# An Approach to Merging of connected network topologies for Improving Ultimate Connectivity in Grid Backdrop 

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#### Abstract

In this paper, we are merging the network topologies for maximum connectivity. Here, we are introducing a method for merging an mconnected network topology and an $n$-connected network topology, where, $m$ is greater than $n$, into a single network topology of connectivity $m$, without disturbing the dwelling links and geographical locations of the nodes. The proposed method is a first of its kind in the area of research concerned with merging networks for ultimate connectivity. The suggested method is made clear in section 2 and illustrated in section 3.


Keywords: Network topology, linking number, interconnection network, ultimate connectivity

## I. INTRODUCTION

Applications of computer communication networks are increasing in every field of human activity. Computer communication networks have many applications in the field of education, business, media, multi-player computer gaming, etc. All these applications demand the design of efficient fault tolerant survivable computer communication network topology with minimum transmission delay, response time and maximum throughput [1] [2]. The energy, memory, transmitting and computing power of the nodes is to be limited in the adhoc design network.

The topological structure of interconnection network can be modeled by a uncomplicated graph, whose vertices represent components of the network and whose edges epitomize physical communication links, where directed edges represents one way communication links and undirected edges represent two way communication links. The incidence function specifies a way that the components of the network are interconnected by links. Such a graph is called the topological structure of the interconnection network, or, in short, network topology. Conversely, any graph can also be considered as a topological structure of some interconnection network. Topologically, graphs and interconnection networks are identical. [3].

## II. PROPOSED METHOD

This section presents the proposed method for merging an $m$-connected network topology and an $n$-connected network topology where $m$ is greater than $n$, into a single network topology of connectivity $m$, without disturbing the existing links and geographical locations of the nodes. To start with, a network graph $G_{m}\left(V_{m}, E_{m}\right)$ of connectivity $m$ and another network graph $G_{n}\left(V_{n}, E_{n}\right)$ of connectivity $n$ are considered. The nodes of both the topologies are numbered by using decimal numbers [4].

Tables $T_{m}$ and $T_{n}$ are constructed for both the network graphs $G_{m}$ and $G_{n}$ with the fields- node, degree of the node and linking number. The linking number of all the nodes of $\mathrm{G}_{\mathrm{m}}$ in table $\mathrm{T}_{\mathrm{m}}$ is initialized to the corresponding degree of the nodes. The linking number of all the nodes of $\mathrm{G}_{\mathrm{n}}$ in table $\mathrm{T}_{\mathrm{n}}$ is initialized to zero. The counter Z is initialized to zero.

If connectivity of both the graph are contra distinct, then cut sets of all cardinalities $\mathrm{C}_{\mathrm{i}}$ are constructed for the network graph $\mathrm{G}_{\mathrm{n}}$, where $n \leq \mathrm{C}_{\mathrm{i}}<m$, i.e $\mathrm{C}_{\mathrm{i}}=n, n+1, n+2, \ldots, m-1[5]$.

For each cutset, corresponding to the cardinality $\mathrm{C}_{\mathrm{i}}$ the components S of $\mathrm{G}_{\mathrm{n}}$ are constructed and the resultant components are arranged in an increasing order of the number of vertices. The counter Z is incremented by one, when the linking number of all the nodes in Table $T_{n}$ is increased by one [6] [7]. The resultant network graph G so obtained will be the network graph with connectivity $m$.

## Algorithm: Network Merging

Input:
(a). $\mathrm{G}_{\mathrm{m}}\left(\mathrm{V}_{\mathrm{m}}, \mathrm{E}_{\mathrm{m}}\right)$ : network with connectivity $m$
(b). $\mathrm{G}_{\mathrm{n}}\left(\mathrm{V}_{\mathrm{n}}, \mathrm{E}_{\mathrm{n}}\right)$ : network with connectivity $n$

Output: $\quad \mathrm{G}(\mathrm{V}, \mathrm{E})$ : Merged network with connectivity $\max (m, n)$
Method: Initialize $G$ to a disconnected network with two components $G_{m}$ and $G_{n}$.
a. Number the nodes of network $\mathrm{G}_{\mathrm{m}}$
b. Number the nodes of network $G_{n}$
c. Construct table $\mathrm{T}_{\mathrm{m}}$ for the network $\mathrm{G}_{\mathrm{m}}$ with following field (Node, Degree of node, Updated degree of node)
d. Initialize updated degree of node of two degree of the corresponding nodes
e. Construct Table $T_{n}$ for the network $G_{n}$ with following field (Node, Degree of node, Updated degree of node)
f. Initialize updated degree of node of $\mathrm{T}_{\mathrm{n}}$ to 0
g. $\quad$ Set $Z=0$.
h. If $(m=n)$ then
(a). Select first $m$ vertices from both $\mathrm{G}_{\mathrm{m}}$ and $\mathrm{G}_{\mathrm{n}}$ and establish links between selected vertices.

## Else

i. Construct cut sets for network $\mathrm{G}_{\mathrm{n}}$ of all cardinalities $\mathrm{C}_{\mathrm{i}}$, such that $n \mathrm{C}_{\mathrm{i}}<m$, where $\mathrm{i}=n, n+1, n+2, \ldots, m-1$
j. For each cardinality $\mathrm{C}_{\mathrm{i}}$ do
(a). Construct the components S corresponding to the cutsets.
(b). Arrange the resultant components S in ascending order of nodes.
(c). For each $S_{j} S$
i. If updated degree of every vertex is $Z$, then select a node from $\mathrm{S}_{\mathrm{i}}$ which has least degree. If there is a tie then select a node ( X ) with least node label index.
ii. Select a vertex from $G_{m}$ with least update degree, if there is a tie, select a node with least degree, if there is a tie select a node ( Y ) with least node label index.
iii. Update the network $G$ by establish a link between X and Y .
iv. Update the update degree corresponding to X and Y in Table 1 and Table 2 respectively.
For End.
If there exist any node (A) in $G$ with degree less then $\mathrm{C}_{\mathrm{i}}+1$, then establish a link between (A) and a node (B) of $\mathrm{G}_{\mathrm{m}}$ (selected as described in step 2), and subsequently update the update degrees corresponding to A and B in table 1 and table 2.

Increment Z by 1 when the updated degree of all the nodes of the network 2 is increased by at least 1 .

For end.
Algorithm end

## III. ILLUSTRATION

In this section, the proposed method is illustrated by taking two network graphs $G_{m}$ and $G_{n}$, where $G_{m}$ is a 5connected and $\mathrm{G}_{\mathrm{n}}$ is 1 -connected network. The merged graph $G$ will be 5 -connected [8].A 5 -connected graph $\mathrm{G}_{\mathrm{m}}$ and 1-connected network graph $\mathrm{G}_{\mathrm{n}}$ are shown in Figure 1 and Figure. 2


Figure: 1 5-connected network graph


Figure: 2 1-connected network graph $\mathrm{G}_{\mathrm{n}}$
For the network graph $\mathrm{G}_{\mathrm{m}}$ the Table1 is constructed with the fields- node, degree of the node, linking number. To start with, the linking number of every node of the graph $G_{m}$ is same as the degree of the node [9][10].

Table: 1 Information table for network graph $\mathrm{G}_{\mathrm{m}}$

|  |  | Linking Number |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |
| a | 5 | e | f | f | g | g |
| b | 5 | e | f | f | g | g |
| c | 5 | e | e | f | f | g |
| d | 5 | e | e | f | f | g |
| e | 5 | e | e | f | f | g |
| f | 5 | e | e | e | f | g |
| g | 5 | e | e | e | f | g |
| h | 5 | e | e | e | f | g |

The main advantage of this method is that it is based on the concept of cut sets. For the network graph $G_{n}$, the Table is constructed with the fields- node, degree of the node, linking number. To start with, the linking number of every node of the graph $G_{n}$ is initialized to zero. The component $\mathrm{S}_{1}$ is considered. This component has only one node, i.e node $\boldsymbol{a}$. The linking number of node $\boldsymbol{a}$ equals 0 , which is less than or equal to $Z$. This node is selected. A node from $G_{m}$ which has least degree is selected. If there is a tie, a node which has the smallest node number is selected. Since, all nodes of $\mathrm{G}_{\mathrm{m}}$ have the same linking number and same degree, the node $\boldsymbol{a}$ of $\mathrm{G}_{\mathrm{m}}$ is selected as it has the least node index label. A link between node $\boldsymbol{a}$ of $\mathrm{G}_{\mathrm{m}}$ and node $\boldsymbol{a}$ of $\mathrm{G}_{\mathrm{n}}$ is established. Simultaneously, the linking number of both the nodes is increased by 1 [11].

The component $S_{2}$ is considered. This component has two nodes namely node $\boldsymbol{e}$ and node $\boldsymbol{f}$. The degrees of nodes $\boldsymbol{e}$ and $\boldsymbol{f}$ are the same. Therefore node number $\boldsymbol{e}$ is selected from this component as $\boldsymbol{e}$ is less than $\boldsymbol{f}$. In $G_{1}$, the linking numbers of nodes $\boldsymbol{b}, \boldsymbol{c}, \boldsymbol{d}, \boldsymbol{e}, \boldsymbol{f}, \boldsymbol{g}, \boldsymbol{h}$ are less than the linking number of node $\boldsymbol{a}$. Further, the degree of every node is the same. Since, node $\boldsymbol{b}$ has the least node index number, it is selected. A link between node $\boldsymbol{b}$ of $\mathrm{G}_{\mathrm{m}}$ and node $\boldsymbol{e}$ of $\mathrm{G}_{\mathrm{n}}$ is established. Simultaneously, the linking number of both the nodes is increased by 1 .The component $S_{3}$ is considered. Here, the linking number of all the nodes is not less than or equal to Z . Therefore, no node is selected from this component. For similar reasons, no node is selected from the component $\mathrm{S}_{4}$ [12].

The resultant graph $G$ from the above iteration is a 2 Connected network graph and is as shown in Figure 3.


Figure: 3 2-connected merged network graph

The counter Z is not incremented as linking number of all the nodes are not increased by one.

The components corresponding to the cut-set of $\mathrm{C}_{2}$ are constructed and the components are arranged in the increasing order of their cardinality. The component $\mathrm{S}_{1}=\{\boldsymbol{a}\}$ is considered. Since, the linking number of the node $\boldsymbol{a}$ in $\mathrm{G}_{\mathrm{n}}$ equals 1 which is greater than Z, node $\boldsymbol{a}$ is not selected. For similar reasons, node $\boldsymbol{e}$ of $\mathrm{S}_{2}$ is not selected. The component $S_{3}=\{\boldsymbol{f}\}$ is considered. The linking number of node $\boldsymbol{f}$ equals 0 which is less than or equal to Z . In $\mathrm{G}_{\mathrm{m}}$, the linking numbers of nodes $\boldsymbol{c}, \boldsymbol{d}, \boldsymbol{e}, \boldsymbol{f}, \boldsymbol{g}$ and $\boldsymbol{h}$ are less than the linking number of node $\boldsymbol{a}$ and $\boldsymbol{b}$. Further, the degree of every node is the same. Since node $\boldsymbol{c}$ has the least node index number, it is selected. A link is established between node $\boldsymbol{c}$ of $\mathrm{G}_{\mathrm{m}}$ and node $\boldsymbol{f}$ of $\mathrm{G}_{\mathrm{n}}$

The component $\mathrm{S}_{6}$ is considered. Here, the linking number of all the nodes is not less than or equal to Z . Therefore, no node is selected from this component. For similar reasons, no node is selected from the component $S_{6}$, $\mathrm{S}_{7}$ and $\mathrm{S}_{8}$. The component $\mathrm{S}_{5}=\{\boldsymbol{b}, \boldsymbol{c}\}$ is considered. Here, the linking numbers of both the nodes are equal to zero, which is less than or equal to counter Z . Therefore, a node from $\mathrm{S}_{5}$ with least degree is selected. Hence, node $\boldsymbol{c}$ is selected. In $\mathrm{G}_{\mathrm{m}}$, the linking numbers of nodes $\boldsymbol{d}, \boldsymbol{e}, \boldsymbol{f}, \boldsymbol{g}$ and $\boldsymbol{h}$ are less than the linking number of node $\boldsymbol{a}, \boldsymbol{b}$ and $\boldsymbol{c}$. Further, the degree of every node is the same. Since node $\boldsymbol{d}$ has the least node index number, it is selected [13]. A link is established between node $\boldsymbol{d}$ of $\mathrm{G}_{\mathrm{m}}$ and node $\boldsymbol{c}$ of $\mathrm{G}_{\mathrm{n}}$.

Since, the degree of every node in the resultant graph should be greater than or equal to 3 after this iteration, it is observed that the degree of the node $\boldsymbol{a}$ of $\mathrm{G}_{\mathrm{n}}$ is 2 . Hence, the node $\boldsymbol{a}$ of $\mathrm{G}_{\mathrm{n}}$ is selected, in $\mathrm{G}_{\mathrm{n}}$. The linking numbers of nodes $\boldsymbol{e}, \boldsymbol{f}, \boldsymbol{g}$ and $\boldsymbol{h}$ are less than the linking number of node $\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}$ and $\boldsymbol{d}$. Further, the degree of every node is the same. Since node $\boldsymbol{e}$ has the least node index number, it is selected. A link is established between node $\boldsymbol{e}$ of $\mathrm{G}_{\mathrm{m}}$ and node $\boldsymbol{a}$ of $\mathrm{G}_{\mathrm{n}}$. The counter Z is not incremented as linking number of all the nodes are not increased by one. The components corresponding to the cut-set of $\mathrm{C}_{3}$ are constructed and the components are arranged in an increasing order of their cardinality. The component $S_{1}=\{a\}$ is considered. Since, the linking number of the node $\boldsymbol{a}$ in $\mathrm{G}_{\mathrm{n}}$ equals b which is greater than Z, node $\boldsymbol{a}$ is not selected. For similar reasons, node $\boldsymbol{e}$ of $S_{3}$ and node $\boldsymbol{f}$ of $\mathrm{S}_{4}$ is not selected.

The component $\mathrm{S}_{2}=\{\mathrm{b}\}$ is considered. The linking number of node $\boldsymbol{b}$ equals 0 which is less than or equal to Z . In $G_{m}$, the linking numbers of nodes $\boldsymbol{c}, \boldsymbol{d}, \boldsymbol{e}, \boldsymbol{f}, \boldsymbol{g}$ and $\boldsymbol{h}$ are less than the linking number of node $\boldsymbol{a}$ and $\boldsymbol{b}$. Further, the degree of every node is the same. Since node $\boldsymbol{c}$ has the least node index number, it is selected. A link is established between node $\boldsymbol{c}$ of $\mathrm{G}_{\mathrm{m}}$ and node $\boldsymbol{d}$ of $\mathrm{G}_{\mathrm{n}}$. Simultaneously, are updated the corresponding tables.

The component $\mathrm{S}_{5}$ is considered. Here, the linking number of all the nodes is not less than or equal to Z . Therefore, no node is selected from this component. For similar reasons, no node is selected from the component $\mathrm{S}_{6}$, $\mathrm{S}_{7}, \mathrm{~S}_{8}, \mathrm{~S}_{9}$ and $\mathrm{S}_{10}[14]$.

Since the degree of every node in the resultant graph should be greater than or equal to 4 after this iteration, it is observed that the degree of the nodes $\boldsymbol{a}, \boldsymbol{c}, \boldsymbol{e}$ and $\boldsymbol{f}$ does not satisfy this condition. Hence, the nodes $\boldsymbol{a}, \boldsymbol{c}, \boldsymbol{e}$ and $\boldsymbol{f}$ of $\mathrm{G}_{\mathrm{n}}$, are selected, in $\mathrm{G}_{\mathrm{m}}$. The linking numbers of nodes $\boldsymbol{g}$ and $\boldsymbol{h}$ are less than the linking number of node $\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}, \boldsymbol{d}, \boldsymbol{e}$ and $\boldsymbol{f}$.

Further, the degree of every node is the same. Since, node $\boldsymbol{g}$ has the least node index number, it is selected. A link is established between node $\boldsymbol{g}$ of $\mathrm{G}_{\mathrm{m}}$ and node $\boldsymbol{a}$ of $\mathrm{G}_{\mathrm{n}}$. Similarly, links are established between node $\boldsymbol{h}$ of $\mathrm{G}_{\mathrm{m}}$ and node $\boldsymbol{c}$ of $\mathrm{G}_{\mathrm{n}}$, node $\boldsymbol{a}$ of $\mathrm{G}_{\mathrm{m}}$ and node $\boldsymbol{e}$ of $\mathrm{G}_{\mathrm{n}}$ and node $\boldsymbol{b}$ of $\mathrm{G}_{\mathrm{m}}$ and node f of $\mathrm{G}_{\mathrm{n}}$. The corresponding tables are updated. The resulting network graph is shown in Figure 4


Figure: 4 4-connected merged network graph
The counter Z is not incremented as linking number of all the nodes are not increased by one. The components corresponding to the cut-set of $\mathrm{C}_{3}$ are constructed and the components are arranged in an increasing order of their cardinality. The component $S_{1}=\{a\}$ is considered. Since, the linking number of the node $\boldsymbol{a}$ in $\mathrm{G}_{\mathrm{n}}$ equals 3 which is greater than Z , node $\boldsymbol{a}$ is not selected. For similar reasons, node $\boldsymbol{b}$ of $S_{2}$, node $\boldsymbol{e}$ of $S_{4}$ and node $\boldsymbol{f}$ of $S_{5}$ is not selected. The component $\mathrm{S}_{3}=\{\mathrm{d}\}$ is considered. The linking number of node $\boldsymbol{d}$ equals 0 which is less than or equal to $Z$. In $G_{m}$, the linking numbers of nodes $\boldsymbol{f}, \boldsymbol{g}$ and $\boldsymbol{h}$ are less than the linking number of node $\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}, \boldsymbol{d}$ and $\boldsymbol{e}$. Further, the degree of every node is the same. Since, node $\boldsymbol{f}$ has the least node index number, it is selected. A link is established between node $\boldsymbol{f}$ of $\mathrm{G}_{1}$ and node $\boldsymbol{b}$ of $\mathrm{G}_{2}$. Simultaneously, the corresponding tables are updated.

Since, the degree of every node in the resultant graph should be greater than or equal to 5 after this iteration, it is observed that the degree of the nodes $\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}, \boldsymbol{e}$ and $\boldsymbol{f}$ of $\mathrm{G}_{2}$ does not satisfy this condition. Hence, the nodes $\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}, \boldsymbol{e}$ and $\boldsymbol{f}$ of $\mathrm{G}_{\mathrm{n}}$ are selected, in $\mathrm{G}_{\mathrm{m}}$. The linking numbers of nodes $\boldsymbol{d}, \boldsymbol{e}, \boldsymbol{f}, \boldsymbol{g}$ and $\boldsymbol{h}$ are less than the linking number of node $\boldsymbol{a}, \boldsymbol{b}$ and $\boldsymbol{c}$. Further, the degree of every node is the same. Since, node $\boldsymbol{d}$ has the least node index number, it is selected. A link is established between node $\boldsymbol{d}$ of $\mathrm{G}_{\mathrm{m}}$ and node $\boldsymbol{a}$ of $\mathrm{G}_{\mathrm{n}}$. Similarly, links are established between node $\boldsymbol{e}$ of $\mathrm{G}_{\mathrm{m}}$ and node $\boldsymbol{b}$ of $\mathrm{G}_{\mathrm{n}}$, node $\boldsymbol{f}$ of $\mathrm{G}_{\mathrm{m}}$ and node $\boldsymbol{c}$ of $\mathrm{G}_{\mathrm{n}}$, node $\boldsymbol{g}$ of $\mathrm{G}_{\mathrm{m}}$ and node $\boldsymbol{e}$ of $\mathrm{G}_{\mathrm{n}}$ and node $\boldsymbol{h}$ of $\mathrm{G}_{\mathrm{m}}$ and node $\boldsymbol{f}$ of $\mathrm{G}_{\mathrm{n}}$. The corresponding tables are updated.

## IV. CONCLUSIONS

The main advantage of this method is that it is based on the concept of cut sets. The connectivity of the merged graph increases by one after every iteration. Hence, we get all possible networks with connectivity between $n$ and $m$. Thus far, methods have been proposed to construct $m$ connected networks and to merge two or more networks into
a single network for maximum connectivity. Once a network is constructed, and a claim is made that the constructed network is $m$-connected, to verify the claim, the next chapter proposes a method to compute the connectivity of a network in a single iteration.

## V. REFERENCES

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