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Network Reliability Evaluation of Fault Tolerant Varietal Hypercube Network

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Abstract: A cube based interconnection network is used to interconnect a large number of standalone processors. The cube based networks have been extensively used in multicomputer systems. As the interconnection network in a communication system cannot be implemented directly with commodity components as its design impacts critically on the performance and cost of the total system. With the increase in size, the complexity of the system increases under critical conditions. So its reliability prediction becomes quite essential and also the topology of interconnection becomes an important issue.
This paper proposes a new, simple and general method to find the maximal incomplete sub cube of Varietal hypercube network when any node fails permanently. A recursive and generalized algorithm has been proposed for predicting the reliability of such system. The method is well illustrated by taking Varietal hypercube as an example and the network reliability of Varietal hypercube network is evaluated.

Keywords: Varietal hypercube , Maximal Incomplete sub cube, discarded region, Network reliability.

I. INTRODUCTION

As parallel computer communication systems are very much popular and commercially widely used in real time applications, therefore considerable interest and increasing efforts have been made to develop such large communication systems. Therefore a wide variety of interconnection networks have been proposed like rectangular meshes, trees, shuffle exchange networks, omega networks and binary cubes[1]and[2]. Due to attractive properties like regularity, symmetry, small diameter, strong connectivity, recursive, construction and partition ability the n-cube topology has enjoyed the largest popularity. These property leads to simple routing, support for wide application spectrum and fault tolerance for communication systems [3].

The n-dimensional cube is composed of 2^n nodes and has n- edges per node, n-bit binary addresses are assign to the nodes to the cubes in such a way that an edge or link connects two nodes if and only if their binary addresses differ by a single bit [3].This Inter connection network supports large numbers of resources with small diameters. The probability of fault in a larger system is given due importance. Whenever a fault arises, an n-cube may operate in a gracefully degradable manner due to the execution of parallel algorithms in smaller fault free sub cubes[6], which are comprises of healthy nodes. In order to maintain cube topology in the presence of faults, researchers have proposed addition of spare nodes thereby replacing the failed components with spares. This results in a much larger system than what is attained by any conventional reconfiguration scheme which identifies only complete sub cube [7]. Also fault tolerance can be achieved by reconfiguring the larger system to smaller sized system after the occurrence of fault [4]. Unlike a complete one, an incomplete cube can be of any arbitrary size, i.e. can be used to interconnect systems with any numbers of processors, making it possible to finish a given batch of jobs faster than it's complete counterpart alone by supporting simultaneous execution of multiple jobs of different sizes by assigning more nodes to execute the job cooperatively. Chen et al. [9] determine sub cubes in a faulty hypercube. Similar research work can be found in literature [6], [8] and [10].Thus reconfiguring a faulty n-cube in to a maximal incomplete cube tends to lower potential performance degradation. This motivates our study to propose a simple, general and recursive method for finding all the incomplete sub cubes of a Varietal hypercube.

With the increase in size, the complexity of the interconnection network increases there by corresponding increase in computational power to maintain acceptable performance under reliable conditions [11] and [12]. For this the reliability prediction of varietal hypercube network is quite essential [13], [14] and [15].

This paper proposes an efficient distributed procedure for locating or identifying all maximal incomplete sub cubes present in a faulty Varietal hypercube. The concept of discarded regions eliminates those nodes impossible to be part of any fault free sub cube containing the given node. There by forming the maximal incomplete sub cube. This method is illustrated through a 3-dimensinal Varietal hypercube. A recursive and generalized algorithm has been proposed for predicting the reliability of Varietal hypercube.

II. NETWORK DETAILS

A. *Varietal Hypercube:*

The *n*-dimensional varietal hypercube, denoted as VH_n, is a labeled graph defined recursively as follows. The VH_1 is the complete graph of two vertices labeled with 0 and 1 respectively. For $n > 1$, VH_n is obtained by joining nodes VH^0_{n-1} and VH^1_{n-1} according to the rule: a vertex u $=0u_{n-1}\, u_{n-2} \ldots u_1$ from VH^0_{n-1} and a vertex $v=1v_{n-1}\, v_{n-2} \ldots v_1$ from VH^1_{n-1} are adjacent in VH_n if and only if

i. $u_{n-1}\, u_{n-2} \ldots u_1 = v_{n-1}\, v_{n-2} \ldots v_1$ *if* $n \neq 3k$ *or*

ii. $u_{n-3} \ldots u_1 = v_{n-3} \ldots v_1$ *and*
$\{(u_{n-1}u_{n-2}, v_{n-1}v_{n-}) \in (0,00),(01,01),(10,11),(11.10)\}$, *if* $n=3k$

A three dimensional Varietal hypercube is shown in Fig.1.

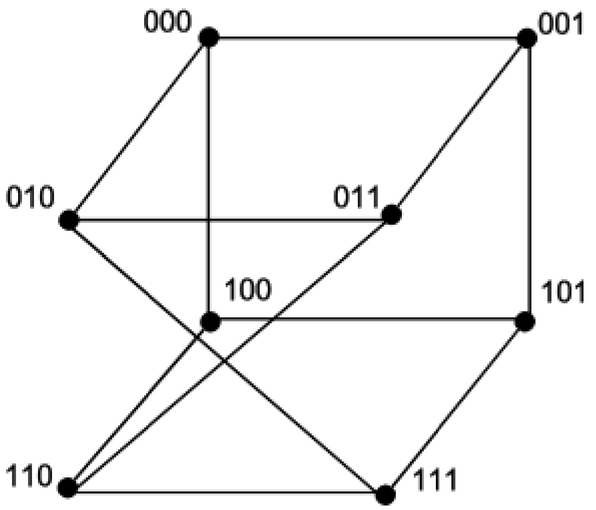

Figure.1: Varietal Hypercube Network (for $n=3$)

III. PROPOSED APPROACH FOR NETWORK RELIABILITY EVALUATION

The following notation and assumptions are made throughout this paper for network reliability evaluation of Varietal hypercube interconnection network.

A. *Notations:*

VHn Varietal Hypercube Interconnection n/w
S Source node
D Destination node
$\otimes$ Discarding operation
n System dimension
N number of nodes in hypercube
u,v,w Adjacent nodes of source node
$\overline{v}$, $\overline{w}$ Antipodal nodes of v, w

B. *Assumptions:*

a. Node failures are statistically independent of each other.
b. The Varietal hypercube interconnection network degrades performance exponentially with time.
c. Repair facility is not available.

C. *Basic Properties:*

a. ***Definition 1:*** A maximal incomplete sub cube is obtained when link is properly added to a maximal sub cube of n-1 dimension so that the destination node D is reached.

Unlike complete Varietal hypercube an incomplete Varietal hypercube can be constructed with any numbers of nodes to avoid the practical restriction of Varietal hypercube topology on the numbers of nodes in a system must be a power of 2. A proper incomplete sub cube in a faulty Varietal hypercube refers to a fault free incomplete sub cube which is not contained entirely in any of the fault free sub cube.

b. ***Lemma 1:*** For an n-dimensional interconnection n/w in one of n adjacent nodes of the source node may be faulty. So the resulting maximal sub cube has a dimension of n-1.

c. ***Proof:*** Consider an n-dimensional Varietal hypercube VHn in which one of the n-adjacent node to source node S is faulty. This leads to disconnection of link to faulty node. Without loss of generality and obeying the symmetry of n- Varietal hypercube. The faulty node must present in VHn. This contradicting definition of a maximal sub cube.

d. ***Definition 2:*** A discarded region in an interconnection network is the smallest sub cube comprises of a faulty node and the antipodal nodes of the (n-1) fault free adjacent nodes.

For a faulty n- Varietal hypercube VHn and a given source node S. It is possible to identify systematically every fault free sub cube which involves the source node S. This is expressed by set P = {Pi/Pi is a fault free sub cube in VHn and Pi involve node S}.This can be done by determining the region which never contribute to any fault free sub cube containing the node S. Each fault results in one such regions known as discarded region which is the smallest sub cube involving both the faulty and the antipodal nodes of adjacent (n-1) nodes. A discarded region is addressed by performing $\otimes$ operation on the labels of the faulty node and the antipodal node where $\otimes$ is the bit operation defined as: it yields 0 (or 1) if the two corresponding bits are "0" (or "1") and it is * if the two corresponding bits differ.

e. ***Theorem-1***: The no. of discarded complete sub cube in VHn of dimension 'n' is equal to the number of nodes present in a fault free maximal in complete sub cube.

f. ***Proof:*** For an inter connection n/w VHn of dimension n , there are exactly n numbers of nodes adjacent to a given source node S. It can be straight forward from the properties of interconnection hypercube n/w that VHn-1 can be obtained by removing 2^{n-1} nodes from VHn. The process can be repeated such that VH_2 can be obtained which shows clearly that VHn is having a hierarchical structure.So , for a given source node S, one out of n adjacent nodes can

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be faulty so that , an interconnection network of lower dimension can be obtained while preserving the hierarchical and regular properties of the n/w. Thus for a given source node(S) and destination node (D) (which can never be faulty) 2^{n-1} +1 nodes can not be faulty. Further, choosing a node N ϵ Adj(s) to be faulty. Finding the antipodal nodes Y^{i}' of X ϵ {Adj(s)}, i =1……(| adj(s)| -1) and carrying out Y $\otimes$ ^{i}'{Adj(s)} leads to n faulty nodes and 2^{n-1}+1 discarded regions, which proves the theorem.

g. ***Lemma 2:*** In an n-dimensional Varietal hypercube, for a given node, the n length independent node is present at a minimum distance of n.

h. ***Proof:*** Consider an n-dimensional Varietal hypercube having 2^{n} number of nodes, keeping source and destination nodes fixed out of the 2^{n} nodes. With out loss of generality and from the properties of the Varietal hypercube topology, the disjoint path from source to destination consists of n-1 number of intermediate nodes. Hence total number of nodes in a disjoint path is n+1.As a link or an edge is defined between two nodes. Therefore the number of path required is n+1-1 i.e. ‘n’. This completes the proof.

IV. PROPOSED METHOD

Let VHn denotes an n-dimensional interconnection network i.e. n-dimensional Varietal hypercube. Each node in VHn is labeled by a n-bit string. For a given source node S, there exists a numbers of adjacent nodes, out of which at least one node is faulty.

Otherwise it will destroy the regularity property of VHn. The addresses of the adjacent nodes are differ in obtained from equation (1) exactly one bit. Assume ‘u’ be the faulty node, ‘v’ and ‘w’ be the non-faulty nodes. Where ‘u’, ‘v’ , ‘w’ are represented as binary strings [4]. $\overline{v}$ and $\overline{w}$ be the antipodal nodes of v and w. Taking bit operation $u \otimes \overline{v}$ and $u \otimes \overline{w}$ results n discarded regions. This leads to formation of an incomplete interconnection network I_{n-1}^{m}.m numbers of nodes in fault free incomplete cube with dimension of n-.A recursive algorithm for generating the reliability expression R for the maximal incomplete Varietal hypercube is provided below.

V. ALGORITHM FOR RELIABILITY EXPRESSION

Reliability (G, S, D, n)

{

If $(n \geq 2)$

{

Adjacent=Adj(S)

Choose a node N from adjacent in $^{n}C_{1}$ *ways.*

$N' = \{\text{Antipodal (N)}\}$

$V_i' = N' \otimes \{V \sim (S \cup D \cup N)\}$

$V' = \{V \sim (S \cup D \cup (Adj(S) \sim N)\}$

for i=1 to $|V'|$

Discard $N' \otimes V_i'$ *region*

G/= (V/,E/)

$R = R \times {}^{n}C_{1}\ p^{n+1}\ q^{2^{n-1}-n}$

Reliability (G/, S, D, n-1)

}

else

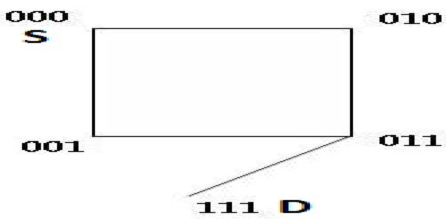
return;

}

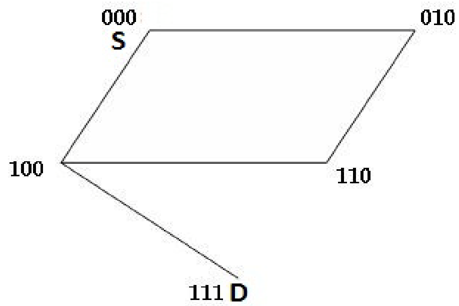
Efficiency: The complexity of the proposed algorithm is found to be $O(n^2)$.

V. ILLUSTRATION

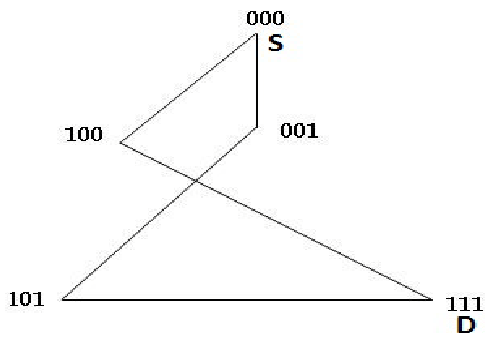
The proposed method is illustrated through a 3D-Varietal hypercube in Fig.1.



2(a) 100 taken as faulty



2(b) 001 taken as faulty



2(c) 010 taken as faulty

Figure.2 (a), (b) and (c) represents incomplete maximal sub cubes of a 3D-Varietal hypercube.

Consider a 3-D Varietal hypercube VH_3 as shown in fig.1 having the source and destination nodes labeled as 000 and 111 respectively. Out of the three adjacent nodes of source node let cube node 001 is faulty, Then the antipodal nodes of the two other adjacent nodes are 011 and 101.A discarded region is addressed simply by performing operation $\otimes$ on the labels of the faulty node and the antipodal nodes. When faulty node is 001,antipodal node of 100 is 011 and 001 $\otimes$ 011 = 0∗1 and 001 $\otimes$ 101 = ∗01.After removing this two discarded regions a maximal incomplete sub cube results which is shown in fig.2(b).The same operation can be performed by taking 100 and 010 cube nodes as faulty nodes. This results in the two other maximal incomplete sub cube as shown in fig. 2(a) & 2(c).

VI. RESULTS AND DISCUSSION

The network reliability of Varietal hypercube (VH_n) is plotted against the mission time under different node failure rates in (Fig.3). It can be observed that the best value of reliability of VH_n is 72% at mission time 100 hours and λ_N =0.001. At high node failure. rate of 0.005, the value of reliability of VH_n is slightly more than 35% even at mission time of 100 hours.

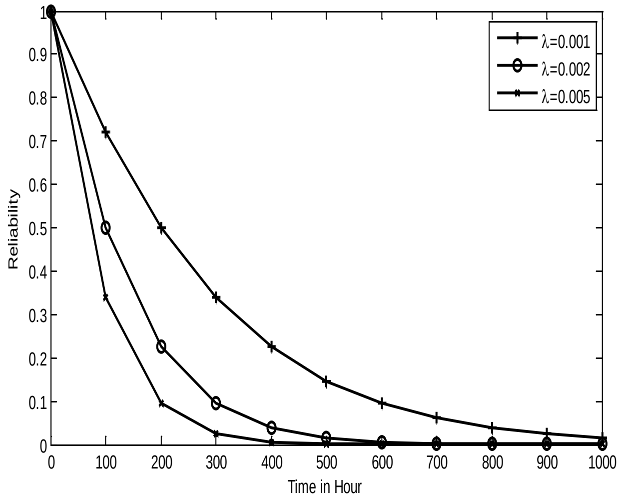


Figure.3 Network Reliability of Varietal Hypercube

Whereas at a node failure rate of 0.002, the VH_n can provide reliability value of only 50% for the same mission time. The value of reliability falls exponentially and becomes less than 10% beyond 500 hours under λ_N =0.005.

VII. CONCLUSIONS

A fast, simple and effective method has been introduced for identifying all maximal fault free incomplete sub cubes from a faulty Varietal hypercube; taking maximum fault tolerance capacity is equal to the system dimension. In this process every fault free node is required to participate in the identification process. An efficient algorithm has been incorporated to find network reliability of the incomplete Varietal hypercube and the complexity of the proposed algorithm is found to be $O(n^2)$. This proposed method is suitable for a Varietal hypercube with arbitrary node failure and can be used for all types of cube based topologies which are operating in a gracefully degradable manner after fault.

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