



Study on the Problem of Student Subsidies Assessment Based on Fuzzy Analytic Hierarchy Process Method

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Abstract: A fuzzy analytic hierarchy process (AHP) method was proposed to determine the student subsidies assessment. Based on the analysis and investigation of factors affecting colleges and universities assessments and difficulties in this process, 5 first grade indexes, 3 second grade indexes and 13 third grade indexes were conducted to establish the index system for synthesis evaluating project. The AHP method was used to construct the hierarchy relationship with influence factors and set up their hierarchy structures, and the weight of every influence factor was computed based on the proved scale (-2,2) EM AHP and judgment matrix fuzzy sets method. After all grades were acquired by means of investigations and students' applications, the matrix of the membership comparative degree was calculated by standardization datum. A synthesis assessment model was made to estimate students rank. We use this model to test the student subsidies assessment of a college, then we got the rank forecast is that, first-class precision 72.63%, second-class precision 34.25%, and third-class 60.0%. The results show distinctly that the fuzzy AHP method is valid and feasible to assess the student subsidies in colleges.

Key words: Subsidies assessment; Fuzzy synthesis assessment method; Analytic hierarchy process (AHP); (-2,2) scale method.

I. INTRODUCTION

With the continuous reform of China's higher education in depth and the constant expansion of college enrollment, the students from economically disadvantaged families take an increasing share of college students. The proportion of the assessment of subsidies has increased after 2007, that makes the quota for the number of impoverished students who get the state-funded more than the number of actual families poorer students, and the excess of the quota promote part of the already needy students added to the list of applications, resulting in poverty and the division of the poverty standard is difficult to define and review of the existing university system in practice, often a lack of quantitative standards. These led to a departure of the original intention of this project.

Xue Dan[1] make a comparison between subsidies rating model based on fuzzy comprehensive evaluation and model based on household income-spending, found that the model based on household income-spending is closer to the actual situation. Wang Yanhua & Du Jianbin[2] use fuzzy comprehensive evaluation method to evaluate scholarship performance, especially make a probing analysis of the performance of fairness and efficiency. Through empirical analysis they discovered that the school grants performance is reasonable, but the positioning of fairness and efficiency exist certain deviation. Wang Lei[3] analyses some commonly used methods and existing problems of scholarship evaluation in university, and introduces analytic hierarchy process (AHP) to establish hierarchic models.

This article makes some research in the factors which the subsidies assessment involved, such as external recognition (classes identified), Internal situation (income, expenses, loans and family situation) and so on, then establish corresponding evaluation index system combined the fuzzy comprehensive evaluation method and hierarchical analysis during the grants evaluation analysis process. We solve the models by using MATLAB, that improve the efficiency of evaluation.

The remainder of this paper is organized as follows. Section 2 introduces Fuzzy Analytical Hierarchy Process. Section 3 determines the student subsidies assessment by using fuzzy analytic hierarchy process (FAHP) method. Section 4 concludes this paper with a brief summary

II. FUZZY ANALYTICAL HIERARCHY PROCESS

The analytic hierarchy process (AHP) [4] is a structured technique for organizing and analyzing complex decisions. Based on mathematics and psychology, it was developed by Thomas L. Saaty in the 1970s and has been extensively studied and refined since then. It provides a systematic hierarchy framework, which facilitates the ideas of decision to become clearer. It has particular application in group decision making[5], and is used around the world in a wide variety of decision situations, in fields such as government, business, industry, healthcare, and education.

Fuzzy comprehensive evaluation which mainly uses in the decision-making that does not require a precise evaluation, is the process of using fuzzy set theory to system

for systematic comprehensive evaluation. That's simple and practicable, and consistency is better; comprehensive value can be calculated via the membership degree matrix.

Fuzzy analytic hierarchy makes the quantitative and objectivity of AHP and the inclusive of fuzzy comprehensive evaluation method organic integration, which is very much applicable in decision-making. When the number of evaluation of objects is huge, we can use the fuzzy comprehensive evaluation method to grade each object directly, and then calculate the relative membership value, determine the weights of each factor via AHP.

The major steps of fuzzy analytic hierarchy process as follow:

- Establish the index system of hierarchy;
- Calculate the weights of factors via AHP;

- Use fuzzy comprehensive evaluation method to determine the relative membership value of the index of schemes.
- Calculate the final evaluation value via membership synthetic method to obtain the best option.

III. SUBSIDIES EVALUATION DECISION IN UNIVERSITIES

A. Establish Evaluation Index System:

Combined with the current widely considered factors in the evaluation process in universities, we establish the index system shown in Figure 1.

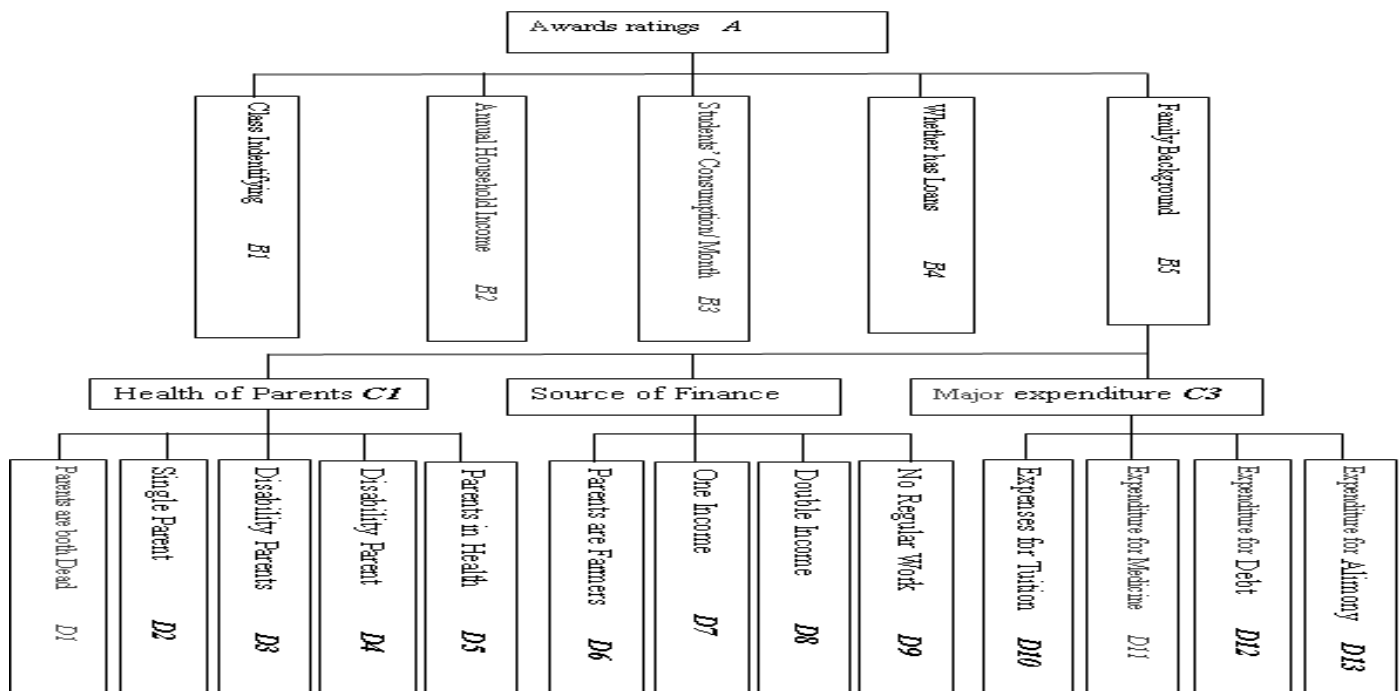


Figure 1 Comprehensive evaluation index system

According to the investigation of university grants application, we know that, factor B_1 is qualitative description, it general could be split into impoverished students and the neediest students in the preliminary assessment. B_2 and B_3 are quantitative description which would be filled in by the students. Factor B_4 can be divided into business loans and national loan two types. Obviously, factor B_5 is also qualitative description, such as, single-parent families, laid-off parents, someone is sick in the family and so on. Considering the characteristics of problems above, we calculate the weight of each factor by using AHP, then translate qualitative language into quantitative values which will be put into the consolidated score formula.

B. Determine index weight

According to the main idea of fuzzy analytic hierarchy process, we use AHP to determine the weight of each index. Procedure is as follows [6]:

a) Establish judgment matrix:

The key to determining the weight of each index is to establish judgment matrix that is a process of converting

human thought into quantitative values. 1-9 scale method in AHP is a method to divide limit of the ordinary people's ability to distinguish things at the same time into nine grades. In this paper, the Analytic Hierarchy Process will be applied to student subsidies assessment, involving parents' situation, source of income and household expenditure and so on, these factors include many aspects, which are different but also closely connection. While 1-9 scale method needs lot of information, it is in generally hard to exactly given. There are some scholars proposed comparative scale(0, 2) and the corresponding transformation algorithm [7], its judgment is: when element A compare with B, if A is more important than B, represented by 2; if A and B are equally important, represented by 1; if A is less important than B, represented by 0. This method needs more simple information, and is easier to collect experts' judgment information, easier to accepted and adapted by experts and decision-makers. But it still exists three deficiencies [8]: 1、loss of judgment information. 2 loss of cumulative dominance. 3、loss of consistency. Therefore we use a compromising scale method - (-2, 2) EM here which gives consideration to (1/9, 9) EM method and (0,2) EM

method.

(-2, 2)EM uses five scale: (-2, -1, 0, 1, 2) to judge the important relationship between two elements in the same level, giving the scale comparative judgment matrix C, then converts matrix C into matrix B, calculates maximum characteristic root $\lambda_{\max}$ and ordering vector W via EM, the specific calculation steps are as follows:

a) Suppose there are n kinds of elements located at the same level, we obtain the judgment matrix by EM method:

$$C = \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1n} \\ c_{21} & c_{22} & \cdots & c_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ c_{n1} & c_{n2} & \cdots & c_{nn} \end{bmatrix}$$

$$c_{ij} = \begin{cases} 2 & \text{The } i^{\text{th}} \text{ element is most important than the } j^{\text{th}} \text{ element} \\ 1 & \text{The } i^{\text{th}} \text{ element is more important than the } j^{\text{th}} \text{ element} \\ 0 & \text{The } i^{\text{th}} \text{ element and } j^{\text{th}} \text{ element are equally important} \\ -1 & \text{The } j^{\text{th}} \text{ element is more important than the } i^{\text{th}} \text{ element} \\ -2 & \text{The } j^{\text{th}} \text{ element is most important than the } i^{\text{th}} \text{ element} \end{cases}$$

Moreover $c_{ii} = 0$, that is each element itself is the same importance.

b) Calculate the important ranking index of each element r_i :

$$r_i = \sum_{j=1}^n c_{ij} \quad i = 1, 2, \dots, n$$

c) Work out judgment matrix B:

$$b_{ij} = \begin{cases} r_i - r_j + 1 & r_i \geq r_j \\ (r_j - r_i + 1)^{-1} & r_i < r_j \end{cases}$$

We get:

$$B = \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{nn} \end{bmatrix}$$

d) Calculate judgment matrix B's characteristic root $\lambda_{\max}$ and characteristic vector $W = (w_1, w_2, \dots, w_n)^T$, then check the consistency.

e) Data Result

For subsidies evaluation factor B5, we qualitatively give the important sequence ordering $C_1 > C_2 > C_3$. We get the comparison matrix and judgment matrix the by Delphi method: solve the weight arithmetic mean comprehensive vector according to the judgment matrix which given by experts. From step (1) we can obtain the comparison matrix of $B_5 - C$.

Table 1 comparison matrix of level $B_5 - C$

$B_5 - C$	C_1	C_2	C_3	r_i
C_1	0	1	2	3
C_2	-1	0	1	0
C_3	-2	-1	0	-3

According to the step (3) we get judgment matrix of level $B_5 - C$, show as Table 2. The results of other comparison matrix and judgment matrix are not listed here.

Table 2 judgment matrix of level $B_5 - C$

$B_5 - C$	C_1	C_2	C_3	M_i	$\bar{W}_i$	W_i
C_1	1	4	7	28	3.0366	0.6955
C_2	1/4	1	4	1	1	0.2290
C_3	1/7	1/4	1	1/28	0.3293	0.0754

Where

$$M_i = \prod_{j=1}^n b_{ij}, \quad \bar{W}_i = \sqrt[n]{M_i}, \quad W_i = \bar{W}_i / \sum_{i=1}^n \bar{W}_i \quad (i=1, 2, \dots, n)$$

b) Consistency check:

$$\text{From the formula } \lambda_{\max} = \frac{1}{n} \sum_{i=1}^n \frac{(AW)_i}{W_i}, \text{ we obtain}$$

characteristic root $\lambda_{\max}$, calculate consistency check index by $CI = (\lambda_{\max} - n) / (n - 1)$. The corresponding average consistency ratio can be calculated via formula $CR = CI / RI$, where RI represents the average consistency index over numerous random entries of same order reciprocal matrices. For $B_5 - C$ level, using the formula above we work out:

(a). Maximum characteristic root $\lambda_{\max} = 3.0764$;

(b). Consistency check index
 $CI = (\lambda_{\max} - n) / (n - 1) = (3.0764 - 3) / (3 - 1) = 0.0382$;

(c). Look-up table we know: $RI = 0.58$,

$CR = CI / RI = 0.0382 / 0.58 = 0.0659 < 0.1$, it illustrates single sort of this layer level has a satisfactory consistency.

In a similar way, for B5, we get the weight and consistency of each standard layer, results are as follows:

(a). For $C_1 - D$, we have

$$W_{C_1-D} = (0.5459, 0.2334, 0.1616, 0.0400, 0.0191), \\ \lambda_{\max} = 5.4054, CI = 0.1014, RI = 1.1200, CR = 0.0905 < 0.1.$$

(b). For $C_2 - D$, we have

$$W_{C_2-D} = (0.3190, 0.1441, 0.0376, 0.4993), \\ \lambda_{\max} = 4.1341, CI = 0.0447, RI = 0.9000, CR = 0.0497 < 0.1.$$

(c). For $C_3 - D$, we have

$$W_{C_3-D} = (0.5709, 0.2691, 0.0800, 0.0800),$$

$$\lambda_{\max} = 4.0606, CI = 0.0202, RI = 0.9000, CR = 0.0225 < 0.1$$

We use the same calculation principles to obtain judgment matrix of $A-B$:

$$W_{A-B} = (0.2786, 0.1088, 0.0745, 0.0196, 0.5186),$$

$$\lambda_{\max} = 5.3589, CI = 0.0897,$$

$$RI = 1.1200, CR = 0.0801 < 0.1.$$

From the results above, we know consistency levels of the all judgment matrix are satisfactory, there is no need to adjust the value of the judgment matrix elements.

c) Hierarchy Total Taxis:

This process is from the highest layer to the lowest layer layer-by-layer. In this paper, we calculate the synthetic weights of $(D_1, D_2, \dots, D_{13})$ to criterion layer factor (B5), and the results of hierarchy total taxis are shown in Table 3:

Table 3 Total ordering of the 13 indicators relative to B5

layer B_5	layer C_i			Weight of Hierarchy total taxis N_i	Result of ordering
	C_1 0.6955	C_2 0.2290	C_3 0.0754		
w_{11}	0.5459			0.3797	1
w_{12}	0.2334			0.1630	2
w_{13}	0.1616			0.1124	3
w_{14}	0.0400			0.0278	4
w_{15}	0.0191			0.0133	5
w_{26}		0.3190		0.0731	6
w_{27}		0.1441		0.0330	7
w_{28}		0.0376		0.0086	8
w_{29}		0.4993		0.1143	9
w_{310}			0.5709	0.0430	10
w_{311}			0.2691	0.0203	11
w_{312}			0.0800	0.0060	12
w_{313}			0.0800	0.0060	13

C. Quantifying Data:

We divide factor B5 into 13 secondary index to collect the information in students' application forms with the principle : if one of (D_1, D_2, D_3, D_4) appears, recorded as 3; if one of

(D_6, D_7, D_8, D_9) appears, recorded as 2; if one of $(D_{10}, D_{11}, D_{12}, D_{13})$ appears, recorded as 1; if the other items are not mentioned, recorded as 0. The form constitutes a matrix D, shown as follow:

Table 4 Information of students' application forms

B_5	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	D11	D12	D13
A1	0	0	0	0	1	2	0	0	0	0	1	0	0
A2	0	0	0	3	0	0	0	0	2	0	1	0	0
A3	0	0	0	3	0	0	0	0	2	0	1	0	0

The first column is students' number, the first row is 13 secondary index.

In factor B1, we record the "neediest students" who are evaluated by the class as 2; record "impoverished students" as 1. For factor B4 which means whether have a loan, if "yes",

record as 2; if not, record as 1. Finally standardize the comprehensive value of B2, B3 and B5 by using the following formula (1).

According to the idea of fuzzy mathematics, we give each student's evaluation value of each index in factor B5,

then establish fuzzy evaluation matrix of relative membership degree based on evaluation value. Suppose evaluation index sample set which consists of m schemes includes n index $\{x(i, j) | i = 1, 2, \dots, n; j = 1, 2, \dots, m\}$,

where each index value is not negative. When determining fuzzy evaluation matrix of relative membership degree of the single index, we need to standardize sampling data $\{x(i, j)\}$ by using the following formula (1).

$$r(i, j) = \begin{cases} x(i, j) / [x_{\max}(i) + x_{\min}(i)], & \text{For forward indicators} \\ [x_{\max}(i) + x_{\min}(i) - x(i, j)] / [x_{\max}(i) + x_{\min}(i)], & \text{For reverse indicators} \end{cases} \quad (1)$$

In formula (1), $x_{\max}(i)$, $x_{\min}(i)$ are respectively the maximum and minimum value of the i^{th} index, $r(i, j)$ is standardized evaluation index. Take 238 students' grants applications of a university for example, each student's family comprehensive evaluation score as follows:

$$f = D \square W_{B-D} = (0.2595, 0.1697, \dots, 0.2452, 0.0365) \quad (2)$$

Where D is given in Table 4, W_{B-D} are weights of 13 secondary index to B_5 .

Similarly, we can obtain fuzzy evaluation matrix of relative membership degree of all the criteria layers, listed part of them in Table 5:

Table 5 Comprehensive evaluation matrix of relative membership degree

	B_1	B_2	B_3	B_4	B_5
A1	0.6667	0.9756	0.7429	0.3333	0.2020
A2	0.6667	0.9707	0.6714	0.3333	0.1321
A3	0.6667	0.9707	0.7143	0.3333	0.1321
	$\vdots$	$\vdots$	$\vdots$		
A236	0.3333	0.6585	0.5143	0.3333	0.1119
A237	0.3333	0.6098	0.5714	0.3333	0.1909
A238	0.3333	0.6098	0.5714	0.6667	0.0284

We can put B and W_{A-B} into formula(2) to calculate all the students' comprehensive value f , take the students' number for x axis, comprehensive value for y axis, it can intuitively observe situation of students' score.

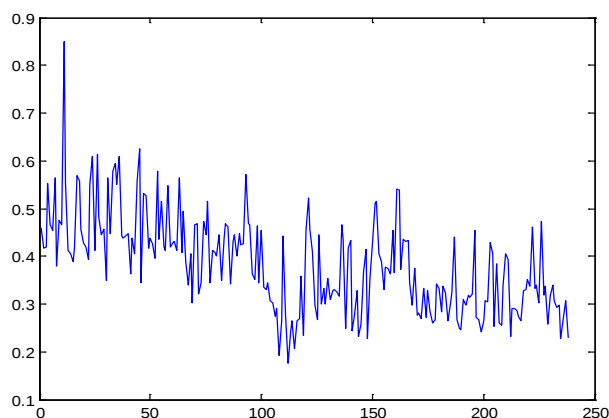


Figure 2 comprehensive value of students

At this point, we order all the comprehensive value which will be divided into three classes: first-class, second-class and third-class, the corresponding student numbers also be divided into three classes as follows:

First-class:

11,45,26,36,24,34,53,33,93,17,7,63,31,44,18,12,4,35,23,58,161,162,47,48,121,55,152,76,151,65,27,9,74,226,94,84,71,70,10,95,136,5,98,85,222,1,29,19,122,6,159,120,100,196,40,90,32,126,75,88,28,81,39,37,110,187,83,50,42,38,54,164,137,166,140,87,165,61,150,203,20,60,92,51,91,59,3,56,21,139,16,49,2,147,78.

Second-class:

62,25,13,57,123,79,64,204,43,153,68,210,14,89,80,52,211,22,154,15,66,206,8,156,157,169,163,146,160,158,41,96,118,130,149,97,220,30,82,46,99,73,167,103,77,179,86,231,224,67,209,182,228,221,101,173,180,128,223,133,155,102,219,132,143,175,218,134,183,186,195,72,230.

Third-class:

193,227,135,194,191,131,232,104,237,201,202,69,105,225,185,129,127,168,192,124,234,233,107,213,214,215,181,176,171,142,170,111,106,216,197,174,172,117,178,198,236,188,125,114,116,109,217,200,184,207,177,229,145,208,205,138,189,190,141,113,199,119,144,212,238,148,235,115,108,112.

The original data has given the results for: first-class: No. 1-95, second-class: No.96-168, third-class: No.169-238. From the above we can evaluate the prediction accuracy of the model: first-class: 72.63%, second-class: 34.25%, third-class: 60.00%.

IV. CONCLUSION

We can find that, it has higher accuracy and more satisfy in evaluating first and third class, lower accuracy in evaluating the second-class. The reason may be on one hand the results of qualitative description selection is not reasonable enough, on the other hand, it exists quantization error. In view of these, we will improve in the future study from two aspects: (1) Amend index system; (2) Improve scale method of judgment matrix: using uncertain type - interval scale method [9].

Fuzzy Analytic Hierarchy Process (FAHP) combines Analytic Hierarchy Process (AHP) and Fuzzy Comprehensive Evaluation, that makes the advantage of both effectively plays. This method is feasible in problem of

program evaluation selection.

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