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Performance of OFDM with I/Q Imbalance and Phase Noise

Md. Kislu Noman*
Lecturer, Dept. of CSE,
Pabna Science and Technology University,
Pabna, Bangladesh
md.k.noman@gmail.com

Md. Ibrahim Abdullah
Associate Professor
Dept. of CSE,
Islamic University, Kushtia, Bangladesh
ibrahim25si@yahoo.com

Md. Aktaruzzaman
Assistant Professor
Dept. of CSE,
Islamic University, Kushtia, Bangladesh
mazaman_iuk@yahoo.com

Abstract: This Orthogonal Frequency Division Multiplexing (OFDM) is a multi-carrier transmission scheme that enables information to be transmitted at high data-rates with high robustness to the effects of noise and fading than single carrier transmission systems. It eliminates the interference between the sub-carriers and increases the spectral efficiency of the system. On the other hand, the OFDM system suffers from different drawbacks. The OFDM transmission exploits the strict orthogonality of each sub-carrier, which makes OFDM sensitive to frequency offsets and phase noise. The goal of this paper is to analyze the OFDM system with phase noise, nonlinear power amplifier and I/Q imbalance. Simulation result shows the BER performance of M-QAM in AWGN channel and Rayleigh fading channel and compares the BER performance of M-QAM.

Keywords: OFDM, I/Q imbalance, Phase Noise, AWGN, Rayleigh fading, BER, QAM.

I. INTRODUCTION

OFDM is a multiplexing technique that subdivides the bandwidth into multiple frequency sub-carriers. In an OFDM system, the input data stream is divided into several parallel sub-streams of reduced data rate (thus increased symbol duration) and each sub-stream is modulated and transmitted on a separate orthogonal sub-carrier. The increased symbol duration improves the robustness of OFDM to delay spread. The sub-carrier frequencies are chosen so that the sub-carriers are orthogonal to each other, meaning that cross-talk between the sub-channels is eliminated and inter-carrier guard bands are not required [1]. This greatly simplifies the design of both the transmitter and the receiver. However OFDM requires very accurate frequency synchronization between the receiver and the transmitter; with frequency deviation the sub-carriers will no longer be orthogonal, causing *Inter-Carrier Interference* (ICI) [2]. Furthermore, the introduction of the cyclic prefix (CP) can completely eliminate *Inter-Symbol Interference* (ISI) as long as the CP duration is longer than the channel delay spread. The CP is typically a repetition of the last samples of data portion of the block that is appended to the beginning of the data payload [5].

The primary advantage of OFDM over single-carrier schemes is its ability to cope with severe channel conditions (for example, attenuation of high frequencies in a long copper wire, narrowband interference and frequency-selective fading due to multi-path) without complex equalization filters. Channel equalization is simplified because OFDM may be viewed as using many slowly modulated narrowband signals rather than one rapidly modulated wideband signal. By converting a single high frequency carrier to several sub-carriers, OFDM enhances the ability to cope with frequency selective fading effects and narrow bandwidth interference. The orthogonal property also greatly simplifies the design of both transmitter and receiver. A receiver can detect every sub-carrier data, which commonly is done via Fast Fourier Transform (FFT). Therefore a separate filter for each sub channel is not required. However, in practice, the sub-carriers are modulated in different amplitude and phase [3].

The OFDM system suffers from different drawbacks. Since the OFDM signal is a combination of several modulated sub carriers, the signal may have large peak power, which makes the Peak-to-Average Power Ratio (PAPR) also large. High PAPR results in reduction of efficiency of the Power Amplifier. The OFDM transmission exploits the strict orthogonality of each sub carrier, which makes OFDM sensitive to frequency offsets and phase noise. The basic modulation algorithms and other adaptive modulation techniques also increase the complexity of computations. In the modern OFDM transceiver, the RF electronic devices have several different impairments known as "dirty RF". The impairments that have major impacts on the system performance are: Nonlinear high power amplifier, Phase noise, PAPR Problem and In-phase and Quadrature (I/Q) imbalances [2] [4]. In this paper we have investigate the bit error rate (BER) of OFDM with I/Q imbalance and phase noise that caused by local oscillator.

This paper is organized as follows. In section 2, we describe the orthogonality theory of OFDM. In section 3, we discuss the I/Q imbalance and phase noise problems. In section 4, we represent the simulation result of OFDM in details. Finally we make conclusions in section 5.

II. ORTHOGONALITY OF OFDM

Two periodic signals are orthogonal when the integral of their product over one period is equal to zero. For the case of continuous time:

$$\int_0^T \cos(2\pi m f_o t)\cos(2\pi n f_o t)dt = 0, m \neq n \qquad (1)$$

For the case of discrete time:

$$\sum_{K=0}^{N-1} \cos\left(\frac{2\pi kn}{N}\right)\cos\left(\frac{2\pi km}{N}\right)df = 0, m \neq n \qquad (2)$$

The orthogonality in the time domain means within an OFDM symbol period, all sub-carriers have integer cycles and the numbers of the cycles between the channels differ by integer numbers. An OFDM signal consists of N orthogonal sub-carriers modulated by N parallel data streams. Each baseband sub-carrier is of the form:

$$\phi_k(t) = e^{j2\pi f_k t} \qquad (3)$$

Where f_k is the frequency of the k th sub-carrier. One baseband OFDM symbol (without a cyclic prefix) multiplexes *N* modulated sub-carriers:

$$s(t) = \frac{1}{\sqrt{N}}\sum_{k=0}^{N-1} x_k \phi_k(t) \qquad 0 < t < NT \qquad (4)$$

Where x_k is the *k*th complex data symbol (typically taken from a PSK or QAM symbol constellation) and NT is the length of the OFDM symbol. The sub-carrier frequencies f_k are equally spaced

$$f_k = \frac{k}{NT} \qquad (5)$$

Which makes the sub-carriers $\phi_k(t)$ on $0<t<NT$ orthogonal.

To maintain orthogonality between sub-carriers, it is necessary to ensure that the symbol time contains one or more multiple cycles of each sinusoidal carrier waveform. In the case of OFDM, the sinusoids of sub-carriers will satisfy this requirement since each is a multiple of a fundamental frequency. Orthogonality is critical since it prevents ICI. ICI occurs when the integral of the carrier products are no longer zero over the integration period, so signal components from one sub-carrier causes interference to neighboring sub-carriers. As such, OFDM is highly sensitive to frequency dispersion caused by Doppler shifts, which results in loss of orthogonality between sub-carriers [2].

Each sub-carrier in an OFDM system is a sinusoid with a frequency that is an integer multiple of a fundamental frequency f_o. Each sub-carrier is like a Fourier series component of the composite signal, an OFDM symbol. The sub-carrier waveform can be expressed as the following equation:

$$\begin{aligned} S(t) &= \cos(2\pi f_o t + \theta_k) \\ &= a_n \cos(2\pi n f_o t) + b_n \sin(2\pi n f_o t) \\ &= \sqrt{a_n^2 + b_n^2}\cos(2\pi n f_o t + \phi_n) \end{aligned} \qquad (6)$$

Where $\phi_n = \tan^{-1}\left(\frac{b_n}{a_n}\right)$

The sum of the sub-carriers (*SB*) is then the baseband OFDM signal:

$$SB(t) = \sum_{n=0}^{N-1}\{a_n \cos(2\pi n f_o t) - b_n \sin(2\pi f_o t)\} \qquad (7)$$

III. I/Q IMBALANCE AND PHASE NOISE EFFECTS

OFDM modulation works on the principle of converting a serial symbol stream to a parallel symbol stream with each symbol from the parallel set modulating a separate carrier. The spacing between the carriers is 1/T where T is the duration of the OFDM symbols (without cyclic prefix). This guarantees orthogonality of the carriers i.e. there is no interference between carriers. The main advantage of this scheme is that one carrier (or set of carriers) may undergo severe fading but other carriers would be able to carry data. Equalization on these narrowband channels is also much easier than equalization of one wideband channel. Inter Symbol Interference (ISI) which affects the signal in the time domain is removed by adding a guard period between symbols, called cyclic prefix.

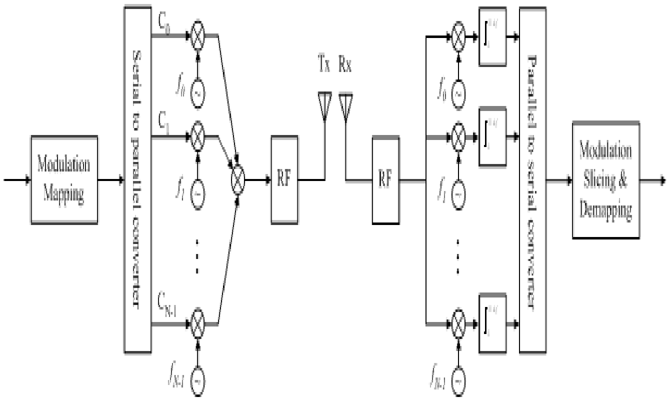

Figure. 1: OFDM Modulator and Demodulator

The implementation of OFDM-based systems suffers from impairments such as in-phase and quadrature-phase (IQ) imbalances equivalently between the real and imaginary parts of the complex signal at both the transmitter (during up conversion) and receiver (during down-conversion). Such imbalances are caused by the analog processing of the radio frequency (RF) signal and can be present at both the transmitter and receiver [6]. In the up and down-conversion of the transceiver, the incoming signal in the I-path is up-converted or down-converted by the local oscillator (LO) at the carrier frequency, while the Q-path signal is up-converted or down-converted with the 90± phase shift. As the IQ modulation is performed in the analog domain, the matching of the I and Q paths is not perfect. The filtering at baseband may not be 100% matched, the 90° phase rotation of the LO signal will be slightly off and the I and Q paths may not be matched with perfectly equal power which results in the so called I/Q imbalance.

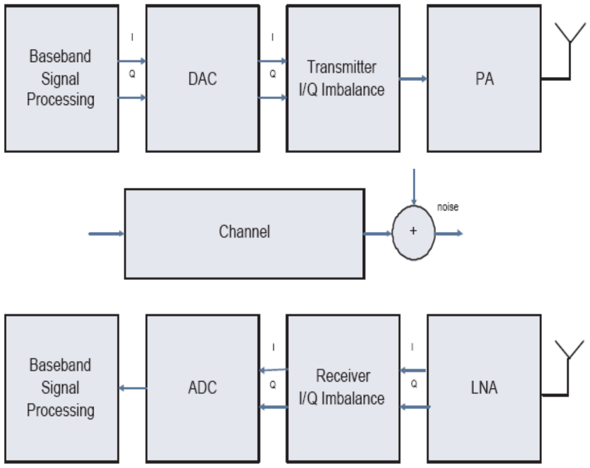

Figure. 2: OFDM transceiver with I/Q imbalance at transmitter and receiver

A model of the OFDM transceiver with I/Q imbalance is shown in fig. 2. First, I/Q imbalance is modeled at the transmitter. Since I/Q imbalance is any mismatch between the I and Q branches from the ideal case, the distorted signal in the time domain can be modeled as [8], [9]:

$$x_d(t) = \mu_t x(t) + v_t x^*(t) \qquad (8)$$

The notation * denotes the complex conjugate and μ_t and v_t are related to the amplitude imbalance γ_t and phase imbalance $\Delta\varphi_t$ between the I and Q branches at the transmitter. μ_t and v_t can be expressed as:

$$\left.\begin{aligned} \mu_t &= \cos(\Delta\varphi_t) + j\gamma_t \sin(\Delta\varphi_t) \\ v_t &= \gamma_t \cos(\Delta\varphi_t) + j\sin(\Delta\varphi_t) \end{aligned}\right\} \qquad (9)$$

Let *r(t)* represents the received complex signal before being distorted by the receiver I/Q imbalance. Using the same model as at the transmitter, the distorted signal can be modeled as

$$r_d(t) = \mu_r r(t) + v_r r^*(t) \qquad (10)$$

Where μ_r and v_r are similarly to μ_t and v_t, and the $r_d(t)$ is the distorted signal after I/Q imbalance at the receiver.

Phase noise is introduced by the Local Oscillators (LO) at both the transmitter and receiver. Phase noise is a random process caused by the frequency fluctuation of LO. It can be described as two multiplicative distortions. However, for a small phase noise bandwidth, the distortion effect approximately equals to the phase noise effect of sum bandwidth of both processes. Fig. 3 shows the effects of the phase noise on the constellation map to describe the notation and noise corruption.

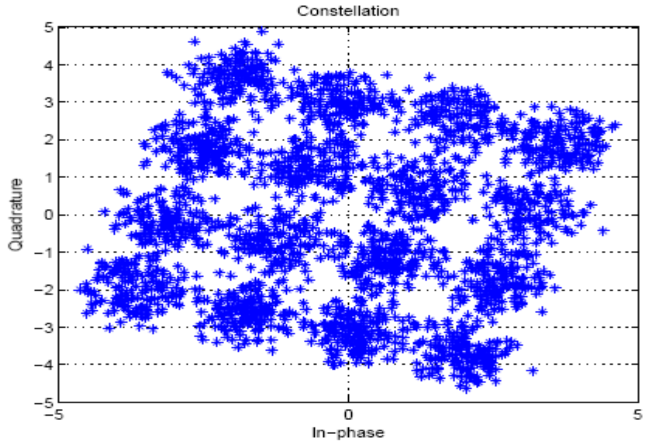

Figure. 3: Effect of the phase noise on the signal constellation

The discrete-time OFDM symbol with phase noise can be expressed as:

$$r(n) = (x(n) \otimes h(n))e^{i\phi(n)} + w(n) \qquad (11)$$

Where *x(n)*, *h(n)* and *w(n)* denote the samples of the transmitted signal, channel impulse response and the channel noise term respectively, and ⊗ denotes convolution. $\phi(n)$ represents the phase noise process at the receiver, and is usually modeled as a Wiener process [7]. After removing the N_{cp} (sample of clock pulse) symbols that correspond to the CP and taking the Discrete Fourier Transform (DFT) on the remaining useful *N* symbols, the demodulated carrier R_k is:

$$R_K = \sum_{n=0}^{N-1} r(n)\exp(-j\frac{2\pi}{N}kn) \qquad (12)$$

With phase noise, the R_k can be expressed as:

$$R_k = X_k H_k I_o + \sum_{l=0,l\neq k}^{N-1} X_l H_l I_{k-l} + W_k \qquad (13)$$

Where $k = -\frac{N}{2} .... \frac{N}{2} - 1$

Here X_k, H_k and W_k represent the transmitted symbol on the *kth* carrier, sampled channel transfer function and frequency domain noise. The term I_k is

$$I_k = \frac{1}{N}\sum_{n-0}^{N-1} e^{j\phi(n)} e^{(-j\frac{2\pi}{n}kn)} \qquad (14)$$

The term I_0 in the first term which stems from the phase noise, and does not depend on the sub-carrier index is referred to as Common Phase Error (CPE). In the OFDM symbol, for a small phase noise, CPE is:

$$I_o \approx e^{j\phi} = 1 + j\phi \qquad (15)$$

Where the angle Φ results from the average of phase noise samples over the symbol period:

$$\Phi = \frac{1}{N}\sum_{n=0}^{N-1}\phi(n) \qquad (16)$$

It can be seen from the equation CPE results from the complex numbers. Therefore, it can be viewed as a rotation on the signal constellation. Since it is a constant for all the sub carriers, it can be corrected by a phase rotation. The summation of the sub-carriers each multiplied by a complex number. The spectral component of phase noise in this error term is randomized; therefore, it cannot be corrected totally.

IV. SIMULATION AND RESULTS

Fig. 4 shows block diagram of the simulator. The modulation method is M-QAM and number of sub-carrier is *N* which is exponent of 2. Then I/Q imbalances are inserted as the main RF impairment of the transmitter LO. The channel is modeled as both AWGN channel and 4-tap Rayleigh fading channel [10]. After signal passing through the channel, phase noise which is modeled as Wiener process and I/Q imbalance are added to the received signal as the impairments of the receiver LO. FFT and demodulation are used to baseband signal processing.

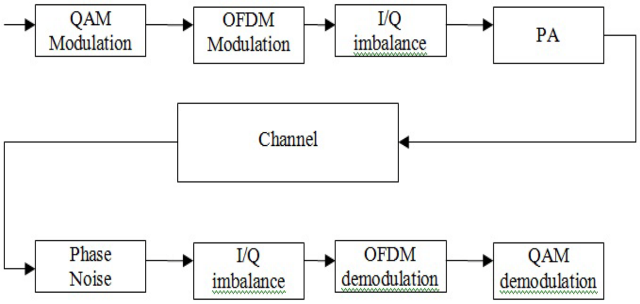

Figure. 4: Block diagram of simulator

Quadrature Amplitude Modulation has been adopted by most wireless communication standards such as WiMAX and LTE. It provides higher bit rates and consequently higher spectral efficiencies. It is usually used in conjunction with Orthogonal Frequency Division Multiplexing (OFDM) which provides a simple technique to overcome the time varying frequency selective channel.

The bit error rates of the four modulation schemes 4-QAM, 16-QAM, 64-QAM and 256-QAM are shown in the figure below. All modulation schemes use Gray coding which gives a few dB of margin in the BER performance. As with the AWGN case each additional bit per symbol requires about 1.5-2 dB in signal to ratio to achieve the same BER.

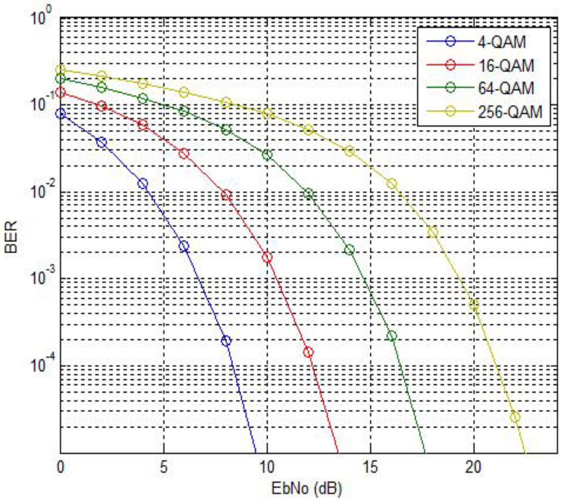

Figure. 5: Bit Error Rate of M-QAM in AWGN

The one-tap Rayleigh fading channel is generated from two orthogonal Gaussian random variables with variance of 0.5 each. The complex random channel coefficient so generated has an amplitude which is Rayleigh distributed and a phase which is uniformly distributed. As usual the fading channel introduces a multiplicative effect whereas the AWGN is additive.

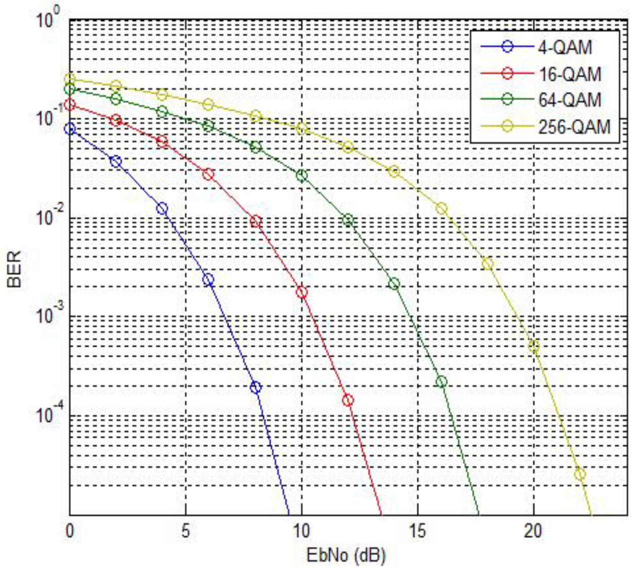

Figure. 6: Bit Error Rate of M-QAM in Rayleigh Fading

The parameters of I/Q imbalance are $\Delta\varphi_t = \Delta\varphi_r = 5^0$, $\gamma_t = \gamma_r = 0.05$, where $\Delta\varphi_t$, $\Delta\varphi_r$, γ_t and γ_r represent phase imbalance at the transmitter, phase imbalance at the receiver, amplitude imbalance at the transmitter and amplitude imbalance at the receiver respectively. The effects of I/Q imbalance in both the AWGN and Rayleigh fading channel [10] are studied. OFDM signals are modulated with 4-QAM, 16-QAM and 64-QAM with 64 sub-carriers and the CP length is 16.

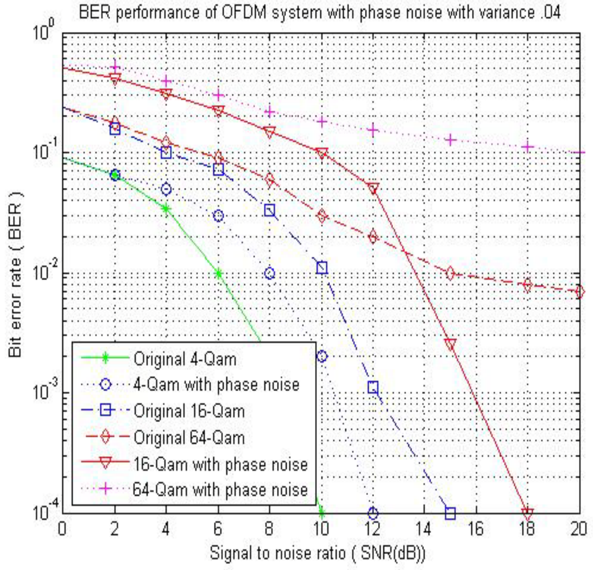

Figure. 7: BER performance of OFDM system with I/Q imbalance at

transmitter and receiver, AWGN channel with variance 0.04.

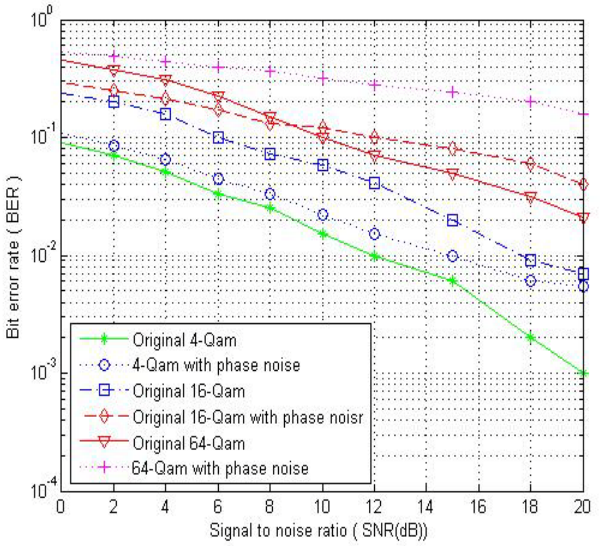

Figure. 8: BER performance of OFDM system with I/Q imbalance at transmitter and receiver, 4-tap Rayleigh fading channel with variance 0.04

Fig. 7 and fig. 8 show the BER performance of the OFDM system with phase noise whose variance is 0.04 in AWGN and Rayleigh fading channel respectively. Since phase noise is modeled as Wiener process, the variance of phase noise is corresponding to the variance of Wiener process [11]. The BER performances are degraded due to phase noise.

V. CONCLUSION

OFDM has been seen as the core technique of the future communication systems because it has many advantages. OFDM transmission has many favorable features, such as robustness against multi-path fading and narrow-band interference, high spectral efficiency and simple channel estimation and equalization, which are why it is an attractive method for wireless communication systems. One very important problem of OFDM is its sensitivity to the RF impairments. Effect of I/Q imbalance and phase noise are studied in this paper. Simulation results demonstrate the BER performance of OFDM system with I/Q imbalance at transmitter and receiver in AWGN channel and 4-tap Rayleigh fading channel. As the symbol rate per carrier increases BER performance decreases. We also notice that as the number of symbol alphabet of QAM increases, i.e. from 4 to 64, the performance of the system suffers more from I/Q imbalance and phase noise.

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