq \|x\| \|y\|$$

III. SYNCHRONIZATION STANDARDS

Theorem Assuming 1,2 and 3 is established,then in the role of adaptive controller(3), the system described by(2)will tend to global asymptotic synchronization.

$$\begin{cases} U_i(t) = -d_i e_i(t) \\ \dot{d}_i(t) = k_i e_i^T(t) e_i(t) \end{cases}, 1 \le i \le N \quad (3)$$

While $k_i (1 \le i \le N)$ and d_i are positive constants.

Proof: Construct the following Lyapunov function:

$$V(t) = V_1(t) + V_2(t) + V_3(t) + V_4(t)$$

$$V_1(t) = \sum_{i=1}^{N} e_i^T(t) e_i(t),$$

$$V_2(t) = \frac{\alpha}{1-\delta} \int_{t-\tau(t)}^{t} \sum_{i=1}^{N} e_i^T(\theta) e_i(\theta) d\theta,$$

$$V_3(t) = \frac{L_l N g}{1-\delta_l} \int_{t-\tau_l(t)}^{t} \sum_{i=1}^{N} e_i^T(\theta) e_i(\theta) d\theta, 0 \le l \le m-1$$

$$V_4(t) = \sum_{i=1}^{N} \frac{\left(d_i - d^*\right)^2}{k_i}$$

then we can get the derivative of $V_1(t) \sim V_4(t)$ along the error system(2)respectively,

$$\dot{V}_1(t) = 2\sum_{i=1}^{N} e_i^T(t) \dot{e}_i(t)$$

$$= 2\sum_{i=1}^{N} e_i^T(t)[f_i(x_i(t), x_i(t-\tau(t))) - f_i(s(t), s(t-\tau(t)))$$

$$+\sigma_0 \sum_{j=1}^{N} g_{(0)ij} H_0(e_j(t-\tau_0(t))) + \sigma_1 \sum_{j=1}^{N} g_{(1)ij} H_1(e_j(t-\tau_1(t))) + \cdots$$

$$+\sigma_{m-1} \sum_{j=1}^{N} g_{(m-1)ij} H_{m-1}(e_j(t-\tau_{m-1}(t))) - d_i e_i(t)]$$

Considering the suppose 1 and utilizing lemma 1, we can get

$$2e_i^T(t)[f_i(x_i(t), x_i(t-\tau(t))) - f_i(s(t), s(t-\tau(t)))]$$

$$\le 2\|e_i(t)\| \|f_i(x_i(t), x_i(t-\tau(t))) - f_i(s(t), s(t-\tau(t)))\|$$

$$\le 2\alpha \|e_i(t)\| (\|e_i(t)\| + \|e_i(t-\tau(t))\|)$$

$$\le 2\alpha \|e_i(t)\|^2 + \alpha(\|e_i(t)\|^2 + \|e_i(t-\tau(t))\|^2)$$

$$= 3\alpha \|e_i(t)\|^2 + \alpha \|e_i(t-\tau(t))\|^2,$$

while $\alpha = \max_{1 \le i \le N} \alpha_i$.

Considering the suppose 2 and utilizing lemma 1, we can get

$$2\sum_{i=1}^{N} \sum_{j=1}^{N} g_{(l)ij} e_i^T(t) H_l(e_j(t - t_l(t)))$$

$$\le 2\sum_{i=1}^{N} \sum_{j=1}^{N} L_l \|g_{(l)ij}\| \|e_i(t)\| \|e_j(t - t_l(t))\|$$

$$\le N g L_l \sum_{i=1}^{N} \|e_i(t)\|^2 + N g L_l \sum_{i=1}^{N} \|e_i(t-\tau_l(t))\|^2,$$

$$0 \le l \le m-1$$

While $g = \max_{1 \le i, j \le N} g_{(l)ij}, 0 \le l \le m-1$,

then

$$\dot{V}_1(t) \le (3\alpha - 2d_i) \sum_{i=1}^{N} \|e_i(t)\|^2 + \alpha \sum_{i=1}^{N} \|e_i(t-\tau(t))\|^2$$

$$+ N g L_l \sum_{i=1}^{N} \|e_i(t)\|^2 + N g L_l \sum_{i=1}^{N} \|e_i(t-\tau_l(t))\|^2$$

Considering the suppose 3, we can get

$$\dot{V}_2(t)=\frac{\alpha}{1-\delta}\sum_{i=1}^{N}e_i^T(t)e_i(t)-\frac{\alpha(1-\dot{\tau}(t))}{1-\delta}\sum_{i=1}^{N}e_i^T(t-\tau(t))e_i(t-\tau(t))$$

$$\le \frac{\alpha}{1-\delta}\sum_{i=1}^{N}\|e_i(t)\|^2-\alpha\sum_{i=1}^{N}\|e_i(t-\tau(t))\|^2$$

$$\dot{V}_3(t)=\frac{L_lNg}{1-\delta_l}\sum_{i=1}^{N}e_i^T(t)e_i(t)-\frac{L_lNg(1-\dot{\tau}_l(t))}{1-\delta_l}\sum_{i=1}^{N}e_i^T(t-\tau_l(t))e_i(t-\tau_l(t))$$

$$\le \frac{L_lNg}{1-\delta_l}\sum_{i=1}^{N}\|e_i(t)\|^2-L_lNg\sum_{i=1}^{N}\|e_i(t-\tau_l(t))\|^2$$

$$\dot{V}_4(t)=2\sum_{i=1}^{N}(d_i-d^*)e_i^T(t)e_i(t)=2d_i\sum_{i=1}^{N}\|e_i(t)\|^2-2d^*\sum_{i=1}^{N}\|e_i(t)\|$$

then

$$\dot{V}(t)=\dot{V}_1(t)+\dot{V}_2(t)+\dot{V}_3(t)+\dot{V}_4(t)$$

$$\le(\frac{4+3\delta}{1-\delta}\alpha+\frac{2-\delta_l}{1-\delta_l}L_lNg-2d^*)\|e_i(t)\|^2$$

Making $d^*>\frac{1}{2}(\frac{4+3\delta}{1-\delta}\alpha+\frac{2-\delta_l}{1-\delta_l}L_lNg)$,we will get $\dot{V}(t)<0$. According to Lyapunov theory, $\lim_{t\to\infty}\|e_i(t)\|=0,1\le i\le N$. So system (2) reaches the global asymptotic synchronization on the role of adaptive controller (3).

IV. SIMULATION

To illustrate the synchronization mechanism above, we select Rössler system and Lorenz system which to be network nodes to carry on synchronization control. Network nodes $m=8$, its distribution 1,3,5,7 is Rössler system and 2,4,6,8 is Lorenz system.

The dynamic equations of Rössler chaotic system is described as:

$$\begin{cases}\dot{x}_1=-y_1-z_1\\ \dot{y}_1=x_1+a_1y_1\\ \dot{z}_1=b_1+x_1z_1-c_1z_1\end{cases}\tag{4}$$

While a_1,b_1,c_1 are the three parameters of the system. When $a_1=0.2,b_1=0.2,c_1=5.7$, system (4) is in a chaotic state.

The dynamic equations of Lorenz chaotic system is described as:

$$\begin{cases}\dot{x}_3=a_3(y_3-x_3)\\ \dot{y}_3=x_3(b_3-z_3)-y_3\\ \dot{z}_3=x_3y_3-c_3z_3\end{cases}\tag{5}$$

While $a_3=10,b_3=28,c_3=\frac{8}{3}$, system (5) is in a chaotic state.

Literature [19] proves the establishment of suppose 1 on using the boundaries of chaotic systems. While coupling strength $\sigma_0=0.2,\sigma_1=0.3$ and the coupling function $h_0(x_j(t))=\begin{pmatrix}\sin x_{j1}(t)\\ \sin x_{j2}(t)\\ \sin x_{j3}(t)\end{pmatrix}$, $h_1(x_j(t))=\begin{pmatrix}\cos x_{j3}(t)\\ \cos x_{j2}(t)\\ \cos x_{j1}(t)\end{pmatrix}$. Obviously, it is consistent with hypothesis 2. Set of the delay functions which is consistent with hypothesis 3: $\tau(t)=0.01\frac{e^t}{e^t+1}$, $\tau_0(t)=0.03\frac{e^t}{e^t+1}$,

$\tau_1(t)=0.05\frac{e^t}{e^t+1}$. Assuming the whole network is aplit into two different sub-networks: ringed and star networks. The connection process of ringed network is $1 \to 2 \to \cdots \to 8 \to 1$. Central node of star network is 1. Then we can design the outcoupling matrix $g_{(0)}$ and $g_{(1)}$ as follows:

$$g_{(0)}=\begin{bmatrix} -2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -2 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -2 \end{bmatrix}$$

$$g_{(1)}=\begin{bmatrix} -4 & 1 & -2 & 3 & -4 & 5 & -6 & 7 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -2 & 0 & 2 & 0 & 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & -3 & 0 & 0 & 0 & 0 \\ -4 & 0 & 0 & 0 & 4 & 0 & 0 & 0 \\ 5 & 0 & 0 & 0 & 0 & -5 & 0 & 0 \\ -6 & 0 & 0 & 0 & 0 & 0 & 6 & 0 \\ 7 & 0 & 0 & 0 & 0 & 0 & 0 & -7 \end{bmatrix}$$

Select $d_i=k_i=1(i=1,2,\cdots,8)$,the initial value of the systems can be selected arbitrarily. The curve of error $e_{ij}(i=1,2,\cdots,8;\ j=1,2,3)$ is shown on figure 1.

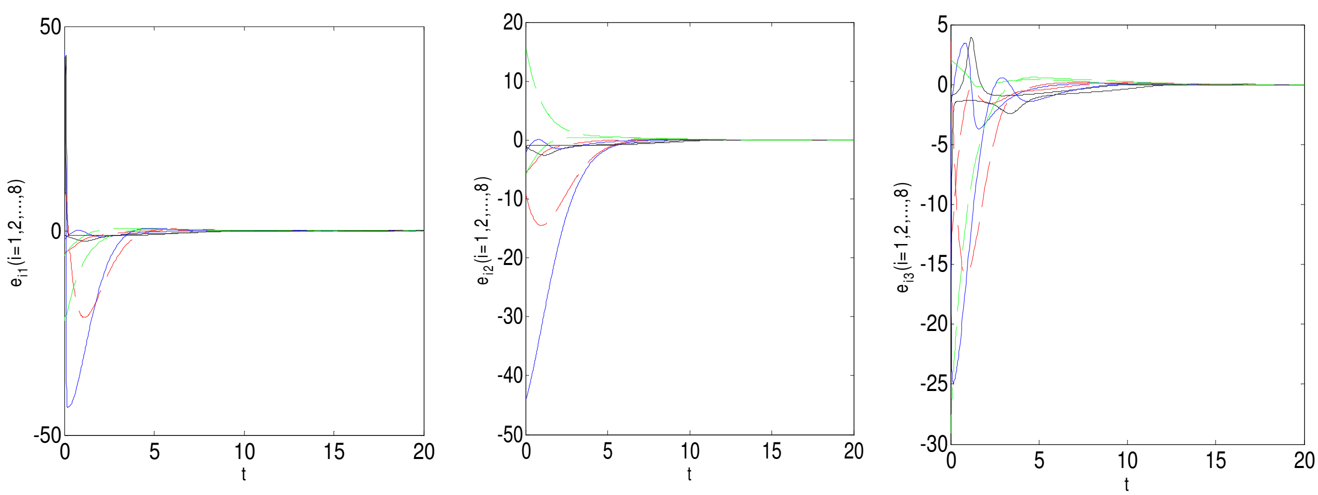

Figure.1 Curve of synchronization error $e_{ij}(i=1,2,\cdots,8;\ j=1,2,3)$ of dynamic network

V. CONCLUSION

In the paper, for complex networks with multi-links and nonlinear coupling, we consider the situation that the networks have nodes with different structures and vary with time delay. Then we realize adaptive synchronization of dynamic network through synchronous controller based on Lyapunov theory and confirm effectiveness and correctness of the method through simulation.

VI. REFERENCES

[1] Erdős, P. Rényi, A. "On Random Graphs I" in Publ. Math. Debrecen, 1959, 6: 290–297.

[2] D. J. Watts,S. H. Strogatz. Collective dynamics of ‘small-world’ Networks [J]. Nature, 1998, 393:440-442.

[3] Albert-László Barabási, Réka Albert. Emergence of Scaling in Random Networks [J]. Science, 1999, 5439(286): 509-512.

[4] S.N. Dorogovtscev, J. F. F. Mendes. Evolution of networks [J]. Adv.Phys. , 2002, 51: 1079-1187.

[5] Barahona M., Pecora L. M.Synchronization in small-world systems [J]. Physical Review, 2004, 89(5): 054101.

[6] Xu Dan, Li Xing, Wang Xiaofan. An investigation on local area control of virus spreading in complex networks [J]. ACTA PHYSICA SINICA. 2007,56 (3):1313-1317.

[7] Li C., Sun W., Kurths J. Synchronization between two coupled complex networks [J].Phys.Rev.E, 2007, 76:046204.

[8] Li C., Chen G. Synchronization in general complex dynamical networks with coupling delays [J]. Physica A. 2004, 343: 263-278.

[9] Zhen Jia, Guangming Deng, Weight identification of a weighted bipartite graph complex dynamical network with coupling delay [J]. Journal of Inequalities and Applications, Volume 2010 (2010), Article ID 805178, 9 pages, doi:10.1155/2010/805178.

[10] Jia Zhen, Lu Junan, et al., Generalized projective synchronization of a class of chaotic (hyperchaotic) systems with uncertain parameters [J]. Chinese Physics, 2007, 16(05): 1246-1251.

[11] Jia Zhen, Linear generalized synchronization of chaotic systems with uncertain parameters[J]. Journal of Systems Engineering and Electronics, 2008, 19 (4):779-784.

[12] Zhang Qunjiao,Lv Junan,Lu Jinhu. Adaptive feedback synchronization of a general complex dynamical network with delayed nodes [J]. IEEE Transactions on the Circuits and Systems, 2008, 55 (2): 183-187.

[13] Lv Jinhu,Henry L. Synchronization: a fundamental phenomenon in complex dynamical networks[J]. IEEE International Symposium on Circuits and Systems, 2005, 50(1): 300-303.

[14] Wu Jianshe, Jiao Licheng. Synchronization in dynamic networks with nonsymmetrical time-delay coupling based on linear feedback controllers [J]. Physica A., 2008, 387: 2111-2119.

[15] Zhou J., Wu Q. J. Exponential stability of impulsive delayed linear differential equations [J]. IEEE Transactions on Circuits and Systems II: Express Briefs, 2009, 56(9): 744-748,

[16] Cai S., Zhou J., et al., Robust impulsive synchronization of complex delayed dynamical networks. [J]. Phys.Lett.A., 2008, 372, 4990-4995.

[17] Wang Lei, Dai Huaping, Dong Hui. Adaptive synchronization of weighted complex dynamical networks with coupling timevarying delays[J]. Physics Letters A, 2008: 3632-3639.

[18] Gao Yang, Li Li Xiang et al. Stability analysis of complex networks with multi-links [J]. ACTA PHYSICA SINICA., 2008, 57(3): 1444-1452.

[19] Gao Yang, Li Li Xiang et al. Adaptive synchronization in united complex dynamical network with multi-links [J]. ACTA PHYSICA SINICA., 2008, 57(4): 2081-2091.

[20] Bian Qiu-Xiang, Yao Hong-Xing. Synchronization of weighted complex networks with multi-links and nonlinear coupling [J]. ACTA PHYSICA SINICA., 2010, 59(5): 3027-3034.

[21] Lv Ling, Zhang Chao. Chaos synchronization of a complex network with different nodes [J]. ACTA PHYSICA SINICA., 2009, 58(3): 1462-1466.

