



Application of Queuing Model and Cost System in Managerial Decision Making

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Abstract: Most management dealing with customers awaiting service find it difficult to determine the efficiency of their service facility and also to measure the performance of their system. Some of these performance measures could be the following; the number of customers in the system, the time spent by customer in the system, the queuing length and the time spent on the queue. The number of the customers being served for a specific time and the probability that there are n customers in the system are included in the performance measures. $M/M/1$ models for performance measure were developed with cost implications and the models were applied to one of the banking industry and the results help us to determine the efficiency and the performance measure in the bank and any business organization.

Keywords: performance, measure, efficiency, customers, bank.

I. INTRODUCTION

When limited service facilities fail to satisfy the demands for service that are made upon them, bottlenecks occur which generate queues or waiting lines. In business, queues have certain economic implications. It may be costly for a company to allow long queues to develop. It will probably be costly to speed up service in order to reduce waiting times and queue lengths, because it would be necessary to employ extra service assistant, service counters or service equipment[2]. Extra cares must be taken by the servers so as not to make mistake that will result to loss by either the company or the customers. Customers may expect to be served within a certain length of time, otherwise they may take their customers else where[3].

As the word turns to a global village characterized by intense and ever increasing competition, operation bank managers continue to experience wrenching changes, which they must keep up with for survival. Bank customers have also become increasingly demanding. Today, they require high quality, low price and immediate service delivery and tomorrow, they want additional components of value from their chosen banker. Since service delivery in banks is personal, customers are either served immediately or join a queue (waiting line) if the system is busy.

Waiting line is what we encounter everywhere we go, while shopping, checking into hotels, at hospitals and clinics e.t.c. In a traditional non-queuing environment, customers can be left confused as to what line to stand in, what counter to go to when called and distracted by noisy crowded environment. When customers wait in queue, there is the danger that waiting time will become excessive leading to the loss of some customers to competitors[4]. At one point or the other, people or individual wait on a line expecting one form of service or the other, some of those waiting lines can be seeing in banks, registration centers, fuelling stations,

phone booths, airports motor parks, restaurants stadia and so on where people have to wait for one from of service or queue [1]. In a brief term, a queue can be defined as an aggregation of items waiting for service function. A queue emerges when the numbers of waiting for the service outweigh the channels, that is responsible for the service, when the arrival pattern is higher than the service pattern. A queue do not only involves people, it also involves machines and other materials due for service or repairs goods or units, individual machine and so on requires service and has to wait because the service is not forthcoming, then a queue is said to exist[5].

When limited service facilities fail to meet the demand for the services that are made upon them, problems occur which produces queues or waiting lines. A queue is characterized by the arrival of unit which require at one or more services facilities[3]. The units that are demanding for one or more service are called customers. Because of the rampant occurrences of queue in most of our service, the concept of queue with special attention on airport using international airport authority, Ilorin as a case study for the course of this project the queue process consist of customers arriving at the service facility then waiting in a line (if all services are busy) eventually receiving service and finally departing from the system facility, this is described as the birth dead process with a population consisting of customers either waiting for the service or currently in service. A birth occurs when a new customers arrives at service facility, a death occurs when a customer departs from the facility[4].

Management may wish to provide a service system which either

- Minimizes the joint cost of servicing customers and idle time wasted by customers or
- Balances the requirement to provide a satisfactory service time with the interest of economy i.e to

provide a reasonably quick service at a relatively low cost.

One of the main approaches to finding solution to a queuing problem is by using queuing model. A simple queue situation will be considered under M/M/1 model.

II. METODOLOGY

A. M/M/1 Model:

An M/M/1 model is a single channel system with poisson arrival (λ) and an exponential services time (μ)

Let λ =average arrival rate

$\lambda\delta t$ =probability that an arrival enters the system between t and $t + \delta t$ time interval.

$1 - \lambda\delta t$ = probability that no arrivals occur within the interval t and $t + \delta t$

μ = average service rate

$\mu\delta t$ = probability of completion of a service between t and $t + \delta t$ time interval

$1 - \mu\delta t$ =probability of no completion of a service between $t +$ higher order of terms in δt

Let n = number of units (customers) in the system (with line and service facility) at time t .

$P_n^{(t)}$ = probability that exactly n customers are in the system at time t .

$P_{n-1}^{(t)}$ = probability that $(n - 1)$ customers are in the system at time t .

$P_{n+1}^{(t)}$ = probability that $(n+t)$ customers are in the system at time t .

$P_n(t+\delta t)$ = probability that n customers are in the system at time $(t + \delta t)$.

In order to determine the properties of the single channel system, it is necessary to obtain an expression for $P_n(t)$.

Assumptions:-

i. Utilization factors $\rho < 1$, and $\lambda < \mu$

ii. $\rho \in (0,1)$

\therefore

By mathematical induction, the probability distribution

$$P_n = \lambda \left[\frac{P_0}{\mu} \right]^n \dots(1)$$

To obtain P_0 , the probability that there are no customer in the system, consider

$$\sum_{n=0}^{\infty} P_n = 1 \implies \sum_{n=0}^{\infty} \left[\frac{\lambda}{\mu} \right]^n = 1 \dots, (2)$$

From (1) & (2) other performance measures were derived

$$E(n) = \frac{\lambda}{\mu - \lambda} \dots(5)$$

$E(n)$ is the average number of customers in the system at any point in time.

Average time spent in the system

$$E(ts) = \frac{E(n)}{\lambda}$$

$$E(ts) = \frac{1}{\mu - \lambda}$$

$$\text{Average queuing length } E(q) = \frac{\lambda^2}{\mu(\mu - \lambda)}$$

$$\text{Average time spent on queue} = \frac{E(q)}{\lambda}$$

B. Cost in Queuing System:

We have two costs, namely:-

Waiting cost and service cost. The total cost is therefore the sum of waiting cost and service cost.

$$\begin{aligned} \text{Total cost } T(c) &= C_w E(n) + C_f \mu \\ &= C_w \left(\frac{\lambda}{\mu - \lambda} \right) + C_f \mu \end{aligned}$$

Where C_w = waiting cost C_f = service cost

To determine the minimum service rate

$$\frac{dT(c)}{d\mu} = \frac{C_w \lambda}{(\mu - \lambda)^2} + C_f = 0$$

$$\mu^* = \lambda + \sqrt{\frac{C_w \lambda}{C_f}}$$

μ^* is the minimum service rate. Experts in costing or management will however determine the C_w and C_f i.e waiting cost/unit/time and service cost /unit/time.

The performance measure developed would enable the management decide on how to control their system. The management will be able to decide on whether to reduce or increase the service facilities available. Looking at the value of the utilization factor at a glance will enable the management to determine efficiency of the system.

III. APPLICATION RESULT

A. Application 1:

Research conducted from Stanbic IBTC bank ilorin gave the following data:-

$\lambda = 24$ customer/hr, $\mu = 32$ customer/hr

a. Analysis:

$$\rho = 24/32 = 0.75$$

$$E(n) = 3 \text{ customers}$$

$$E(q) = 2.25 \text{ customers} = 2 \text{ customers}$$

$$E(ts) = 30 \text{ mins}$$

$$E(tq) = 0.38 \text{ hr} = 22.8 \text{ mins}$$

b. Comment:

If the service rate is increase the utilization factor will also be reduce but if the service rate is decrease the bank needs to open another service point.

B. Application 2:

In a bookstore, book supplies draw books from bookshelves, which is presently staffed by one cashier. The bookstore manager feels worried about the time spent by the cashier in issuing out receipts to customers and he wants to find out it by employing one assistant cashier, his worries could be eliminated. In his investigation, he finds out that:

- a. a simple queue exists
- b. each book supplier is paid ₦20 per hour
- c. the cashier receives ₦30 per hour and can answer 24 customers per hour

- d. an assistant cashier, if employed would earn ₦25 per hour and the number of customers answered would be raised to 30 per hour
- e. avergely 20 customers visit the book store per hour will it worth while to employ the service of an assistant cashier?

a) Solution

Consider fist, the existing system (before employing an assistant cashier)

Average arrival rate = $\lambda = 40$ per hour

Average service rate = $\mu = 48$ per hour

\therefore Traffic intensity = $P = \frac{\lambda}{\mu} = \frac{40}{48} = \frac{5}{6}$

\therefore Number in system = $\rho = \frac{5/6}{1 - 5/6}$

$= 5/6 \times 6/1 = 5$

\therefore Book suppliers cost per hour = ~~₦20~~ $\times 5 = \text{₦}100$ consider next, the new system after employing an assistant cashier.

Traffic intensity = $\rho = 20/30 = 2/3$

Number in system = $\rho = \frac{2/3}{1 - 2/3} = \frac{2/3 \times 3/1}{1 - 2/3} = 2$

\therefore Book suppliers cost = ~~₦20~~ $\times 2 = \text{₦}40$ per

Assistant cashier's cost = ~~₦25~~ per hour

\therefore Total cost = ~~₦40~~ + ~~₦25~~ = ~~₦65~~ per hour

\therefore Net savings per hour = ~~₦100~~ - ~~₦65~~
~~₦35~~ per hour

The employment of one assistant cashier could worth while.

IV. CONCLUSION

From the result, of the first Application, one can see that Stanbic IBTC bank Ilorin is not expected to increase the service facility because the utilization factor is under control. The result of Application 2 shows that the employment of one assistant cashier would worth while.

V. REFERENCES

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