



Robust Hybrid Feedback Control Design for Ramp Metering using Sliding Mode Control

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Abstract: The paper presents the feedback control design for controlling the inflow into the freeways to reduce traffic congestion using hybrid dynamics based on sliding control methodology. Earlier approaches for designing ramp metering based on discretized linearized methods or non linear designs using lumped parameters have limitations which are overcome using sliding mode control. The sliding controller design provides a systematic approach to the problem of maintaining stability and consistent performance. One of the most common handicaps for applying sliding mode control to real applications is chattering problem. This problem has been dealt by using boundary layer approach. Accordingly, we propose a novel sliding mode control without chattering. The proposed sliding mode control removes the chattering phenomenon by replacing a sign function with a continuous function. The simulation results for the model are also presented.

Keywords: Freeways, hybrid dynamics, sliding control, ramp metering, chattering.

I. INTRODUCTION

The steadily increasing traffic congestions on freeways worldwide have led to the use of several control mechanisms. Basically, these are formed by controlling the number of vehicles entering the freeway from an on-ramp (ramp metering), and/or by changing the free speed limit of the vehicles between the specified sections of the road (variable speed limiting). Ramp metering is the most common type of control mechanism. It has been recognized as an effective way for relieving freeway congestion which is typically the result of either a surge of demand during peak commuting hours or a temporary reduction of the road capacity.

Designing of ramp metering improves the traffic flow on the freeways by controlling the flow of traffic from the entrance ramp. Ramp metering has been studied for more than 45 years now [1], [2], [3] and [4]. Optimizing techniques studying ramp metering are covered in [5], [6] and [7]. Simulation based analysis on ramp metering is given in [8] and [9]. A local feedback control law for on-ramp metering is studied in [10]. Fuzzy logic based freeway ramp control is covered in [11]. Ramp metering has been used in most of the developed countries like France [12], Germany [13], Italy [14], United Kingdom [15], USA [16] and New Zealand [17].

The objective of the ramp meter design is to control the inflow into the freeways so as to reduce the congestion and jams on the highways. Ramp meters can be pre-programmed or can operate in an actuated mode using real time data. The feedback control law can be designed based on the actual instantaneous traffic parameters so that the traffic flow can be

maximized and congestion is avoided. A freeway with entrance ramp is shown in Fig. 1.

Freeway traffic flows can be controlled by lumped parameter system approach owing to the space discretized system models. Earlier models based on lumped parameters fail to utilize the rarefaction behavior of the traffic. These results in zero outflow from a section when traffic density is at jam density, thereby meaning that traffic would never come out of jam. Moreover, these models do not satisfy the entropy conditions required by the distributed parameter hybrid dynamic model. This is the major limitation for the control design in the past.

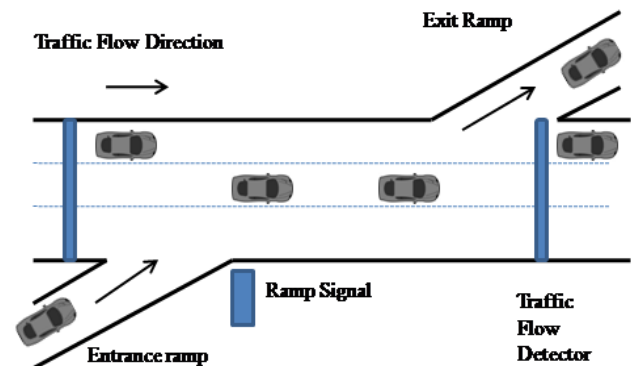


Figure 1. Freeway with Entrance Ramp.

But Godunov based model for feedback ramp metering [18] reproduces the rarefaction behavior and a feedback control design for ramp metering system is presented which

provides asymptotic behavior for the closed loop system. The entropy consistent lumped parameter model uses the Godunov based switching ODEs to produce a hybrid dynamic model. A uniformly stable feedback control law for the various switching states has been designed.

But in real world, there are modeling inaccuracies which have an adverse effect on the non linear control systems. Robust control is one of the major approaches to deal with model uncertainties and a simple approach to robust control is sliding control methodology. Our contribution in this paper is to modify the control laws to achieve the trade-off between tracking performance and parametric uncertainties. The sliding controller design presented provides a systematic approach to the problem of maintaining stability and consistent performance. One of the most common handicaps for applying sliding mode control to real applications is chattering problem. This problem has been dealt by using boundary layer approach. Accordingly, we propose a novel sliding mode control without chattering. The proposed sliding mode control removes the chattering phenomenon by replacing a sign function with a continuous function. It is also designed to move the state to the sliding surface in the infinite time without chattering. Simulation results for the model are also presented.

II. MATHEMATICAL MODEL

In Lighthill-Whitham-Richards (LWR) model [19] and [20], the traffic state is represented from a macroscopic point of view by the function $\rho(t, x)$ which represents the traffic density (number of vehicles per unit length of road) at position x and time t . The flux or flow of rate, f (number of vehicles per unit time passing point x at a time t) which is the product of traffic density and the traffic speed v , i.e. $f = \rho v$. The dynamics of the traffic are represented by a conservation law expressed as:

$$\frac{\partial}{\partial t} \rho(t, x) + \frac{\partial}{\partial x} f(x, t) = 0 \quad (1)$$

or

$$\frac{\partial}{\partial t} \rho + \frac{\partial}{\partial x} (\rho v) = 0 \quad (2)$$

Greenshield's model [21] and [22] predicts the uninterrupted traffic flow and explains the trends that are observed in a real traffic flows. It assumes a linear relationship between traffic density and traffic speed.

$$v(\rho) = v_f \left(1 - \frac{\rho}{\rho_{max}}\right) \quad (3)$$

where v_f is the free flow speed and ρ_{max} is the maximum density. Free flow is the speed when traffic density is zero because there are no vehicles on the roadway. This free flow speed is the maximum speed. As the density increases, the flow also increases to some maximum flow conditions. When the traffic density reaches maximum, it corresponds to a traffic jam and the speed is zero. The maximum flow occurs when

the traffic is flowing at half of free flow speed. Fig. 2 gives the fundamental diagram of Greenshield's model.

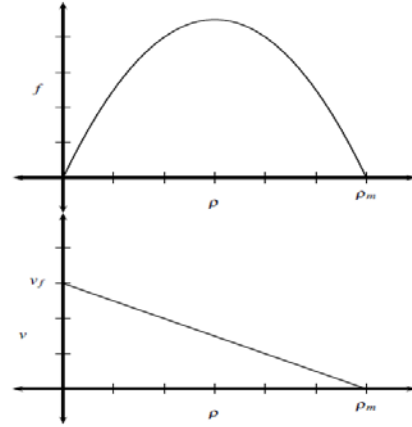


Figure 2. Fundamental diagram of Greenshield's model.

A space discretized model of (1) or (2) for the ramp metering is presented in Fig. 3. Here $u(t)$ is the ramp inflow into the freeway, f_i is the upstream inflow and f_o is the downstream outflow.

Assuming unit length for the section, the ODE for the space discretized model for the ramp metering, is given by

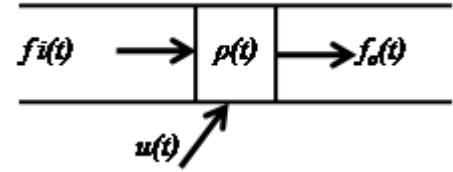


Figure 3. Space Discretized Model.

$$\frac{d\rho(t)}{dt} = f_i(t) + u(t) - f_o(t) \quad (4)$$

As per Greenshield's model, the ~~flow~~ traffic f_o is given by

$$f_o(t) = v_f \rho(t) \left(1 - \frac{\rho(t)}{\rho_{max}}\right) \quad (5)$$

Substituting (5) in (4) establishes that when traffic density is equal to jam density and the value of $u(t)$ becomes zero, the rate of increase in traffic density is positive. For positive inflow, the density can increase according to (4). When the traffic density is equal to jam density for the section, two issues need to be addressed:

- The inflow from upstream can increase the density above the jam value, and
- The outflow is zero from the section not allowing for the traffic to be dissipated to downstream.

The LWR model does not have these limitations as can be seen in Fig. 4 which shows the traffic characteristics where traffic density is shown on the x-axis, time on the y-axis and traffic density is assumed to be piecewise constant. Here the upstream traffic density ρ_0 is lower, middle section has jam density ρ_m , and downstream has $\rho = 0$ (zero density). When the time increases, the shockwave travels upwards and the rarefaction is towards the downstream thereby dissipating the

jam.

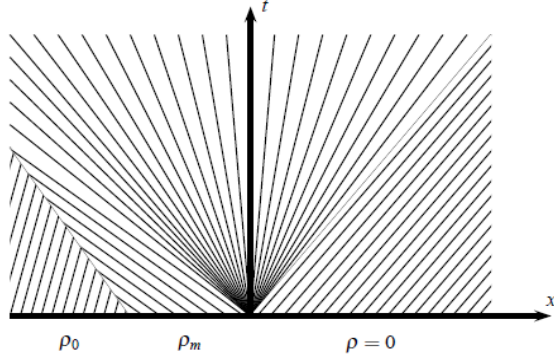


Figure 4. Traffic Characteristics.

Godunov's model is used to address these two issues. Godunov proposed a way to make use of the characteristic information within the framework of a conservative method. Rather than attempting to follow characteristics backwards in time, Godunov suggested solving Riemann problems [22] forward in time. Solutions to Riemann problems are relatively easy to compute, give substantial information about the characteristic structure, and lead to conservative methods since they are themselves exact solutions of the conservation laws and hence conservative.

The Godunov method is based on solving the Riemann problem. A Riemann problem in the theory of hyperbolic equations is a problem in which the initial state of the system is defined as:

$$\rho(x; t = 0) = \begin{cases} \rho_{lt} & \text{for } x \leq 0 \\ \rho_{rt} & \text{for } x > 0 \end{cases} \quad (6)$$

In other words: the initial state is constant for all negative x , and constant for all positive x , but differs between left and right. For solving the Riemann problem, the initial condition is a piecewise constant function with two values ρ_{lt} for the upstream (left) and ρ_{rt} for downstream (right) densities [23]. From the junction of the two densities either a shockwave or a rarefaction wave can emanate. A shockwave develops if $f'(\rho_{lt}) > f'(\rho_{rt})$ [32]. A rarefaction develops if $f'(\rho_{lt}) < f'(\rho_{rt})$. The rarefaction can be entirely to the left, or to the right or in the middle as shown in Fig. 5.

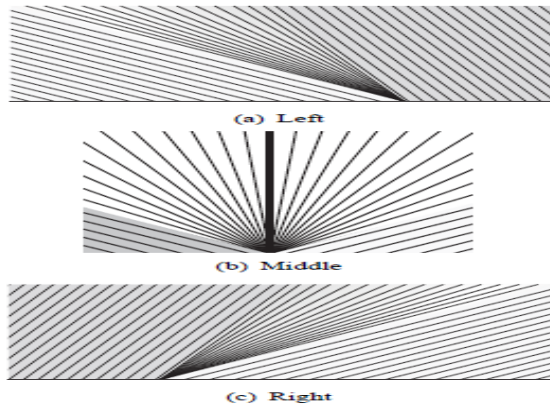


Figure 5. Left, Middle and Right Rarefaction.

The speed of the shockwave is given by (7), in which, $x_s(t)$ is the position of the shockwave as a function of time. If the shock speed is positive, then the flow at junction between the two traffic densities will be a function of upstream traffic density, whereas if the shock speed is negative, then the in flow at junction between the two traffic densities will be a function of downstream traffic density.

$$s = \frac{dx_s(t)}{dt} = \frac{|f(\rho_{lt}) - f(\rho_{rt})|}{\rho_{lt} - \rho_{rt}} \quad (7)$$

The shockwave and rarefaction conditions analysis gives us the Godunov based ODE model for traffic. The ODE for Godunov method is in line with the conservation law, and is given by (8), where we have assumed unit length for the section. As shown in Fig. 6, the flow $f_i(t)$ will be a

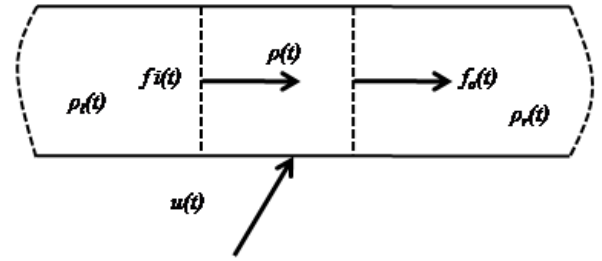


Figure 6. Godunov Based Model.

$$\frac{d\rho(t)}{dt} = f_i(t) - f_o(t) + u(t) \quad (8)$$

function of upstream density ρ_{lt} and downstream density ρ_{rt} and shall be given by (9) where a new function F is obtained from Godunov method. Here the upstream and downstream traffic densities are with respect to the left junction.

$$f_i(t) = F(\rho_{lt}, \rho) \quad (9)$$

For the right junction, the out flow $f_o(t)$ is given by (10).

$$f_o(t) = F(\rho, \rho_{rt}) \quad (10)$$

The function $F(\rho_{lt}, \rho_{rt})$ is given by the Godunov method in (11) [23].

$$F(\rho_{lt}, \rho_{rt}) = f(\rho^\#(\rho_{lt}, \rho_{rt})) \quad (11)$$

Here, the flow-dictating density $\rho^\#$ is obtained from the following [23]:

- Case 1 $f'(\rho_{lt}), f'(\rho_{rt}) \geq 0 \Rightarrow \rho^\# = \rho_{lt}$
- Case 2 $f'(\rho_{lt}), f'(\rho_{rt}) \leq 0 \Rightarrow \rho^\# = \rho_{rt}$
- Case 3 $f'(\rho_{lt}) \geq 0 \geq f'(\rho_{rt}) \Rightarrow \rho^\# = \rho_{lt}$,
if $s > 0$, otherwise $\rho^\# = \rho_{rt}$
- Case 4 $f'(\rho_{lt}) < 0 < f'(\rho_{rt}) \Rightarrow \rho^\# = \rho_s$

Here ρ_s is obtained as the solution to $f'(\rho_s) = 0$

In each of the first three cases, the value is either ρ_{lt} or ρ_{rt} . Note in particular that in cases 1 and 2, it is irrelevant whether the solution is a shock or rarefaction, since the value of ρ_s is the same in either case. This shows that using Godunov's method with entropy-violating Riemann solutions does not necessarily lead to entropy-violating numerical solutions. In case 4, ρ_s is neither ρ_{lt} nor ρ_{rt} , but is some intermediate value ρ_s satisfying the Godunov dynamics.

Godunov model described above calculates the density of the free way segment which is calculated on the basis of densities on both the sides viz. upstream traffic density (left) and downstream traffic density (right) i.e. the known boundary conditions. But in real life situations, the traffic density is not known and there are modeling inaccuracies or structured uncertainties. Robust control is one of the major approaches to deal with model uncertainty.

A. Sliding Control Methodology:

A simple approach to robust control is the sliding control methodology [26] in which n^{th} order differential equation is replaced by equivalent first order differential equation and then perfect performance can be achieved in the presence of arbitrary parameter inaccuracies. For the class of systems to which it applies, sliding controller design provides a systematic approach to the problem of maintaining stability and consistent performance in the face of modeling imprecision.

The idea behind the sliding control is to pick up a well behaved function of the tracking error, s , and then select the feedback control law u such that s^2 remains a Lyapunov like function of the closed loop system, despite the presence of model imprecision and of disturbances. The controller design procedure then consists of two steps. First, a feedback control law u is selected so as to verify sliding condition. However, in order to account for the presence of modeling imprecision and of disturbances, the control law has to be discontinuous across the sliding surface, $S(t)$. Since the implementation of the associated control switching is necessarily imperfect, this leads to chattering. In practice, chattering is undesirable since it involves high control activity and further may excite high frequency dynamics neglected in the course of modeling. Thus, in a second step the discontinuous control law u is suitably smoothed to achieve an optimal trade-off between control bandwidth and tracking precision: while the first step accounts for parametric uncertainty, the second step achieves robustness to high frequency unmodeled dynamics.

Consider the first order differential system,

$$\dot{x}(t) = f(t) + u(t) \quad (12)$$

where $x(t)$ is the output of interest, $f(t)$ is not exactly known and $u(t)$ is the control input.

As the dynamics of f which can be non-linear or time varying is unknown, we take estimated value as $\hat{f}$.

The error estimated on f is assumed to be bounded by the function

$$F = F(x, \dot{x}) : |\hat{f} - f| \leq F \quad (13)$$

So the time varying surface $s(t)$ is written as

$$s(t) = x(t) - x_d(t) \quad (14)$$

where $s(t)$ is the tracking error and x_d is the desired state.

The simplified first order problem of keeping the scalar s at zero can be achieved by choosing the control law of (12) such that outside of $s(t)$ is

$$\frac{1}{2} \frac{d}{dt} s^2 \leq -\eta |s| \quad (15)$$

where, η is strictly a positive constant. $\frac{d}{dt} s^2$ denotes that the squared distance to surface measured by s^2 ($s \dot{s}$) decreases along all trajectories.

The behavior of sliding condition is shown in Fig. 7:

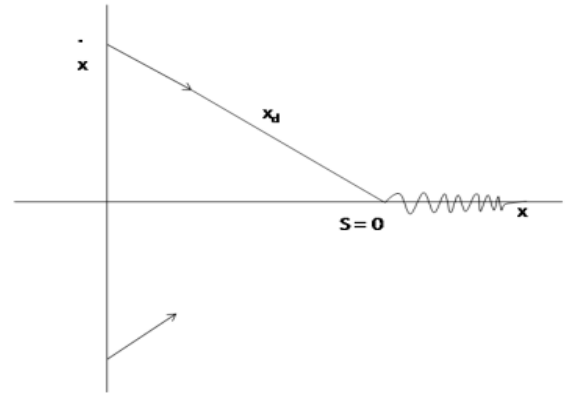


Figure 7. Sliding Mode Exponential Convergence.

So (15) can be written as:

$$s \dot{s} \leq -\eta |s| \quad (16)$$

III. HYBRID DYNAMICAL MODEL AND CONTROL DESIGN

The ODE model for the ramp metering system can be written as:

$$\frac{d\rho(t)}{dt} = F(\rho_{lt}, \rho) - F(\rho, \rho_{rt}) + u(t) \quad (17)$$

This is a switched hybrid system [25], where the switching happens autonomously based on the values of ρ_{lt} , ρ , and ρ_{rt} . The function $F(\rho_{lt}, \rho)$ can have three distinct values, $f(\rho_{lt})$, $f(\rho)$, or $f(\rho_s)$. Similarly, $F(\rho, \rho_{rt})$ can have three distinct values. Hence, the dynamics can be written as:

$$\frac{d\rho(t)}{dt} = G_q(\rho_{lt}, \rho, \rho_{rt}) + u(t) \quad (18)$$

where $q \in \{1, 2, \dots, 9\}$ and the different G_q functions can be obtained from (11), (17) & (18).

The following feedback linearization based model for the ramp metering control that attempts to keep the mainline traffic density at ρ_s , which is taken to be the flow maximizing density is given in (19). For the Greenshield's model this critical density is $\rho_m/2$.

$$u(t) = -G_q(\rho_{lt}, \rho, \rho_{rt}) - k(\rho(t) - \rho_s), k > 0 \quad (19)$$

We propose the control law using sliding mode as

$$u(t) = -G_q(\rho_{lt}, \rho, \rho_{rt}) - k \operatorname{sgn}(s(t)), k > 0 \quad (20)$$

where $\operatorname{sgn}(s(t))$ is defined as

$$\operatorname{sgn}(s(t)) = \begin{cases} +1 & \text{if } s(t) \geq 0 \\ -1 & \text{if } s(t) < 0 \end{cases} \quad (21)$$

and $s(t) = \rho(t) - \rho_s$, which is the sliding surface.

This control algorithm causes chattering which must be eliminated for the controller to perform properly. This can be achieved by smoothing out the control discontinuity in the thin boundary layer neighboring the switching surface by introducing the saturation function. Saturation function is a continuous approximation of sign function [26].

The saturation function $\operatorname{sat}(s(t), \varphi)$ can be defined as:

$$\operatorname{sat}(s(t), \varphi) = \begin{cases} +1 & \text{if } s(t) \geq \varphi \\ -1 & \text{if } s(t) \leq -\varphi \\ \frac{s(t)}{\varphi} & \text{otherwise} \end{cases} \quad (22)$$

The saturation function determined by the system dynamics is proposed for the use inside the boundary layer to reduce chattering around the switching surface and using a continuous control within the boundary layer. Accordingly, we further modify the control law proposed in (20) after incorporating the saturation function to remove chattering and is given in (23).

$$u(t) = -G_q(\rho_{lt}, \rho, \rho_{rt}) - k \operatorname{sat}(s(t), \varphi), k > 0 \quad (23)$$

IV. SIMULATIONS

The control law developed has been implemented by using MATLAB. The feedback linearization based model for the ramp metering control is consistent with the conservation law as well as the Godunov conditions which are used for making the hybrid model. Further, sliding mode control is applied using a sign function in the control law. But this leads to chattering which is undesirable. The chattering has been removed by using saturation function modifying the control law.

The ordinary differential equation (ODE) given in (17) which uses the hybrid control scheme developed in this study is implemented in this simulation. The lower and upper limits of traffic flow are taken as zero and 75% of the maximum flow is applied at the inflow of the control.

The simulation was run with different initial values of traffic density viz. $\rho_0 = 50, 20, 10$ with jam density, $\rho_m = 86$ and the simulation results are depicted in the following figures. As shown in the ‘Density using Hybrid Control’ plot, the traffic density converges to the desired critical density that maximizes the flow. The desired result is obtained as the steady state traffic density of 43 (i.e. $\rho_m/2$) is achieved. The chattering phenomenon is clearly shown in the ‘Density using

Sliding Mode’ plot. The chattering occurs at the steady state traffic density of 43 due to uncertainties. The chattering reduction using the saturation function is also shown in ‘Density using Saturation Function with Sliding Mode Control’ plot. Here the value of φ is taken as 2.0 for applying boundary layer conditions.

The simulation results are shown in Fig. 8 to Fig. 16.

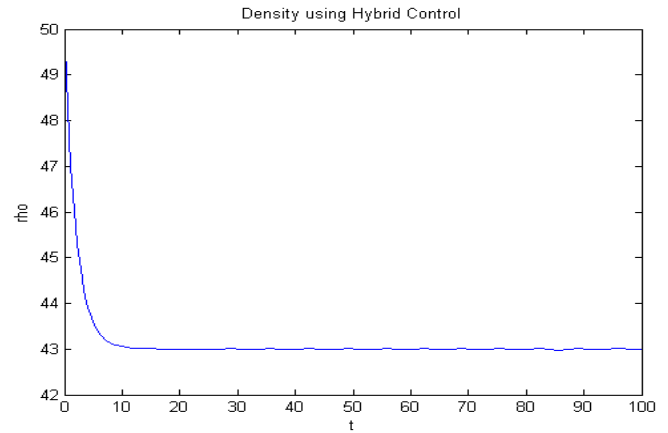


Figure 8. Freeway segment traffic density using the hybrid based control, $\rho_0 = 50$.

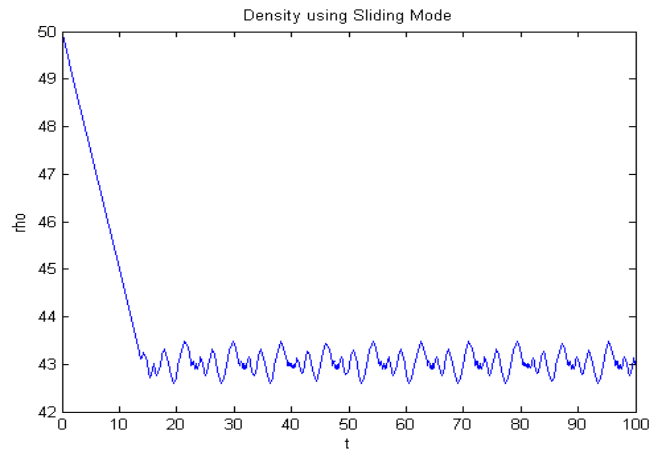


Figure 9. Freeway segment traffic density using the sliding mode control depicting chattering phenomenon, $\rho_0 = 50$.

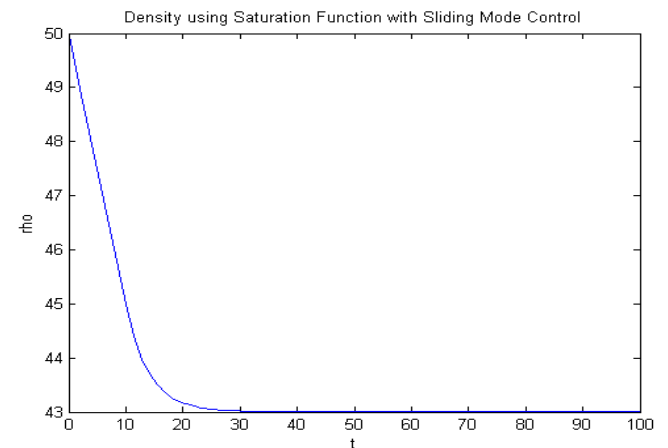


Figure 10. Freeway segment traffic density using chattering reduction of sliding mode control by adopting non linear saturation function, $\rho_0 = 50$.

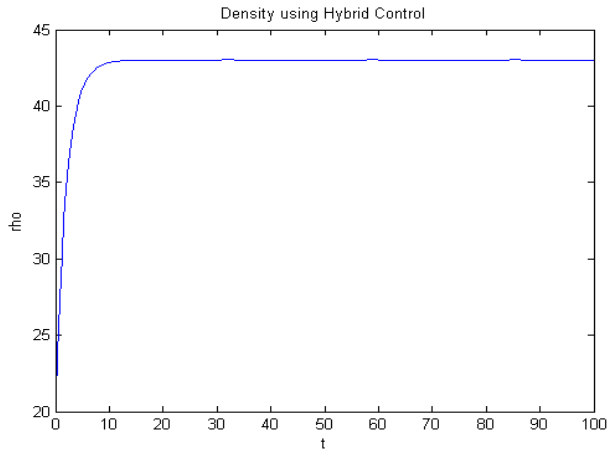


Figure 11. Freeway segment traffic density using the hybrid based control, $\rho_0 = 20$.

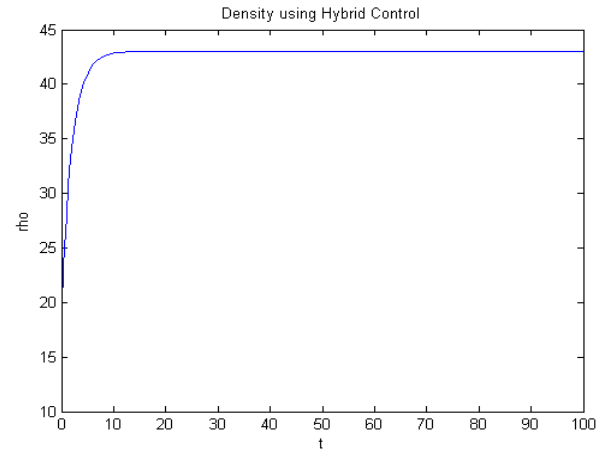


Figure 14. Freeway segment traffic density using the hybrid based control, $\rho_0 = 10$.

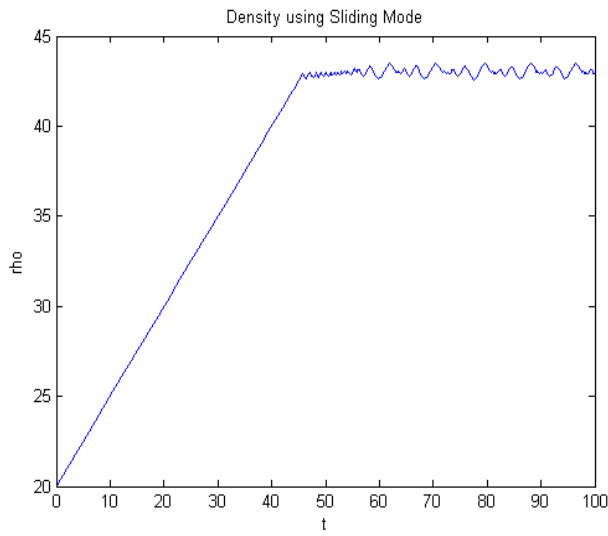


Figure 12. Freeway segment traffic density using the sliding mode control depicting chattering phenomenon, $\rho_0 = 20$

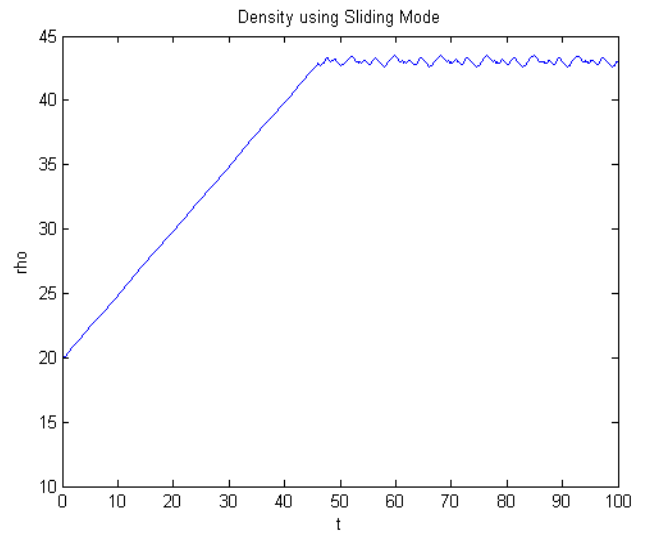


Figure 15. Freeway segment traffic density using the sliding mode control depicting chattering phenomenon, $\rho_0 = 10$.

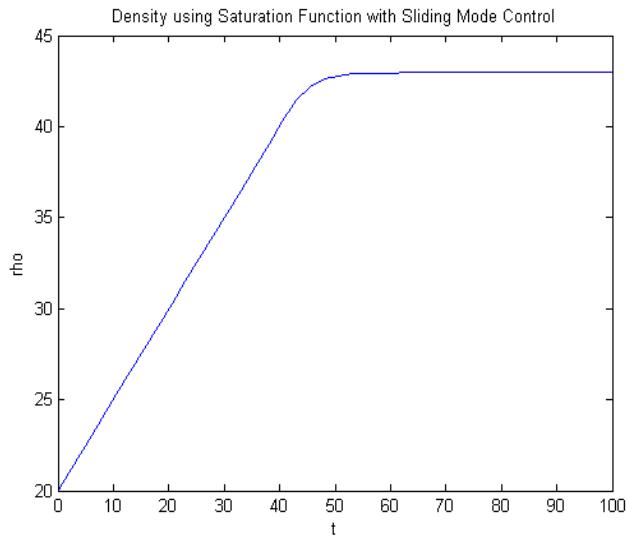


Figure 13. Freeway segment traffic density using chattering reduction of sliding mode control by adopting non linear saturation function, $\rho_0 = 20$.

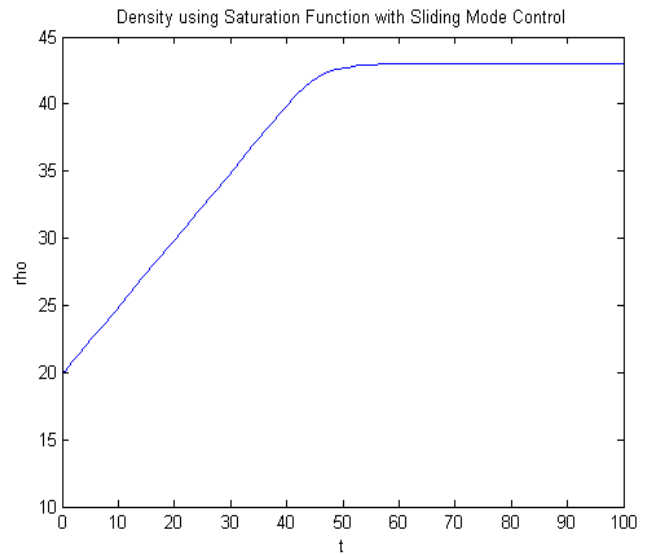


Figure 16. Freeway segment traffic density using chattering reduction of sliding mode control by adopting non linear saturation function, $\rho_0 = 10$.

V. CONCLUSIONS

The paper presents a sliding mode feedback control design of an isolated ramp based on Godunov dynamics. Sliding Mode Control is used to achieve the trade-off between tracking performance and parametric uncertainties. The sliding controller design presented provides a systematic approach to the problem of maintaining stability and consistent performance. One of the most common handicaps for applying sliding mode control to real applications is chattering problem. This problem has been dealt by using boundary layer approach. Accordingly, a novel sliding mode control without chattering has been presented. Simulation results for the model are also presented.

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