Volume 5, No. 6, July-August 2014

ISSN No. 0976-5697

International Journal of Advanced Research in Computer Science

RESEARCH PAPER

Available Online at www.ijarcs.info

DRI Essential Vertices in Caterpillar Trees

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Abstract: The total number of diametral paths reachable from a vertex v is called the *Diametral Reachable Index* of that vertex. In this paper an effort is made to analyse some special properties of DRI (Diametral Reachable Index) of vertices of caterpillar. The vertex whose removal changes the DRI of some other vertices is called DRI essential vertex. The set of DRI essential vertices, whose removal changes the diameter of the graph, is called diameter critical set. An algorithmic approach is also discussed to find DRI essential vertices and the cardinality of the set of diameter critical vertices.

Keywords: DRI, DRI essential, Diameter Critical, MDCV set, DCV number, Diametral.

I. INTRODUCTION

Graph theory is now a very important area of research as it is useful in developing many applications in networking, medical field, transport, chemical field etc. An algorithmic approach with graph theory is of high importance in the study of different problems and their analysis. The concept of distance i.e. the shortest path has very interesting applications in computer networks as well as transport networks. However developing a generalised algorithm is very challenging, and hence we are considering some special classes of graphs for our study and the analysis of the concept of DRI.

Some of the basic terminologies needed to understand the proposed work are explained briefly in this section.

A *graph* $G(V, E)$ is an ordered pair of sets. Elements of V are called *vertices* or *nodes*, and elements of E are called *edges* or *lines*. We refer V to as the vertex set of G, with E being the edge set [1]. The subsets of vertices are referred as *clusters*.

The *distance* between two vertices in a graph is the length of a shortest path between them. For a vertex v in a graph G, the eccentricity $e(v)$ of v is the distance between v and the farthest vertex from v in G. The minimum of eccentricity among the vertices of G is its radius and the maximum eccentricity is the diameter [2].

The vertices whose value is equal to maximum eccentricity are called *diametral vertices*. The paths originating from diametral vertices, whose length is equal to diameter, are called *diametral paths* [2].

A *caterpillar* is a tree in which every graph vertex is on a central stalk or only one graph edge away from the stalk, in other words, removal of its endpoints leaves a path graph. A tree is a caterpillar iff all nodes of degree are surrounded by at most two nodes of degree two or greater [2][3].

The total number of diametral paths reachable from a vertex v is called the *Diametral Reachable Index* of that vertex, denoted $DRI(v)$ [4].

The readers can further refer [5] for some basic properties of DRI.

II. PROPOSED WORK

Before we proceed with details of DRI essential vertices let us consider an example of a caterpillar tree T. Whenever we delete the vertices of a graph the edges incident on that vertex will be deleted. The effect of deleting the vertices is seen on the DRI of some vertices as well as on diameter of the graph.

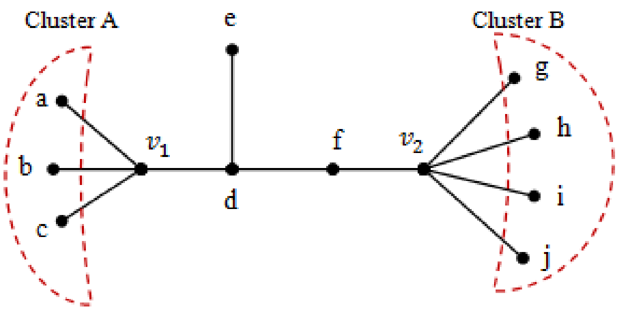

Figure 1. Example of Caterpillar T

The Figure 1 shows a caterpillar tree T; we see that the diametral vertices are a, b, c, g, h, i, j. The diameter of T is 5. The diametral vertices are categorized in to 2 clusters (sets) A and B. The cluster A has 3 vertices a, b and c- i.e. $A = \{a, b, c\}$. The cluster B has 4 vertices g, h, i and j- i.e. $B = \{g, h, i, j\}$. Also observe that cluster A is adjacent to vertex v_1 and cluster B is adjacent to v_2. The degree of $v_1 = |A|$ and degree of $v_2 = |B|$. We can easily compute the DRI values of the vertices of cluster A & B easily.

The number of diametral paths starting from each vertex of cluster A is equal to as $|B|$. That means from each vertex of cluster A, there exists a unique diametral path to each vertex of cluster B, and vice versa. Therefore it can be simply said that, total number of diametral paths originating from cluster A as

$= |A| \cdot |B|$ …… (1)

Now, by considering the vertices of cluster B as start vertices and the vertices of cluster A as end vertices, we compute the total number of diametral paths as

$= |B| \cdot |A|$ …… (2)

Therefore simply we can say that total number of unique diametral paths of T as $= |A| \cdot |B$

The same can be seen in the above caterpillar. The different diametral paths originating from cluster A are:

a. a, v_1, d, f, v_2, g
b. a, v_1, d, f, v_2, h
c. a, v_1, d, f, v_2, i
d. a, v_1, d, f, v_2, j
e. b, v_1, d, f, v_2, g
f. b, v_1, d, f, v_2, h
g. b, v_1, d, f, v_2, i
h. b, v_1, d, f, v_2, j
i. c, v_1, d, f, v_2, g
j. c, v_1, d, f, v_2, h
k. c, v_1, d, f, v_2, i
l. c, v_1, d, f, v_2, j

The different diametral paths originating from cluster B are:

a. g, v_1, d, f, v_2, a
b. g, v_1, d, f, v_2, b
c. g, v_1, d, f, v_2, c
d. h, v_1, d, f, v_2, a
e. h, v_1, d, f, v_2, b
f. h, v_1, d, f, v_2, c
g. i, v_1, d, f, v_2, a
h. i, v_1, d, f, v_2, b
i. i, v_1, d, f, v_2, c
j. j, v_1, d, f, v_2, a
k. j, v_1, d, f, v_2, b
l. j, v_1, d, f, v_2, c

Note also that, the number of diametral paths from vertices of cluster A

$$\left.\begin{array}{ll} a = 4 = |B| & \text{i.e, } DRI(a) = |B| \\ b = 4 = |B| & \text{i.e, } DRI(b) = |B| \\ c = 4 = |B| & \text{i.e, } DRI(c) = |B| \end{array}\right\} \ldots (3)$$

from vertices of cluster B

$$\left.\begin{array}{ll} g = 3 = |A| & \text{i.e, } DRI(g) = |A| \\ h = 3 = |A| & \text{i.e, } DRI(g) = |A| \\ i = 3 = |A| & \text{i.e, } DRI(g) = |A| \\ j = 3 = |A| & \text{i.e, } DRI(g) = |A| \end{array}\right\} \ldots (4)$$

Based on these observations, following 3 properties can be stated:

Proposition 1: For each $v \in A$, there exist $|B|$ number of diametral paths and hence $DRI(v) = |B|$

Proposition 2: For each $v \in B$, there exists $|A|$ number of diameter paths. Therefore $DRI(v) = |A|$.

With these basics we shall now deal with DRI essential vertices in the next section.

III. DRI ESSENTIAL VERTICES

In this section we present the effect of removal (deletion) of diametral vertices. Whenever a vertex is deleted from a graph the diameter either increases or decreases or remains same. Here we consider only the deletion of diametral vertices and its effects on DRI of other vertices. Deletion of non diametral end vertices has no effects on the DRI of vertices or diameter of T as long as tree remains connected.

Consider the caterpillar of Figure 1,

Step 1: Delete the vertex a. The tree will be as shown in figure 2.

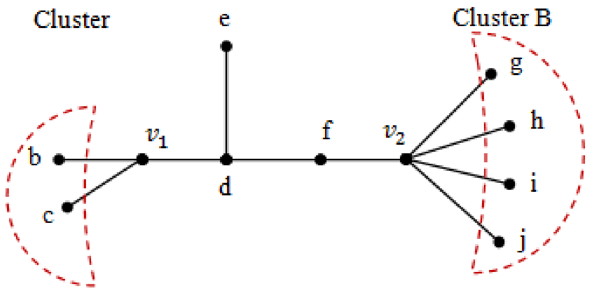

Figure 2. Caterpillar Tree after deleting vertex a

Now we see that cluster A has 2 vertices and B has 4 vertices

$\therefore$ Total Paths $= |A| * |B| = |2| * |4| = 8.$

All the diametral paths from a are deleted. All the diametral paths to vertex a are also deleted.

Total number of paths to $a = |B|$ as we have seen in last section equation (3).

$\therefore$ Total number of diametral paths deleted due to $a = |B| = 4$

Step 2: Delete one more vertex ie, b.

Total number of diametral paths deleted due to $b = |1| \cdot |4|$

Step 3: After deletion the last vertex c

Total diametral paths $= 0 \cdot |4| = 0$. Here the diameter itself is changed and hence no diametral paths exists after deletion of c.

The vertices whose deletion changes the DRI value of some other vertices are called *DRI essential* vertices. All the diametral vertices of caterpillar are DRI essential vertices. If we delete any of the diametral vertices belonging to any one of the cluster, then the DRI of the vertices present in another cluster will be changed. With respect to the vertices of cluster B the vertices of cluster A are essential and vice versa.

When graph has no diametral path means its diameter has changed. Yes, at step 3, the diameter of graph reduces by 1, and hence no diametral paths with earlier diameter value. Suppose that we delete all the vertices of cluster A. Then the diameter of the graph itself changes. The set which consists of minimum number of vertices whose deletion changes the diameter of the tree is called *Minimum Diameter Critical Vertex (MDCV)* set and the cardinality of this set is referred as *Diameter Critical Vertex (DCV)* number. Therefore the cluster A of figure 1, is called the MDCV set which is non empty and DCV number is 3.

IV. ALGORITHM

The algorithm given here finds the MDCV set and DCV number for the given caterpillar graph. The graph is read as adjacency matrix. The variables used are as given in the comments. The C programming based pseudocode is used.

// u, v, v_1, v_2 are vertices.
// $dv[1 \ldots k]$, the array of diametral vertices.
// $adjacent(u)$ returns the adjacent vertex of u.
// $deg[u]$ – stores the degree of vertex u.
// $A[\,], B[\,]$ - Clusters
// i, j - index for arrays initialised to 1.

a. Read the caterpillar graph as adjacency matrix.
b. Find the eccentricity of each vertex.
c. Compute the diameter of the caterpillar.

d. Find all the diametral vertices and store in array $dv[\,]$.
e. Find the cluster A and B, vertex v_1 and v_2 by using the following pseudocode:

$u = dv[1]$; // consider the first diametral vertex.
$v = adjacent(u)$;
$v_1 = v$; // v_1 is found.
$deg[v_1] + +$;
$A[i] = u$; //Add the vertex u to cluster A
$i + +$;
$for\ each\ u \in dv[2 \ldots k]$
{
$v = adjacent(u)$;
$if(v == v_1)$
{
//Add the vertex u to cluster A
$A[i] = u$;
$i + +$;
$deg[v_1] + +$;
}
$else$
{ $v_2 = u$;
//Add the vertex u to cluster B
$B[j] = u$;
$j + +$;
$deg[v_2] + +$;
}
}

6. Compare the number of vertices in each cluster.
The number of vertices can be easily obtained by degree of v_1 and v_2.
$if(deg[v_1] == deg[v_2])$
{
Display ("Both clusters have same number of vertices and hence both are MDCV and DCV number= $deg[v_1] = deg[v_2]$.");
}
$else\ if(deg[v_1] < deg[v_2])$
{
Display ("MDCV is the cluster A");
Display the content of array $A[]$.
DCV=$deg[v_1]$.
Display the value of DCV.
$else$
{
Display ("MDCV is the cluster B");
Display the content of array B[].
DCV=$deg[v_2]$.
Display the value of DCV.
}

7. Display the DRI essential vertices with respect to each cluster.
Display ("the DRI essential vertices w.r.t cluster B are the vertices of cluster A").
$for\ i = 1\ to\ \deg[v_1]$
Print A[i]
Display ("the DRI essential vertices w.r.t cluster A are the vertices of cluster B").
$for\ i = 1\ to\ \deg[v_2]$
Print B[i]

8. End of the algorithm.

A. *Analysis:*

Finding the cluster A and B takes at most n steps, where n is the number of vertices. This is same as finding MDCV set. Finding the DCV number can be done easily with one comparison and therefore it takes $O(a)$ where a is some constant. Therefore the worst case time complexity of the algorithm is $O(n)$. The details regarding algorithms and analysis can be seen in [6][7].

V. CONCLUSION

The DRI essential and diameter critical vertices are explained here with respect to caterpillar tree. These can be further checked to verify with trees (non caterpillar) and some other class of graphs such as cycles, planar graphs, etc. Another ambiguous concept kept open to consider is the trivial tree, i.e. tree with one vertex. In this case diameter of the tree is zero, where as, whether to consider the DRI(v) as one or Zero is the ambiguous part, as both can be shown to be true and both can be shown to be false. The algorithm that is designed is also the simplest one with linear complexity for running time.

VI. ACKNOWLEDGEMENT

Our sincere thanks to Dr. H. B. Walikar, Professor, Department of Computer Science, Karnatak University, Dharwad, for his valuable suggestions.

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