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Anti-synchronization of Liu-Chen-Liu and Liu-Su-Liu Chaotic Systems by Active Nonlinear Control

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Abstract: The purpose of this paper is to study chaos anti-synchronization of identical Liu-Chen-Liu chaotic systems (Liu, Chen and Liu, 2007), identical Liu-Su-Liu chaotic systems (Liu, Su and Liu, 2006), and non-identical Liu-Chen-Liu and Liu-Su-Liu chaotic systems using active nonlinear control. Sufficient conditions for achieving anti-synchronization of the identical and different Liu-Chen-Liu and Liu-Su-Lu chaotic sytsems using active nonlinear control are derived based on Lyapunov stability theory. Since the Lyapunov exponents are not required for these calculations, the nonlinear feedback control method is effective and convenient to anti-synchronize identical and different Liu-Chen-Liu and Liu-Su-Liu chaotic systems. Numerical simulations are also given to illustrate and validate the anti-synchronization results for Liu-Chen-Liu and Liu-Su-Liu chaotic systems.

Keywords: Chaos Anti-synchronization, Nonlinear Control, Liu-Chen-Liu System, Liu-Su-Liu System, Active Control.

I. INTRODUCTION

Chaotic systems are dynamical systems that are highly sensitive to initial conditions. This sensitivity is popularly referred to as the *butterfly effect* [1].

Since the pioneering work of Pecora and Carroll [2], chaos synchronization has attracted a great deal of attention from various fields and it has been extensively studied in the last two decades [2-17]. Chaos theory has been explored in a variety of fields including physical [3], chemical [4], ecological [5] systems, secure communications [6-8] etc. In the recent years, various schemes such as PC method [2], OGY method [9], active control [10-12], adaptive control [13-14], time-delay feedback approach [15], backstepping design method [16], sampled-data feedback synchronization method [17], sliding mode control [18], etc. have been successfully applied for chaos synchronization. Recently, active control has been applied to anti-synchronize identical chaotic systems [19-20] and different hyperchaotic systems [21].

In most of the chaos anti-synchronization approaches, the master-slave or drive-response formalism is used. If a particular chaotic system is called the *master* or *drive* system and another chaotic system is called the *slave* or *response* system, then the idea of the anti-synchronization is to use the output of the master system to control the slave system so that the states of the slave system have the same amplitude but opposite signs as the states of the master system asymptotically. In other words, the sum of the states of the master and slave systems are expected to converge to zero asymptotically when anti-synchronization appears.

This paper has been organized as follows. In Section II, we give the problem statement and our methodology. In Section III, we discuss the chaos anti-synchronization of two identical Liu-Chen-Liu systems ([22], 2007). In Section IV, we discuss the chaos anti-synchronization of two identical Liu-Su-Liu chaotic systems ([23], 2006). In Section V, we discuss the anti-synchronization of Liu-Chen-Liu and Liu-Su-Liu chaotic systems. In Section VI, we present the conclusions of this paper.

II. PROBLEM STATEMENT AND OUR METHODOLOGY

Consider the chaotic system described by the dynamics

$$\dot{x} = Ax + f(x) \tag{1}$$

where $x \in \mathbf{R}^n$ is the state of the system, A is the $n \times n$ matrix of the system parameters and $f : \mathbf{R}^n \to \mathbf{R}^n$ is the nonlinear part of the system. We consider the system (1) as the *master* or *drive* system.

As the *slave* or *response* system, we consider the following chaotic system described by the dynamics

$$\dot{y} = By + g(y) + u \tag{2}$$

where $y \in \mathbf{R}^n$ is the state vector of the response system, B is the $n \times n$ matrix of the system parameters, $g : \mathbf{R}^n \to \mathbf{R}^n$ is the nonlinear part of the response system and $u \in \mathbf{R}^n$ is the controller of the response system.

If $A = B$ and $f = g$, then x and y are the states of two *identical* chaotic systems. If $A \neq B$ and $f \neq g$, then x and y are the states of two *different* chaotic systems.

For the anti-synchronization of the chaotic systems (1) and (2) using active control, we design a feedback controller u which anti-synchronizes the states of the master system (1) and the slave system (2) for all initial conditions of the systems.

If we define the *anti-synchronization error* as

$$e = y + x, \tag{3}$$

then the anti-synchronization error dynamics is obtained as

$$\dot{e} = By + Ax + g(y) + f(x) + u \tag{4}$$

Thus, the global anti-synchronization problem is essentially to find a feedback controller u so as to stabilize the error dynamics (4) for all initial conditions $e(0) \in \boldsymbol{R}^n$, i.e.

$$\lim_{t \to \infty} \|e(t)\| = 0 \tag{5}$$

for all initial conditions $e(0) \in \boldsymbol{R}^n$.

We use Lyapunov stability theory as our methodology. We take as a candidate Lyapunov function

$$V(e) = e^T P e, \tag{6}$$

where P is a positive definite matrix.

Note that $V : \boldsymbol{R}^n \to \boldsymbol{R}^n$ is a positive definite function by construction. We assume that the parameters of the master and slave systems are known and that the states of both systems (1) and (2) are measurable.

If we find a feedback controller u so that

$$\dot{V}(e) = -e^T Q e, \tag{7}$$

where Q is a positive definite matrix, then $\dot{V} : \boldsymbol{R}^n \to \boldsymbol{R}^n$ is a negative definite function.

Thus, by Lyapunov stability theory [24], the error dynamics (4) is globally exponentially stable and hence the condition (5) will be satisfied for all initial conditions $e(0) \in R^n$. Then the states of the master system (1) and slave system (2) will be globally exponentially anti-synchronized.

III. Anti-Synchronization of Identical Liu-Chen-Liu Chaotic Systems

In this section, we apply the active nonlinear control technique for the anti-synchronization of two identical Liu-Chen-Liu chaotic systems ([22], 2007).

Thus, the master system is described by the Liu-Chen-Liu dynamics

$$\begin{aligned} \dot{x}_1 &= a(x_2 - x_1) \\ \dot{x}_2 &= bx_1 + kx_1x_3 \\ \dot{x}_3 &= -cx_3 - hx_1x_2 \end{aligned} \tag{8}$$

where x_1, x_2, x_3 are the states of the system and a, b, c, h, k are positive parameters of the system.

The slave system is also described by the Liu-Chen-Liu dynamics as

$$\begin{aligned} \dot{y}_1 &= a(y_2 - y_1) + u_1 \\ \dot{y}_2 &= by_1 + ky_1y_3 + u_2 \\ \dot{y}_3 &= -cy_3 - hy_1y_2 + u_3 \end{aligned} \tag{9}$$

where y_1, y_2, y_3 are the states of the system and

$$u = \begin{bmatrix} u_1 & u_2 & u_3 \end{bmatrix}^T$$

is the nonlinear controller to be designed.

The Liu-Chen-Liu system (8) is a new 3-D chaotic system proposed by Liu, Chen and Liu ([22], 2007).

The Liu-Chen-Liu system (8) is chaotic when

$$a = 10,\ b = 40,\ c = 2.5,\ h = 1 \text{ and } k = 16.$$

Figure 1 illustrates the chaotic portrait of the Liu-Chen-Liu system, which has a reversed butterfly-shaped chaotic attractor.

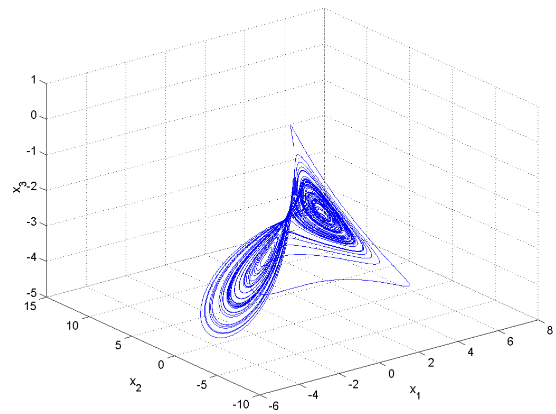

Figure 1. Chaotic Portrait of the Liu-Chen-Liu System (8)

The anti-synchronization error e is defined by

$$e_i = y_i + x_i, \qquad (i = 1, 2, 3) \tag{10}$$

The error dynamics is obtained as

$$\begin{aligned} \dot{e}_1 &= a(e_2 - e_1) + u_1 \\ \dot{e}_2 &= be_1 + k(y_1y_3 + x_1x_3) + u_2 \\ \dot{e}_3 &= -ce_3 - h(y_1y_2 + x_1x_2) + u_3 \end{aligned} \tag{11}$$

To find an anti-synchronizing controller, we first let

$$\begin{aligned} u_2 &= u_{2a} + u_{2b} \\ u_3 &= u_{3a} + u_{3b} \end{aligned} \tag{12}$$

where

$$\begin{aligned} u_{2b} &= -k\left(y_1y_3 + x_1x_3\right) \\ u_{3b} &= h(y_1y_2 + x_1x_2) \end{aligned} \tag{13}$$

Substituting (12) and (13) into (11), we obtain

$$\begin{aligned} \dot{e}_1 &= a(e_2 - e_1) + u_1 \\ \dot{e}_2 &= be_1 + u_{2a} \\ \dot{e}_3 &= -ce_3 + u_{3a} \end{aligned} \tag{14}$$

Next, we consider the candidate Lyapunov function

$$V(e) = \frac{1}{2} e^T e = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2\right) \tag{15}$$

which is a positive definite function on $\boldsymbol{R}^3$.

A simple calculation gives

$$\begin{aligned} \dot{V}(e) = {} & -ae_1^2 - ce_3^2 + (a+b)e_1e_2 + e_1u_1 \\ & + e_2u_{2a} + e_3u_{3a} \end{aligned} \tag{16}$$

Therefore, we choose

$$\begin{aligned} u_1 &= -(a+b)e_2 \\ u_{2a} &= -e_2 \\ u_{3a} &= 0 \end{aligned} \tag{17}$$

Substituting (17) into (14), the error dynamics simplifies to

$$\begin{aligned} \dot{e}_1 &= -ae_1 - be_2 \\ \dot{e}_2 &= be_1 - e_2 \\ \dot{e}_3 &= -ce_3 \end{aligned} \tag{18}$$

Substituting (17) into (16), we obtain

$$\dot{V}(e) = -ae_1^2 - e_2^2 - ce_3^2 \tag{19}$$

which is a negative definite function on $\boldsymbol{R}^3$ since a and c are positive constants.

Hence, by Lyapunov stability theory [24], the error dynamics (18) is globally exponentially stable.

Combining (12), (13) and (17), the anti-synchronizing nonlinear controller u is obtained as

$$\begin{aligned} u_1 &= -(a+b)e_2 \\ u_2 &= -e_2 - k(y_1y_3 + x_1x_3) \\ u_3 &= h(y_1y_2 + x_1x_2) \end{aligned} \tag{20}$$

Thus, we have proved the following result.

Theorem 1. The identical Liu-Chen-Liu systems (8) and (9) are exponentially and globally anti-synchronized for any initial conditions with the nonlinear controller u defined by (20).

Numerical Results

For the numerical simulations, the fourth-order Runge-Kutta method with time-step 10^{-6} is used to solve the systems using MATLAB.

For the Liu-Chen-Liu system (8), the parameter values are taken as those which result in the chaotic behaviour of the system, *viz.* $a = 10, b = 40, c = 2.5, h = 1$ and $k = 16.$

The initial values of the master system (8) are taken as

$$x_1(0) = 7,\ x_2(0) = 3,\ x_3(0) = 12$$

while the initial values of the slave system (9) are taken as

$$y_1(0) = 2,\ y_2(0) = 10,\ y_3(0) = 9$$

Figure 2 shows the anti-synchronization between the states of the master system (8) and the slave system (9).

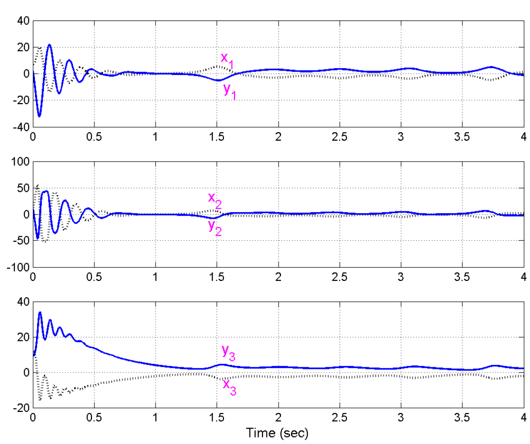

Figure 2. Anti-synchronization of Identical Liu-Chen-Liu Chaotic Systems

IV. ANTI-SYNCHRONIZATION OF IDENTICAL LIU-SU-LIU CHAOTIC SYSTEMS

In this section, we apply the active nonlinear control technique for the anti-synchronization of two identical Liu-Su-Liu chaotic systems ([23], 2006).

Thus, the master system is described by the Liu-Su-Liu dynamics

$$\begin{aligned} \dot{x}_1 &= \alpha(x_2 - x_1) \\ \dot{x}_2 &= \beta x_1 + x_1x_3 \\ \dot{x}_3 &= -\gamma x_3 - x_1x_2 \end{aligned} \tag{21}$$

where x_1, x_2, x_3 are the states of the system and α, β, γ are positive parameters of the system.

The slave system is also described by the Liu-Su-Liu dynamics as

$$\begin{aligned} \dot{y}_1 &= \alpha(y_2 - y_1) + u_1 \\ \dot{y}_2 &= \beta y_1 + y_1y_3 + u_2 \\ \dot{y}_3 &= -\gamma y_3 - y_1y_2 + u_3 \end{aligned} \tag{22}$$

where y_1, y_2, y_3 are the states of the system and

$$u = [u_1 \quad u_2 \quad u_3]^T$$

is the nonlinear controller to be designed.

The Liu-Su-Liu system (21) is a new 3-D chaotic system derived by Liu, Su and Liu ([23], 2007). The Liu-Su-Liu system (21) is chaotic when

$$\alpha = 10,\ \beta = 35 \quad \text{and} \quad \gamma = 1.4.$$

Figure 3 illustrates the chaotic portrait of the Liu-Su-Liu chaotic system (21).

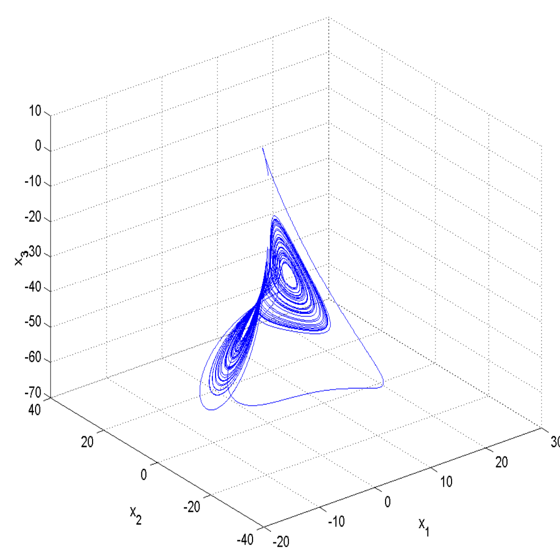

Figure 3. Chaotic Portrait of the Liu-Su-Liu System (21)

The anti-synchronization error e is defined by

$$e_i = y_i + x_i, \qquad (i = 1, 2, 3) \tag{23}$$

The error dynamics is obtained as

$$\begin{aligned} \dot{e}_1 &= \alpha(e_2 - e_1) + u_1 \\ \dot{e}_2 &= \beta e_1 + y_1 y_3 + x_1 x_3 + u_2 \\ \dot{e}_3 &= -\gamma e_3 - y_1 y_2 - x_1 x_2 + u_3 \end{aligned} \tag{24}$$

To find an anti- synchronizing controller, we first let

$$\begin{aligned} u_2 &= u_{2a} + u_{2b} \\ u_3 &= u_{3a} + u_{3b} \end{aligned} \tag{25}$$

where

$$\begin{aligned} u_{2b} &= -y_1 y_3 - x_1 x_3 \\ u_{3b} &= y_1 y_2 + x_1 x_2 \end{aligned} \tag{26}$$

Substituting (25) and (26) into (24), we obtain

$$\begin{aligned} \dot{e}_1 &= \alpha(e_2 - e_1) + u_1 \\ \dot{e}_2 &= \beta e_1 + u_{2a} \\ \dot{e}_3 &= -\gamma e_3 + u_{3a} \end{aligned} \tag{27}$$

Next, we consider the candidate Lyapunov function

$$V(e) = \frac{1}{2} e^T e = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2\right) \tag{28}$$

A simple calculation gives

$$\begin{aligned} \dot{V}(e) = -\alpha e_1^2 - \gamma e_3^2 + (\alpha+\beta) e_1 e_2 + e_1 u_1 \\ + e_2 u_{2a} + e_3 u_{3a} \end{aligned} \tag{29}$$

Therefore, we choose

$$\begin{aligned} u_1 &= -(\alpha+\beta) e_2 \\ u_{2a} &= -e_2 \\ u_{3a} &= 0 \end{aligned} \tag{30}$$

Substituting (30) into (27), the error dynamics simplifies to

$$\begin{aligned} \dot{e}_1 &= -\alpha e_1 - \beta e_2 \\ \dot{e}_2 &= \beta e_1 - e_2 \\ \dot{e}_3 &= -\gamma e_3 \end{aligned} \tag{31}$$

Substituting (30) into (29), we also obtain

$$\dot{V}(e) = -\alpha e_1^2 - e_2^2 - \gamma e_3^2 \tag{32}$$

which is a negative definite function on $\boldsymbol{R}^3$ since α and γ are positive constants.

Hence, by Lyapunov stability theory [24], the error dynamics (31) is globally exponentially stable.

Combining (25), (26) and (30), the anti-synchronizing nonlinear controller u is obtained as

$$\begin{aligned} u_1 &= -(\alpha+\beta) e_2 \\ u_2 &= -e_2 - y_1 y_3 - x_1 x_3 \\ u_3 &= y_1 y_2 + x_1 x_2 \end{aligned} \tag{33}$$

Thus, we have proved the following result.

Theorem 2. The identical Liu-Su-Liu systems (21) and (22) are exponentially and globally anti-synchronized for any initial conditions with the nonlinear controller u defined by (33).

Numerical Results

For the numerical simulations, the fourth-order Runge-Kutta method with time-step 10^{-6} is used to solve the systems using MATLAB.

For the Liu-Su-Liu system (20), the parameter values are taken as those which result in the chaotic behaviour of the system, *viz.* $\alpha = 10$, $\beta = 35$ and $\gamma = 1.4$.

The initial values of the master system (21) are taken as

$$x_1(0) = 7,\; x_2(0) = 9,\; x_3(0) = 15$$

while the initial values of the slave system (22) are taken as

$$y_1(0) = 14,\; y_2(0) = 10,\; y_3(0) = 4$$

Figure 4 shows the anti-synchronization between the states of the master system (21) and the slave system (22).

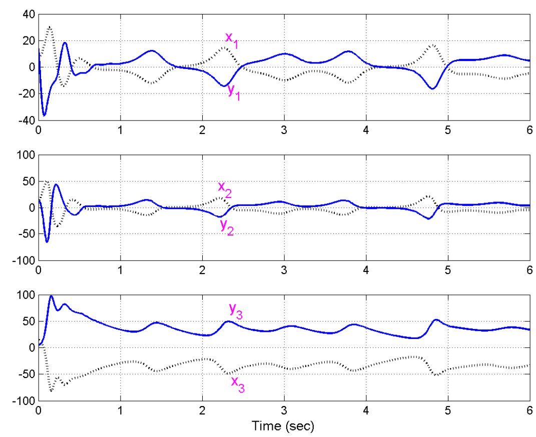

Figure 4. Anti-Synchronization of Identical Liu-Su-Liu Chaotic Systems

V. Anti-Synchronization of Liu-Chen-Liu and Liu-Su-Liu Chaotic Systems

In this section, we apply the active nonlinear control technique for the anti-synchronization of non-identical Liu-Chen-Liu and Liu-Su-Liu chaotic systems. As the master system, we consider the Liu-Chen-Liu chaotic system ([22], 2007) described by

$$\begin{aligned} \dot{x}_1 &= a(x_2 - x_1) \\ \dot{x}_2 &= b x_1 + k x_1 x_3 \\ \dot{x}_3 &= -c x_3 - h x_1 x_2 \end{aligned} \tag{34}$$

As the slave system, we consider the Liu-Su-Liu chaotic system ([23], 2006) described by

$$\begin{aligned} \dot{y}_1 &= \alpha(y_2 - y_1) + u_1 \\ \dot{y}_2 &= \beta y_1 + y_1 y_3 + u_2 \\ \dot{y}_3 &= -\gamma y_3 - y_1 y_2 + u_3 \end{aligned} \tag{35}$$

where all the parameters are positive real constants and $u = [u_1 \quad u_2 \quad u_3]^T$ is the nonlinear controller to be designed.

The anti-synchronization error e is defined by

$$e_i = y_i + x_i, \qquad (i = 1, 2, 3) \tag{36}$$

The error dynamics is obtained as

$$\begin{aligned}\dot{e}_1 &= \alpha(e_2 - e_1) + (a-\alpha)(x_2 - x_1) + u_1 \\ \dot{e}_2 &= \beta e_1 + (b-\beta)x_1 + y_1 y_3 + kx_1x_3 + u_2 \\ \dot{e}_3 &= -\gamma e_3 + (\gamma - c)x_3 - y_1 y_2 - hx_1x_2 + u_3\end{aligned} \tag{37}$$

To find an anti-synchronizing controller, we first let

$$\begin{aligned}u_1 &= u_{1a} + u_{1b} \\ u_2 &= u_{2a} + u_{2b} \\ u_3 &= u_{3a} + u_{3b}\end{aligned} \tag{38}$$

where

$$\begin{aligned}u_{1b} &= -(a-\alpha)(x_2 - x_1) \\ u_{2b} &= -(b-\beta)x_1 - y_1y_3 - kx_1x_3 \\ u_{3b} &= -(\gamma - c)x_3 + y_1y_2 + hx_1x_2\end{aligned} \tag{39}$$

Substituting (38) and (39) into (37), we get

$$\begin{aligned}\dot{e}_1 &= \alpha(e_2 - e_1) + u_{1a} \\ \dot{e}_2 &= \beta e_1 + u_{2a} \\ \dot{e}_3 &= -\gamma e_3 + u_{3a}\end{aligned} \tag{40}$$

Next, we consider the candidate Lyapunov function

$$V(e) = \frac{1}{2}e^T e = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2\right) \tag{41}$$

A simple calculation gives

$$\begin{aligned}\dot{V}(e) = -\alpha e_1^2 - \gamma e_3^2 + (\alpha+\beta)e_1e_2 + e_1u_{1a} \\ + e_2u_{2a} + e_3u_{3a}\end{aligned} \tag{42}$$

Therefore, we choose

$$\begin{aligned}u_{1a} &= -(\alpha+\beta)e_2 \\ u_{2a} &= -e_2 \\ u_{3a} &= 0\end{aligned} \tag{43}$$

Substituting (43) into (40), the error dynamics simplifies to

$$\begin{aligned}\dot{e}_1 &= -\alpha e_1 - \beta e_2 \\ \dot{e}_2 &= \beta e_1 - e_2 \\ \dot{e}_3 &= -\gamma e_3\end{aligned} \tag{44}$$

Substituting (43) into (42), we obtain

$$\dot{V}(e) = -\alpha e_1^2 - e_2^2 - \gamma e_3^2 \tag{45}$$

which is a negative definite function on $\boldsymbol{R}^3$ since α and γ are positive constants.

Hence, by Lyapunov stability theory [24], the error dynamics (44) is globally exponentially stable.

Combining (38), (39) and (43), the anti-synchronizing nonlinear controller u is obtained as

$$\begin{aligned}u_1 &= -(\alpha+\beta)e_2 - (a-\alpha)(x_2 - x_1) \\ u_2 &= -e_2 - (b-\beta)x_1 - y_1y_3 - kx_1x_3 \\ u_3 &= -(\gamma - c)x_3 + y_1y_2 + hx_1x_2\end{aligned} \tag{46}$$

Thus, we have proved the following result.

Theorem 3. The non-identical Liu-Chen-Liu system (34) and Liu-Su-Liu system (35) are exponentially and globally anti-synchronized for any initial conditions with the nonlinear controller u defined by (46).

Numerical Results

For the numerical simulations, the fourth-order Runge-Kutta method with time-step 10^{-6} is used to solve the systems using MATLAB.

For the Liu-Chen-Liu system (34), the parameter values are taken as those which result in the chaotic behaviour of the system, *viz.* $a = 10,\ b = 40,\ c = 2.5,\ h = 1$ and $k = 16.$

For the Liu-Su-Liu system (35), the parameter values are taken as those which result in the chaotic behaviour of the system, *viz.* $\alpha = 10,\ \beta = 35$ and $\gamma = 1.4.$

The initial values of the Liu-Chen-Liu system (34) are taken as

$$x_1(0) = 14,\ x_2(0) = 5,\ x_3(0) = 8$$

while the initial values of the Liu-Su-Liu system (35) are taken as

$$y_1(0) = 5,\ y_2(0) = 12,\ y_3(0) = 4$$

Figure 5 shows the anti-synchronization between the states of the Liu-Chen-Liu system (34) and Liu-Su-Liu system (35).

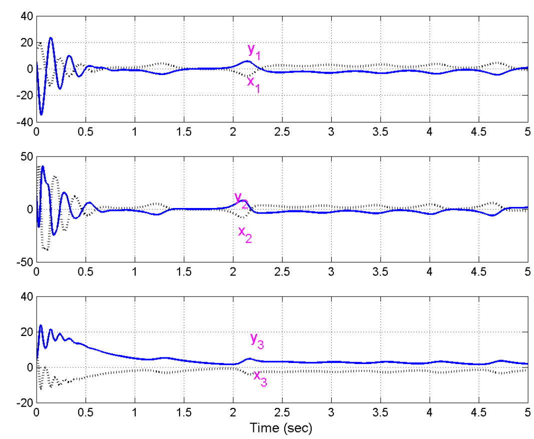

Figure 5. Anti-synchronization of Liu-Chen-Liu and Liu-Su-Liu Chaotic Systems

VI. CONCLUSIONS

In this paper, we have used active control method based on Lyapunov stability theory to achieve global chaos anti-synchronization for the following 3-D chaotic systems.

(A) Identical Liu-Chen-Liu chaotic systems (2007)

(B) Identical Liu-Su-Liu chaotic systems (2006)

(C) Non-Identical Liu-Chen-Liu and Liu-Su-Liu systems.

Since the Lyapunov exponents are not required for these calculations, the active control method is very effective and convenient to achieve global chaos anti-synchronization for identical and different Liu-Chen-Liu and Liu-Su-Liu chaotic systems. Numerical simulations are also given to illustrate and validate the proposed active control method for the global chaos anti-synchronization of the chaotic systems addressed in this paper.

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