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Study of New-Fangled Julia Sets Controlled by TAN Function

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Abstract: In this paper we consider the dynamics of complex transcendental function i.e. tan function using iterative procedure known as Ishikawa Method. Fractals are created and examined for integer and non-integer values. New Julia sets are generated for different values of parameters defined by Ishikawa iteration and transcendental controlled function. Here different dendrites, spiral lobes are considered and our study relies on the analysis of Julia sets.

Keywords: Julia Sets, Fractals, Ishikawa iteration, Complex Numbers, Transcendental function.

I. INTRODUCTION

Complex Graphics of nonlinear dynamical systems have been a focus of research nowadays. These graphics of complex plane is considered under Fractal Theory. A Fractal is a statistical shape that is difficult and detailed at every level of magnification, as fit as self-similar. Fractal is defined as a set, which is self–similar under magnification [1]. Self-similarity is defined as looking the same structure over all ranges of scale, i.e. a small section of a fractal can be viewed as a part of the larger fractal. Self-similarity is established in shapes that have repetitive patterns on smaller scales. Among Euclidean shapes line and a square are self-similar. By enlarging and replicating the line can construct the square and reducing the square can make the line [2]. Here line has dimension equal to 1, and the square has dimension equal to 2. It seems possible that a structure between a line and a square i.e. a jagged line, which will have a non-integer dimension value between 1 and 2. Fractals provide us with facility of having a non-integer dimension (Hausdorff-Besicovitch dimension) over traditional Euclidean dimension having integer dimension.

Fractal Theory is an inspiring branch of applicable Mathematics and Computer Science. Benoit Mandelbrot (1924-2010) is recognized as the father of fractal geometry. He gave the word fractal in the late 1970s. He explain geometric fractals as "a rough or fragmented geometric shape that can be divided into parts, every one of which is a reduced-size duplicate of the whole".[3] There are many variety of fractals found in nature in the form of many usual objects such as mountains, coastlines, trees ferns and clouds[2,4]. They all are fractals in nature and can be represented on a computer by a recursive algorithm of computer graphics.

The Julia sets and the Mandelbrot sets are two most important images under various researches in the field of fractal theory [5]. In 1918, French Mathematician Gaston Julia (1893-1978) [6] investigated the iteration process of a complex function and attained a Julia set, whereas the Mandelbrot set was given by Benoit B. Mandelbrot [2] in 1979.

II. PRELIMINARIES

Since Mandelbrot's was success in making the research of fractals and their applications popular. Many people have learned to define and create fractals. A Fractal is prepared by repeating a simple process again and again. They essentially have the property of 'Self-Similarity' i.e. a fractal is a never ending pattern that repeats itself at different scales either strictly or statistically. Today many self-similar beautiful images can be generated easily on a personal computer and have become a popular field of computer graphics skill. Here we have used the transformation function $Z \rightarrow (Z^n+C)$, $n>=2.0$ and $C=\tan(1/\#pixel^p)$, $p>=1.0$ for generating fractal images with respect to Ishikawa iterates, where z and c are the complex quantities and n, p are real numbers. Each of these fractal images is constructed as a two-dimensional array of pixels. Each pixel is represented by a pair of (x, y) coordinates. The complex quantities z and c can be represented as:

$$Z=Z_x+iZ_y$$

$$C=C_x+iC_y$$

Where $i=\sqrt{(-1)}$ and Z_x , C_x are the real parts and Z_y & C_y are the imaginary parts of Z and C , respectively. The pixel coordinates (x, y) may be associated with (C_x , C_y) or (Z_x , Z_y). Based on this concept, the fractal images can be classified as follows:

z-Plane fractals, where in (x,y) is a function of (z_x,z_y).

c-Plane fractals, where in (x,y) is a function of (c_x,c_y).

In the literature, the fractals for n=2 in z plane are termed as the Mandelbrot set while the fractals for n=2 in c plane are known as Julia sets [7].

A. Mann's Iteration : One Step Iteration:

Iteration means to repeat a process again and again. Starting with the initial value, the output is fed back to the process. The procedure is repeated until the result or goal is reached. Fractal geometries are generated in an iterative manner, which leads to a self-similar structure. Mann's iteration technique is a one-step iteration technique given by William Robert Mann (1920-2006), a mathematician from Chapel Hill, North Carolina. The iteration technique involves one step for iteration, and is given as [8,9]:

$$x_{n+1} = s.f(x_n) + (1-s)x_n\,, \quad \text{where } n\geq 0 \text{ and } 0 < s < 1$$

B. shikawa Iteration : Two Step Iteration:

Another iterative procedure is an Ishikawa iteration [10,11] technique which is a two-step iteration method known after Ishikawa. Let X be a subset of complex number and $f : X \to X$ for all $x_0 \in X$, we have the sequence numbers for $\{x_n\}$ and $\{y_n\}$ in X according to following way:

$$y_n = S'_n\, f(x_n) + (1 - S'_n)x_n$$

$$x_{n+1} = S_n f(y_n) + (1 - S_n)x_n$$

Where, $0 \le S'_n \le 1$, $0 \le S_n \le 1$ and S'_n & S_n are both convergent to non-zero number.

III. GENERATION OF RELATIVE SUPERIOR JULIA SETS

We present here some Relative Superior Julia sets [13,14] for the function $Z \rightarrow (Z^n+C)$, $n>=2.0$, $C=\tan(1/\#pixel^p)$, $p>=1.0$ for integer and some non-integer values. The parameter s and s' also changes the structure and attractiveness of fractals. Following are some of the Julia sets for different values of p and n for quadratic, cubic and bi-quadratic functions.

A. Relative Superior Julia Sets for Quadratic function:

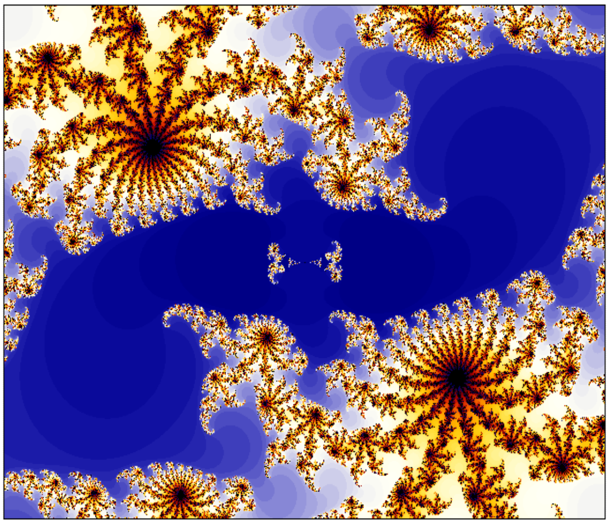

Figure. 1 RSJS for s=s'=1, p=1, n=2, c= 0.3375 + 0.625i

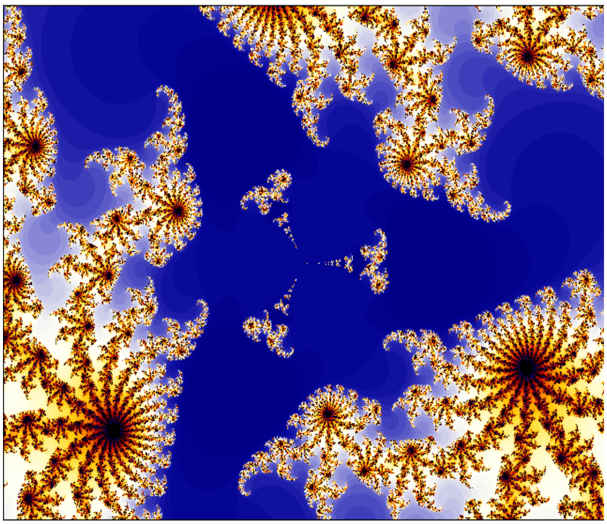

Figure. 2 RSJS for s=s'=1, p=1.5, n=2, c= 0.3375 + 0.625i

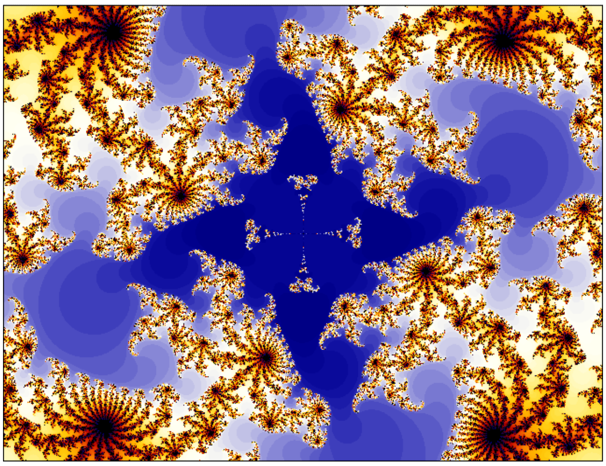

Figure. 3 RSJS for s=s'=1, p=2, n=2, c= 0.3375 + 0.625i

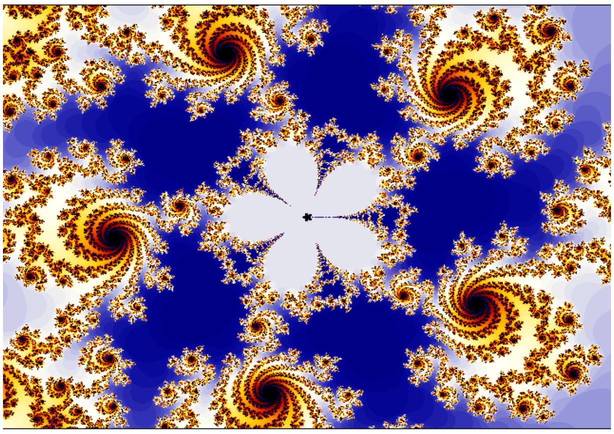

Figure. 4 RSJS for s=s'=1, p=2.5, n=2, c= 0.1 - 0.6125i

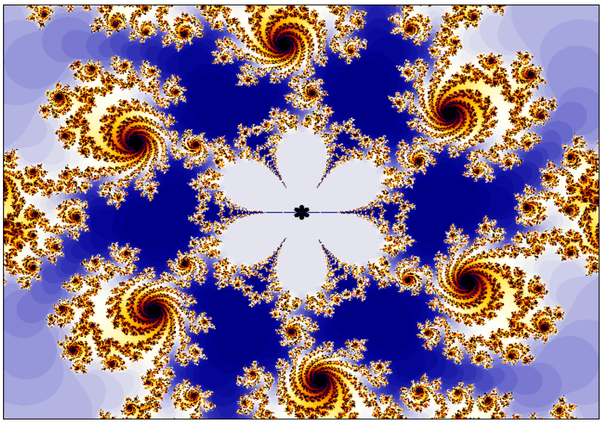

Figure. 5 RSJS for s=s'=1, p=3, n=2, c= 0.1 - 0.6125i

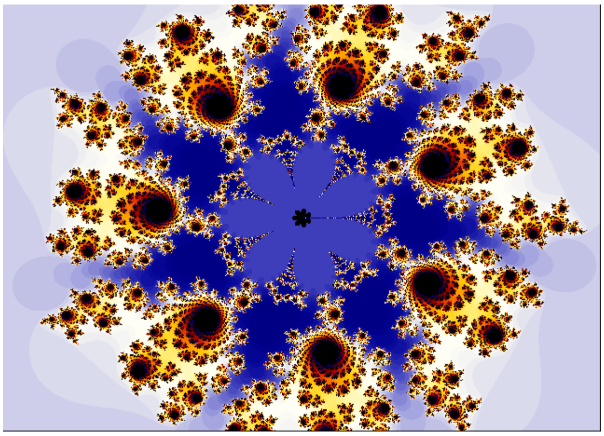

Figure. 6 RSJS for s=s'=1, p=3.5, n=2, c= 0.162 + 0.575i

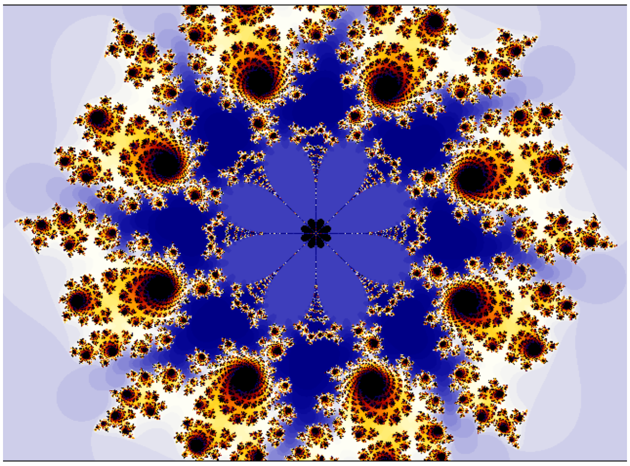

Figure. 7 RSJS for s=s'=1, p=4, n=2, c= 0.162 + 0.575i

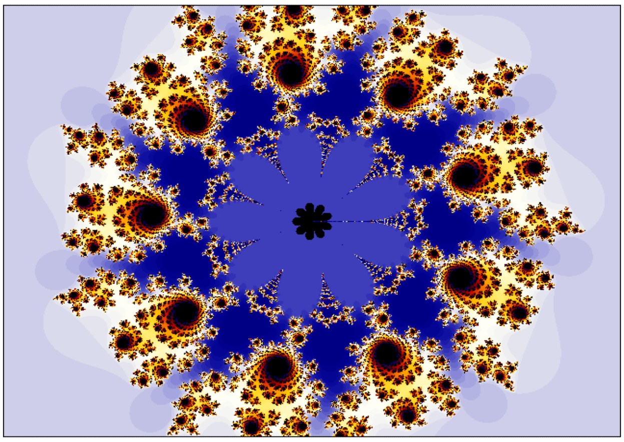

Figure. 8 RSJS for s=s'=1, p=4.5, n=2, c= 0.162 + 0.575i

B. Relative Superior Julia sets for Cubic function:

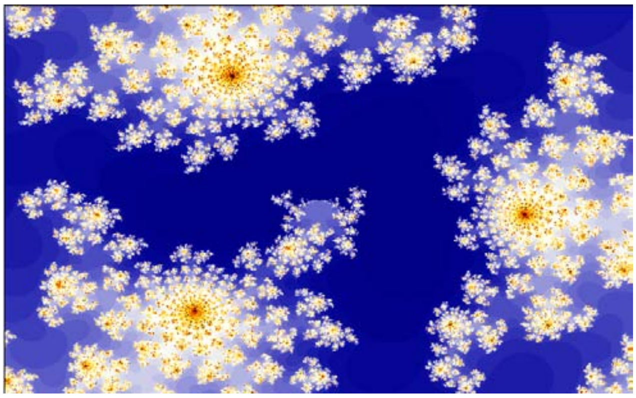

Figure. 9 RSJS for s=s'=1, p=1, n=3, c= - 0.5375 + 0.4125i

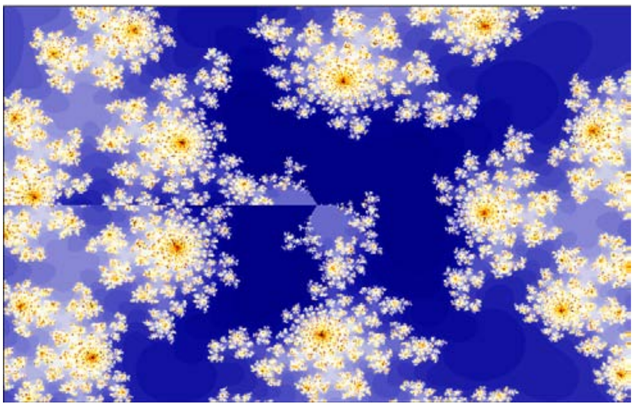

Figure. 10 RSJS for s=s'=1, p=1.5, n=3, c= - 0.5375 + 0.4125i

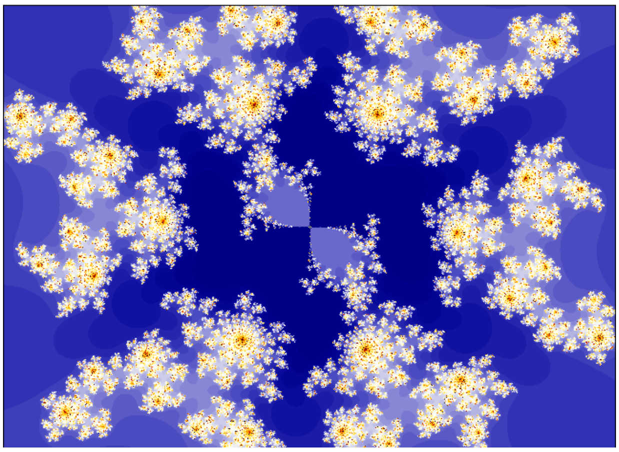

Figure. 11 RSJS for s=s'=1, p=2, n=3, c= - 0.5375 + 0.4125i

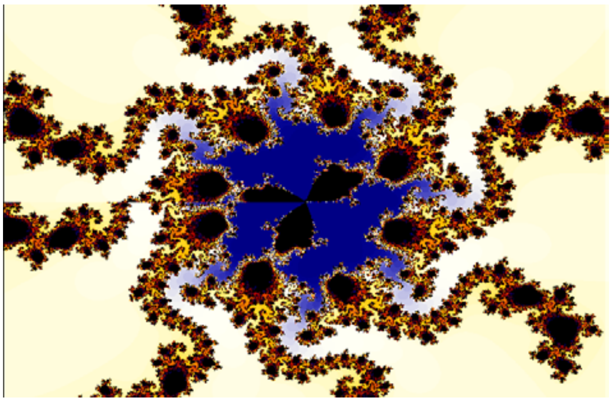

Figure. 12 RSJS for s=s'=1, p=2.5, n=3, c= 0.45 - 0.55i

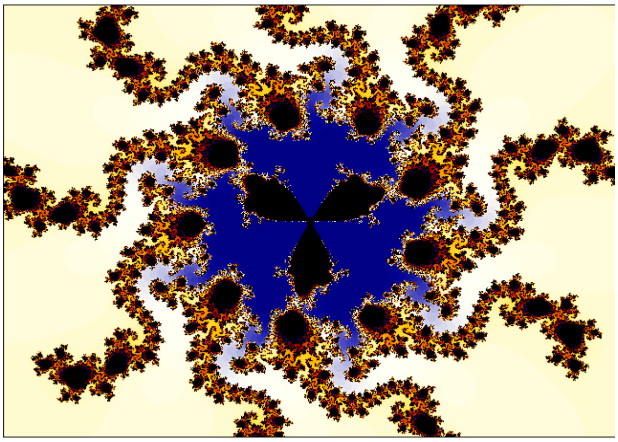

Figure. 13 RSJS for s=s'=1, p=3, n=3, c= 0.45 - 0.55i

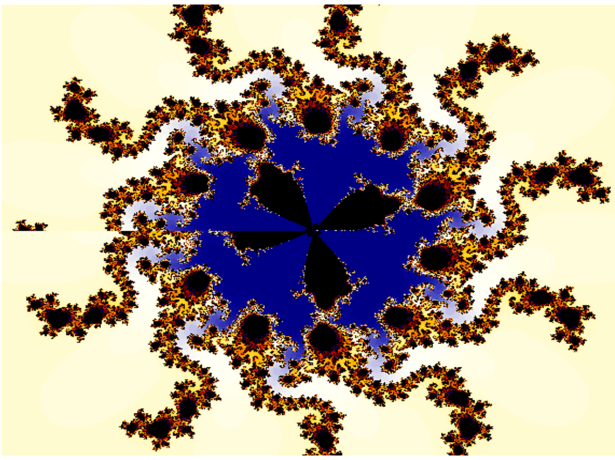

Figure. 14 RSJS for s=s'=1, p=3.5, n=3, c= 0.45 - 0.55i

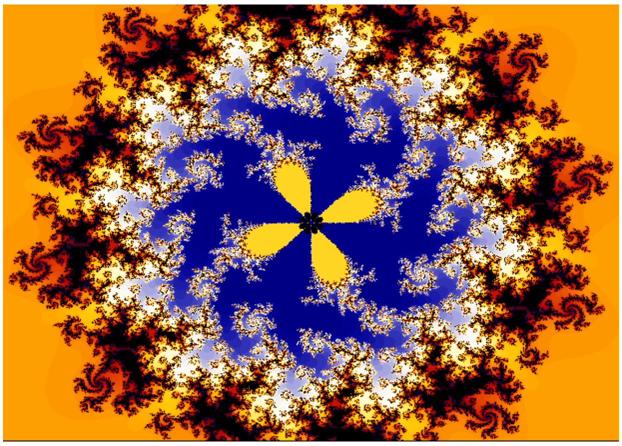

Figure. 15 RSJS for s=s'=1, p=4, n=3, c= - 0.5625 - 0.2375i

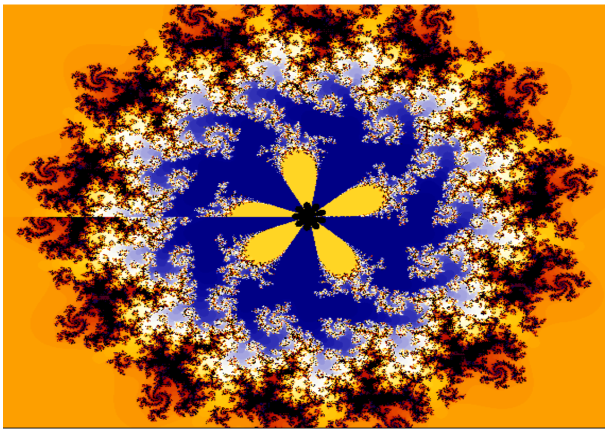

Figure. 16 RSJS for s=s'=1, p=4.5, n=3, c= - 0.5625 - 0.2375i

C. *Relative Superior Julia sets for Bi-Quadratic function:*

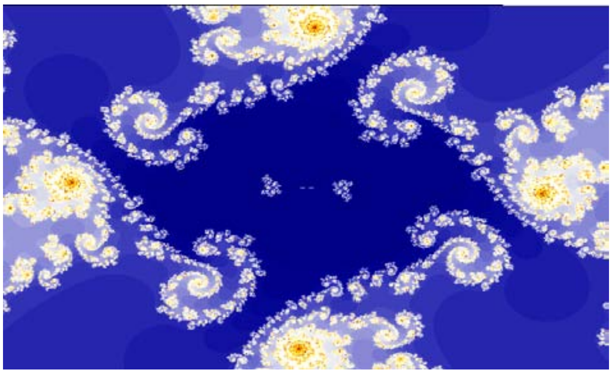

Figure. 17 RSJS for s=s'=1, p=1, n=4, c= 0.4875 - 0.65i

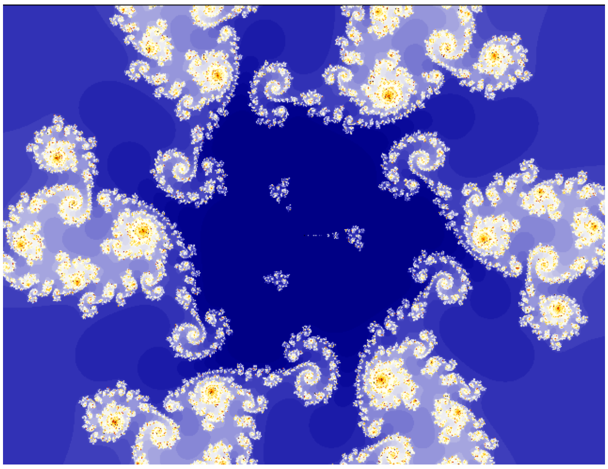

Figure. 18 RSJS for s=s'=1, p=1.5, n=4, c= 0.4875 - 0.65i

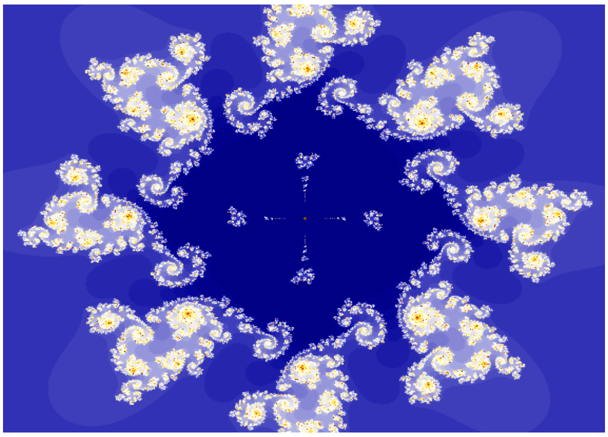

Figure. 19 RSJS for s=s'=1, p=2, n=4, c= 0.4875 - 0.65i

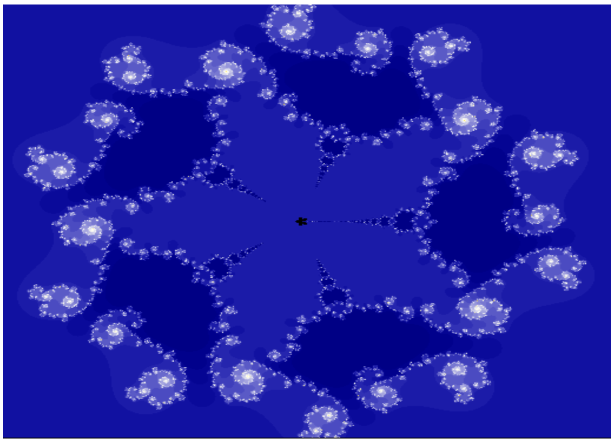

Figure. 20 RSJS for s=s'=1, p=2.5, n=4, c= 0.81 - 0.125i

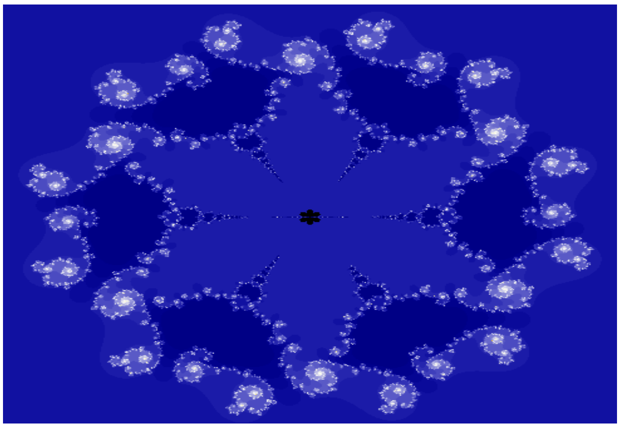

Figure. 21 RSJS for s=s'=1, p=3, n=4, c= 0.81 - 0.125i

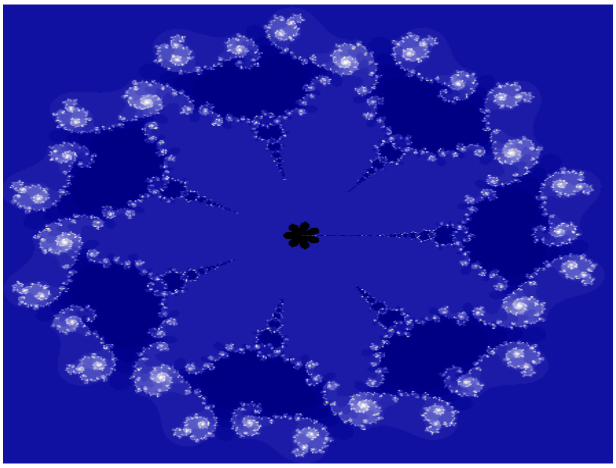

Figure. 22 RSJS for s=s'=1, p=3.5, n=4, c= 0.81 - 0.125i

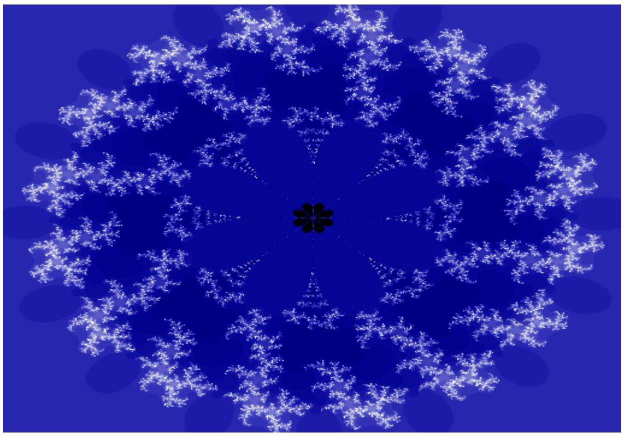

Figure. 23 RSJS for s=s'=1, p=4, n=4, c= - 0.475 + 0.5125i

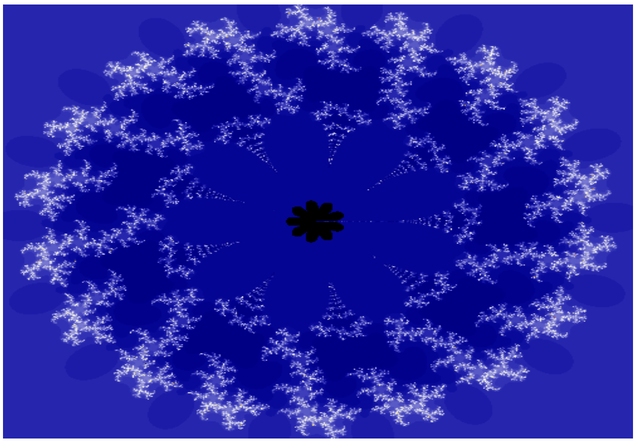

Figure. 24 RSJS for s=s'=1, p=4.5, n=4, c= - 0.475 + 0.5125i

IV. CONCLUSION

In the complex dynamics polynomial function for $z \rightarrow (z^n+c)$, where n>=2.0, control transcendental function is $c=\tan(1/\#pixel^p)$, p>=1.0. The fractals generated with power p and n are found as rotationally symmetric. We have analysis superior iterates at different power of n and p as shown in fig. There are spiral dendrites or tails attached with the centre of main body.

The controlling function tan for value c exhibits new characteristics for the generating fractals. Here we have presented the geometric properties of fractals along different axis. The fractals generated above depend on the parameter p. From above observation and analysis 2p similar image of spirals or tails are generated for above tan controlling function. For non-integer value of p bifurcation and new images are created from left side step by step till the upper integer value is met.

For integer values of p and n, Julia Sets are symmetrical. The generated superior Julia sets resemble to beautiful images, full of colorful, spiral, dendrites, seahorse tail, and crowns, with symmetry and depending on parameter p, n.

In looking at these above images have certainly noticed a type of self-similarity -- that is, the structure and shape of the image is repeated throughout smaller portions of the image. This self-similarity is characteristic of Julia sets.

V. REFERENCES

[1]. B. B. Mandelbrot," Fractals and Chaos" Berlin: Springer. pp. 38, *ISBN 978-0-387-20158-0. "A fractal set is one for which the fractal (Hausdorff-Besicovitch)* dimension strictly exceeds the topological dimension", 2004.

[2]. M. F. Barnsley, R. L. Devaney, B. B. Mandelbrot, H. O. Peitgen, D. Saupe, and R. F. Voss, "The Science of Fractal Images", Springer – Verlag 1988.

[3]. B. B. Mandelbrot, "*The fractal geometry of nature." Macmillan. ISBN 978-0-7167-1186-5,* 1983.

[4]. H. Peitgen and P. H. Richter, "The Beauty of Fractals", Springer-Verlag, Berlin, 1986.

[5]. G. Edgar, "Classics on Fractals", Boulder, CO: Westview Press. *ISBN 978-0-8133-4153-8,* 2004.

[6]. G. Julia, " Sur I' iteration des functions rationnelles", J.Math Pure Appli. 8 (1918), 737-747.

[7]. K. W. Shirriff, "An investigation of fractals generated by $z \rightarrow z^{-n} + c$ ", Computers and Graphics, pp. 603-607, 13, 4,1993.

[8]. A. Negi, and M. Rani, "Midgets of Superior Mandelbrot Set", Chaos, Solitons and Fractals, July 2006.

[9]. S. Shukla and A. Negi," Study of Newbie Fractal controlled by log function", International Journal of Computer Technology and Applications (IJCTA), Vol 4 (2), pp.341-346.

[10]. R. Rana, Y. S. Chauhan and A. Negi, "Ishikawa Iterates for Logarithmic function", International Journal of Computer Applications (IJCA), Volume 15, No.5, February 2011.

[11]. S. Ishikawa, "Fixed points by a new iteration method", Proc. Amer. Math. Soc.44, (1974), pp.147-150.

[12]. A. Negi, "Generation of Fractals and Applications", Thesis, Gurukul Kangri Vishwavidyalaya, 2005.

[13]. M. Rani, and V. Kumar, "Superior Mandelbrot Set", J.Koreans Soc. Math. Edu. Ser. D (2004) 8(4), pp. 279-291.

[14]. M. Rani, "Iterative procedures in fractals and chaos", Ph.D. Thesis, Department of Computer Science, Faculty of Technology, Gurukul Kangri Vishvavidyalaya, Hardwar, 2002.

[15]. R. L. Daveney, "An Introduction to Chaotic Dynamical Systems", Springer-Verlag, New York. Inc.1994.