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Predictor Corrector Method of Numerical Analysis-New Approach

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Abstract: In this paper, we discuss Predictor and Corrector method of Numerical analysis and find some new results. We also derive five-term formula for integration. Then we obtain Predictor formula by neglecting six and higher order differences and obtain corrector formula by using five-term integration formula.

Keywords: first order differential equation; predictor; corrector

I. INTRODUCTION

Many problems in Physical Sciences, material sciences and technology can be transforming into differential equations. We learn about ODEs that are linear (constant or variable coefficient), homogeneous or inhomogeneous, separable, etc. Other ODEs not belonging to one of these classes may also be solvable by special one-offtricks. Majority of ODE's do not have solutions that can be expressed in terms of simple functions [1]. The analytical methods of solving differential equations are applicable only to a limited class of equations .B.S. Grewal [2] discussed in his book that quite often differential equations appearing in physical problems do not belong to any of these familiar types and one is obliged to resort to numerical methods.

Let us consider the first order differential equation.

$$\frac{dy}{dx} = f(x,y), Given\, y(x_0) = y_0$$

If $x_n and x_{n+1}$ be two consecutive mesh points, we have $x_{n+1} = x_n + h$

From Euler's method, we have

$$y_{n+1} = y_n + hf(x_n, y_n)$$

$= y_n + hf[x_0 + nh, y_n]$;

n = 0,1, 2, 3, …… ………(1)

The modified Euler's method gives

$y_{n+1} = y_n + \frac{h}{2}[f(x_n, y_n) + f(x_{n+1}, y_{n+1})]\ldots$ ……...(2)

The value of y_{n+1} is first estimated by using (1), then this value is inserted on the right hand side of (2), gives a better approximation of y_{n+1}.This value of y_{n+1} is again substituted in (2) to find a still better approximation of y_{n+1}.This step is repeated till two consecutive values of y_{n+1}agree. This technique of refining an initially estimate of y_{n+1} by means of a more accurate formula is known as Predictor-Corrector method [4]. A well known two such methods are Milne's method and Adams method. In Milne's method, four prior values are needed for finding the value of y at x_i. For finding predictor formula Milne neglect, fourth and higher order differences and for corrector formula Milne uses Simpson's one third formula. Simpson's one-third formula is derived under the assumption that differences of order higher than second vanishes.

In this paper, It is assumed that six values of y are given corresponding to six equally spaced values of x.

Given $\frac{dy}{dx} = f(x,y)$and

$x = x_0, y = y_0$;
$x = x_1, y = y_1$;
$x = x_2, y = y_2$;
$x = x_3, y = y_3$;
$x = x_4, y = y_4$;
$x = x_5, y = y_5$.

Where

$x_1 = x_0 + h, x_2 = x_1 + h,\ x_3 = x_2 + h,\ x_4 = x_3 + h,$
$x_5 = x_4 + h$

We calculate

$f_0 = f(x_0, y_0)$,
$f_1 = f(x_1, y_1) = f(x_0 + h, y_1)$,
$f_2 = f(x_2, y_2) = f(x_0 + 2h, y_2)$,
$f_3 = f(x_3, y_3) = f(x_0 + 3h, y_3)$,
$f_4 = f(x_4, y_4) = f(x_0 + 4h, y_4)$,
$f_5 = f(x_5, y_5) = f(x_0 + 5h, y_5)$.

Then to find$y_6 = y(x_6) = y(x_0 + 6h)$ i.e. value of y at $x = x_6$.

Newton's forward interpolation formula is [3]

$$f(x,y) = f_0 + n\Delta f_0 + \frac{n(n-1)}{2!}\Delta^2 f_0 + \frac{n(n-1)(n-2)}{3!}\Delta^3 f_0 + \frac{n(n-1)(n-2)(n-3)}{4!}\Delta^4 f_0 + \frac{n(n-1)(n-2)(n-3)(n-4)}{5!}\Delta^5 f_0 + \cdots \ldots\ldots\ldots\ldots(3)$$

$$\because \frac{dy}{dx} = f(x,y)$$

$$\therefore \int_{y_0}^{y_6} dy = \int_{x_0}^{x_6} f(x,y)dx$$

$$\therefore y_6 = y_0 + \int_{x_0}^{x_0+6h} f(x,y)dx$$

$$= y_0 + \int_{x_0}^{x_0+6h} f(x,y)dx$$

$$= y_0 + \int_{x_0}^{x_0+6h}[f_0 + n\Delta f_0 + \frac{n(n-1)}{2!}\Delta^2 f_0 + \frac{n(n-1)(n-2)}{3!}\Delta^3 f_0 + \frac{n(n-1)(n-2)(n-3)}{4!}\Delta^4 f_0 + \frac{n(n-1)(n-2)(n-3)(n-4)}{5!}\Delta^5 f_0 + \cdots.]dx$$

Putting $x = x_0 + nh; dx = hdn.$

Neglecting six and higher order difference.

$$y_6 = y_0 + h\int_0^6 [f_0 + n\Delta f_0 + \frac{n^2-n}{2}\Delta^2 f_0$$
$$+\frac{n^3-3n^2+2n}{6}.\Delta^3 f_0$$
$$+\frac{n^4-6n^3+11n^2-6n}{24}\Delta^4 f_0$$
$$+\frac{n^5-10n^4+35n^3-50n^2+24n}{120}\Delta^5 f_0+\cdots]dn$$

$$= y_0 + h[nf_0 + n^2\frac{\Delta f_0}{2} + \left(\frac{n^3}{6}-\frac{n^2}{4}\right)\Delta^2 f_0$$
$$+\left(\frac{n^4}{24}-\frac{n^3}{6}+\frac{n^2}{6}\right)\Delta^3 f_0$$
$$+\left(\frac{n^5}{120}-\frac{6n^4}{96}+\frac{11n^3}{72}-\frac{3n^2}{24}\right)\Delta^4 f_0$$
$$+\left(\frac{n^6}{720}-\frac{2n^5}{120}+\frac{35n^4}{480}-\frac{50n^3}{360}+\frac{12n^2}{120}\right)\Delta^5 f_0+\cdots.]_{n=0}^{n=6}$$

$$= y_0 + h[6f_0 + 18(E-1)f_0 + \left(\frac{6^3}{6}-\frac{36}{4}\right)(E-1)^2 f_0 +$$
$$+\left(\frac{6^4}{24}-36+6\right)(E-1)^3 f_0$$
$$+\left(\frac{6^5}{120}-\frac{6^5}{96}+\frac{11\times 6^3}{72}-\frac{3\times 6^2}{24}\right)(E-1)^4$$
$$+\left(\frac{6^6}{720}-\frac{2\times 6^5}{120}+\frac{35\times 6^4}{480}-\frac{50\times 6^3}{360}+\frac{12\times 6^2}{120}\right)(E-1)^5]$$
$$= y_0 + h[6f_0 + 18(E-1)f_0 + 27(E^2-2E+1)f_0$$
$$+24(E^3-3E^2+3E-1)f_0$$
$$+\frac{123}{10}(E^4-4E^3+6E^2-4E+1)f_0$$
$$+\frac{33}{10}(E^5-5E^4+10E^3-10E^2+5E-1)f_0]$$
$$= y_0 + h[6f_0 + 18(f_1-f_0) + 27(f_2-2f_1+f_0)$$
$$+24(f_3-3f_2+3f_1-f_0)$$
$$+\frac{123}{10}(f_4-4f_3+6f_2-4f_1+f_0)$$
$$+\frac{33}{10}(f_5-5f_4+10f_3-10f_2+5f_1-f_0)$$
$$= y_0 + \frac{h}{10}(33f_1-42f_2+78f_3-42f_4+33f_5)$$
$$y_6 = y_0 + \frac{h}{10}[33(f_1+f_5)+78f_3-42(f_2+f_4)]$$
……………(I)

This is Predictor formula for finding y_6.

For finding corrector formula, we first find five-term formula for integration as under-

Let

$$\int_{x_0-2h}^{x_0+2h} y\,dx$$
$$= a_{-2}y_{-2} + a_{-1}y_{-1} + a_0y_0 + a_1y_1 + a_2y_2\ldots$$
………(A)

Where the unknowns $a_{-2,}a_{-1},\ a_0,\ a_1, a_2$ are determined by making (A) exact for $y(x) = 1, x, x^2, x^3, x^4$ respectively.

So Putting

$y(x) = 1, x, x^2, x^3, x^4$ respectively.We obtain

$$a_{-2} + a_{-1} + a_0 + a_1 + a_2$$
$$= \int_{x_0-2h}^{x_0+2h} dx = (x)_{x_0-2h}^{x_0+2h} = 4h\ldots\ldots(4)$$

$$a_{-2}(x_0-2h) + a_{-1}(x_0-h)$$
$$+a_0x_0 + a_1(x_0+h) + a_2(x_0+2h)$$
$$= \int_{x_0-2h}^{x_0+2h} x\,dx = \frac{(x^2)_{x_0-2h}^{x_0+2h}}{2} = \frac{[(x_0+2h)^2-(x_0-2h)^2]}{2}$$
………..(5)

$$a_{-2}(x_0-2h)^2 + a_{-1}(x_0-h)^2$$
$$+a_0{x_0}^2 + a_1(x_0+h)^2 + a_2(x_0+2h)^2$$
$$= \int_{x_0-2h}^{x_0+2h} x^2dx = \frac{(x^3)_{x_0-2h}^{x_0+2h}}{3} = \frac{[(x_0+2h)^3-(x_0-2h)^3]}{3}$$
…………(6)

$$a_{-2}(x_0-2h)^3 + a_{-1}(x_0-h)^3$$
$$+a_0{x_0}^3 + a_1(x_0+h)^3 + a_2(x_0+2h)^3$$
$$= \int_{x_0-2h}^{x_0+2h} x^3dx = \frac{(x^4)_{x_0-2h}^{x_0+2h}}{4} = \frac{[(x_0+2h)^4-(x_0-2h)^4]}{4}$$
………………(7)

$$a_{-2}(x_0-2h)^4 + a_{-1}(x_0-h)^4$$
$$+a_0{x_0}^4 + a_1(x_0+h)^4 + a_2(x_0+2h)^4$$
$$= \int_{x_0-2h}^{x_0+2h} x^4dx = \frac{(x^5)_{x_0-2h}^{x_0+2h}}{5} = \frac{[(x_0+2h)^5-(x_0-2h)^5]}{5}$$
………………(8)

Shifting origin to x_0 and taking $x_0 = 0$..Then above five equations becomes

$$a_{-2} + a_{-1} + a_0 + a_1 + a_2 = 4h \quad \ldots\ldots.(9)$$
$$-2a_{-2} - a_{-1} + a_1 + 2a_2 = 0\ldots \quad \ldots\ldots..(10)$$
$$4a_{-2} + a_{-1} + a_1 + 4a_2 = \frac{16h}{3} \ldots \quad \ldots\ldots(11)$$
$$-8a_{-2} - a_{-1} + a_1 + 8a_2 = 0 \quad \ldots\ldots..(12)$$
$$16a_{-2} + a_{-1} + a_1 + 16a_2 = \frac{64}{5}h\ldots(13)$$

Subtracting equation (10) from equation (12) we get $-6a_{-2} + 6a_2 = 0 \Rightarrow a_{-2} = a_2$.

From (13) –(11)

$$12a_{-2} + 12a_2 = \frac{64}{5}h - \frac{16h}{3} = \frac{112}{15}h$$

Putting $a_{-2} = a_2$.

$24\ a_2 = \frac{112}{15}h \Rightarrow a_2 = \frac{14}{45}h = a_{-2}$.

Since $a_{-2} = a_2$. From equation (12) .$a_{-1} = a_1$.

Putting values of $a_{-2},\ a_2 \& a_{-1} = a_1$ in (11) we find values of $a_{-1} \& a_1$.

$a_{-1} = a_1 = \frac{64}{45}h$.

Putting values of $a_{-2},\ a_2,\ a_{-1}, a_1$in equation (9), we get $a_0 = \frac{24}{45}h$

$$\therefore a_0 = \frac{24}{45}h, a_{-1} = a_1 = \frac{64}{45}h, a_2 = a_{-2} = \frac{14}{45}h$$
$$\int_{x_0-2h}^{x_0+2h} y\,dx = a_{-2}\,y_{-2} + a_{-1}y_{-1} + a_0y_0 + a_1y_1 + a_2y_2$$
$$= h\left[\frac{14}{45}y_{-2} + \frac{64}{45}y_{-1} + \frac{24}{45}y_0 + \frac{64}{45}y_1 + \frac{14}{45}y_2\right]$$
$$\therefore \int_{x_0-2h}^{x_0+2h} y\,dx$$
$$= \frac{2}{45}h[7y_{-2} + 32y_{-1} + 12y_0 + 32y_1 + 7y_2]$$
……..(14)

This is five-step integration formula.

Applying this five step formula for first order differential equation

$$\frac{dy}{dx} = f(x,y).$$
$$dy = f(x,y)dx$$

$\int_{y_2}^{y_6^{(1)}} dy = \int_{x_0+2h}^{x_0+6h} f(x,y)dx.$

$\therefore y_6^{(1)} - y_2 = \int_{x_0+2h}^{x_0+6h} f(x,y)dx.$

$\therefore y_6^{(1)}$

$$= y_2 + \frac{2}{45}h[7f_2 + 32f_3 + 12f_4 + 32f_5 + 7f_6] \quad \text{............(II)}$$

$y_6^{(1)}$is first corrected value of y_6, where

$$f_6 = f(x_0 + 6h, y_6)$$

$y_6$is predicted value of $y at x = x_0 + 6h$.

Similarly second corrected value of y_6 is

$y_6^{(2)} = y_2 + \frac{2}{45}h[7f_2 + 32f_3 + 12f_4 + 32f_5 + 7f_6^{(1)}]$.

Where

$f_6^{(1)} = f(x_0 + 6h, y_6^{(1)})$and so on.

When two corrected values become same, then this will be correct value of $y at x = x_0 + 6h$.

Similarly we can find y_7, y_8..........etc. by using predictor formula (I) and corrector formula (II)

II. RESULT AND DISCUSSION

Here we see that predict value of $y_6$when the values of y_0, y_1, y_2, y_3, y_4andy_6 are known can be obtained by predictor formula

$y_6 = y_0 + \frac{h}{10}(33f_1 - 42f_2 + 78f_3 - 42f_4 + 33f_5)$. And corrector formula for y_6 is

$\therefore y_6^{(1)}$

$$= y_2 + \frac{2}{45}h[7f_2 + 32f_3 + 12f_4 + 32f_5 + 7f_6]$$

Thus we can write predictor formula as under

$y_{n+1} = y_{n-5} + \frac{h}{10}(33f_{n-4} - 42f_{n-3} + 78f_{n-2} - 42f_{n-1} + 33f_n)$. Where n=5,6,7,……………

And corrector formula can be written as

$y_{n+1}^{(1)}$

$= y_{n-3} + \frac{2}{45}h[7f_{n-3} + 32f_{n-2} + 12f_{n-1} + 32f_n + 7f_{n+1}]$.

Here we have considered the differences up to 5th order, because we fit a polynomial of degree six.

In this paper, we also derive five-step integration formula

$$\int_{x_0-2h}^{x_0+2h} y dx$$

$= \frac{2}{45}h[7y_{-2} + 32y_{-1} + 12y_0 + 32y_1 + 7y_2]$ which is very useful for integrating a function when we divide range of integration in five parts.

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