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Fuzzy Membership Function Generation: A Modified Approach with Improved Performance

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Abstract: This paper presents a new approach to generate fuzzy membership functions. We propose to use learning vector quantization (LVQ) before using FCM to initialize it with number and initial location of cluster centers. We also propose to use modified fuzzy C-means algorithm (MCM) to provide modified shapes of membership functions at extremes. The simulation has shown that the proposed algorithm has better performance as it provide the optimum solution, speeds up the rate of convergence, does not require to specify the number of clusters beforehand, does not require to specify the partition matrix randomly and generates membership functions that provide a smooth variation of the control action when the parameters are at extreme.

Keywords: Fuzzy C-Means (FCM), Learning vector Quantization (LVQ), Modified Fuzzy C-Means (MCM).

I INTRODUCTION

Data clustering is the process of grouping together similar multi-dimensional data vectors into **a** number of clusters. Clustering algorithms have been applied to **a** wide range of problems, including exploratory data analysis, data mining**,** image segmentation and mathematical programming. Clustering techniques have been used successfully to address the scalability problem of machine learning and data mining algorithms, where prior to, and during training, training data is clustered, and samples from these clusters are selected for training, thereby reducing the computational complexity of the training process, and even improving generalization performance.

Clustering of numerical modeling data forms the basis of many classification and system algorithms. FCM is a common method used to put up clustering and has been used in wide variety of applications [1], such as image recognition, control, molecular biology application such as protein folding and 3D molecular structure etc. The quality of solution of the FCM algorithm, like that of most non-linear optimization problems, depends strongly on the choice of initial values i.e. the number of cluster centers and their initial locations. If the initial values i.e. the numbers of clusters and the cluster center locations are not proper, the solution obtained will not be optimal. A two stage fuzzy c-means algorithm was proposed [2] and multi stage fuzzy c-means algorithm was also proposed [3]. The quantity of operations in these algorithms is very large and the possibility of obtaining the optimal solution is very small, but local optimal solution can be gained easily. After that genetic algorithm was used to initialize the parameters of FCM [4]. The method mentioned above can partly solve the problem of plugging into local optima, yet number of clusters required is to be specified beforehand. Subtractive clustering was used to give the prior information to FCM [5-6] but it requires the partition matrix to be specified randomly and the subtractive clustering requires number of computational steps to give prior information.

In this paper, we propose to use a combination of learning vector quantization (LVQ) [7-8] that initializes FCM with number of clusters and initial cluster center locations and modified fuzzy C-means algorithm [9] that modifies the shapes of membership functions at extremes. The proposed algorithm, thus, provide optimum solution, speeds up the rate of convergence, automatically generates the optimum number of cluster and the initial cluster center locations and provide a smooth variation of the control action when parameters are at extremes. Results show that the proposed method has much potential.

II ALGORITHMS

Fuzzy C-Means

Fuzzy C-means clustering (FCM), relies on the basic idea of Hard C-means clustering (HCM)[1], with the difference that in FCM each data point belongs to a cluster to a degree of membership grade, while in HCM every data point either belongs to a certain cluster or not. So FCM employs fuzzy partitioning such that a given data point can belong to several groups with the degree of belongingness specified by membership grades between 0 and 1. However, FCM still uses a cost function that is to be minimized while trying to partition the data set.

The data is partitioned into c clusters and prototype vectors representing the cluster centers as follows

$$v_i = [\, v_{i1}, v_{i2}, \ldots, v_{im}]^t \in R^n,\ i = 1,2,\ldots\ldots\ldots, c.$$

where v_i is the i^{th} cluster center, m is dimension of a prototype and n is number of data points.

The membership matrix represented by $U \in \Re^{c \times n}$ is allowed to have elements with values between 0 and 1. However, the summation of degrees of belongingness of a data point to all clusters is always equal to unity:

$$\sum_{i-1}^{c} \mu_{ik} = 1, \forall k = 1,2,........,n \qquad (1)$$

where $\mu_{ik} = [0,1]$ represents the membership degree of the data vector x_k in the i^{th} cluster.

The cost function J_m for FCM is defined as:

$$j_m(U,v) = \sum_{k=1}^{n}\sum_{i=1}^{c}(\mu_{ik})^m (d_{ik})^2 \qquad (2)$$

$$\forall k,i : 0 \le \mu_{ik} \le 1; \qquad (3)$$

$$\forall i : 0 \le \sum_{i=1}^{n} \mu_{ik} \le n; \qquad (4)$$

$$\forall k : \sum_{i=1}^{c} \mu_{ik} = 1; \qquad (5)$$

Where

$$d_{ik} = d(x_k - v_i) = \left[\sum_{j=1}^{m}(x_{kj} - v_{ij})^2\right]^{1/2} \qquad (6)$$

is the distance between k^{th} data point and the i^{th} cluster center, n is the number of data points, c is the number of clusters to be formed, v is the center vector and μ_{ik} is the membership of k^{th} data point in the i^{th} cluster. Table 1 shows the steps of fuzzy c-means algorithm.

Table 1 Fuzzy C-Means (FCM) Algorithm

1. Fix c ($2 \le c \le n$), number of clusters and weighting parameter, m ϵ [1, ∞]. Initialize the partition matrix $U^{(0)} = [\mu_{ik}]_{c \times n}$.
2. Calculate the c centers, as follows

$$v_{ij}^{(r)} = \frac{\sum_{k=1}^{n} \mu_{ik}^{m} \times x_{kj}}{\sum_{k=1}^{n} \mu_{ik}^{m}} \qquad (7)$$

3. Update the partition matrix U for r^{th} iteration as follows

$$\mu_{ik}^{(r+1)} = \left[\sum_{j=1}^{c}\left(\frac{d_{ik}}{d_{jk}}\right)^{2/(m-1)}\right]^{-1} \qquad (8)$$

4. If $\left\| U^{(r+1)} - U^{(r)} \right\| \le \varepsilon_L$, stop; otherwise set r = r+1 and return to step 2.

Some important observations about FCM are:

i) FCM always converges for m>1.
ii) FCM finds local minima of the objective function, because it is derived from the gradient of the objective function.
iii) The results of FCM not only depend on the parameter m and c, but also on the initial guess of partition matrix, which affects speed and local minima.

Modified Fuzzy C-Means

The membership functions generated using FCM are generalized soft partitions. These clusters if used for control signal generation may give inaccurate results for input parameters at data extremes. So a modification is done to FCM to place the membership functions at extreme with modified shapes. The steps followed in the modified Fuzzy C-Means algorithm are same as fuzzy C-Means algorithm, with the addition of two more steps, as shown in table 2.

This modification improves the representation of the real extreme conditions using soft partitions obtained by FCM and thus, a smooth variation of the control signal is obtained for the whole range of parameters.

Table 2 Modified Fuzzy C-Means Algorithm

After applying the fuzzy c-means algorithm follow these steps.

5. Find the cluster centers v_1 and v_2 , where v_1 is the cluster center with minimum distance from first data point and v_2 is the cluster center with maximum distance from first data point.
6. For v_1:
   If μ_{11}=0.0 then add a cluster in the beginning, let it be called cluster number "0" such that
   $\mu_{0k} = 1 - \mu_{1k}$ for $x_k < v_1$;
   and the number of clusters are given as z=c+1
   else $\mu_{0k} = 1$ for $x_k < v_1$
   For v_2:
   If μ_{cn}=0.0 then add one more cluster at the end, let it be called cluster "z" such that
   $\mu_{zk} = 1 - \mu_{ck}$ for $x_k > v_2$;
   and the number of clusters are given as z=c+1
   else $\mu_{zk} = 1$ for $x_k > v_2$
   Where k = 1,2,……..,n

Learning Vector Quantization

In LVQ algorithm there is no need to specify the number of clusters. In this algorithm the first input pattern will force the creation of cluster to hold it. Whenever a new pattern is encoded, the Euclidean distance between it and any allocated cluster is calculated. If this distance is smaller than the threshold distance the new pattern is assigned to that cluster otherwise a new cluster is allocated to this new pattern. Basically this is based on Learning Vector Quantization technique of Neural Network, which gives you the center of clusters.

Let the p^{th} vector as X^p and j^{th} cluster as C_j the Euclidean distance d is calculated as

$$d = \left\| X^p - C_j \right\| = \left[\sum_{i=1}^{n}(X_i^p - C_{ji})^2\right]^{1/2} \qquad (9)$$

The steps of the algorithm are given in table 3.

Table 3 Learning Vector Quantization (LVQ) Algorithm

Step 1) Take first input pattern as first cluster center.
Step 2) Calculate the Euclidean distance between input vector and cluster center.
Step 3) Find the cluster closest to the input as:

$$\left\| X^p - C_k \right\| < \left\| X^p - C_j \right\|$$

Where j=1,2,……….,M and j≠k. And M is number of allocated clusters.
Step 4) Once the closest cluster k has been determined, the distance $\| X^p - C_k \|$ must be tested against the threshold distance ρ as

i. $\| X^p - C_k \| < \rho$ the input pattern assigned n^{th} cluster.
ii. $\| X^p - C_k \| > \rho$ a new cluster is allocated to p

Step 5) Update the cluster center using following:

$$C_k = \frac{1}{N_k} \sum X \quad \text{where } x \in S_n \quad (10)$$

Step 6) Go to step 2 with new input pattern. If no new input pattern then exit.

Now we have number of clusters with their centers. Using these cluster centers, we can calculate the membership value of each data point with in each cluster.

It is observed that the algorithm has sensitivity to sequence of presentation of input and arbitrary selection of threshold distance at which new cluster are created.

III COMBINING LEARNING VECTOR QUANTIZATION AND MODIFIED FUZZY C-MEANS ALGORITHM

The combination of learning vector quantization and modified C-means algorithm requires execution of the learning vector quantization first, to obtain the number and initial cluster center locations. After getting these two are given to modified fuzzy C-means algorithm. In modified fuzzy C-means algorithm, the partition matrix U is not initialized randomly; instead, the initial center locations obtained from the learning vector quantization are used to get the initial partition matrix. Thus, this combination leads to faster convergence of the algorithm to obtain soft clusters of the data with their membership grades in different clusters.

The parameters to be defined before executing the algorithm are: Threshold distance ρ, and the maximum acceptable limit ε_L where to stop the modified C-means algorithm. The steps for the proposed algorithm are given in table 4.

IV RESULTS

The proposed algorithm was simulated using MATLAB 6.1, to create membership functions for the rapid Nickel-Cadmium (Ni-Cd) battery charger, developed in [10-11]. The main objective of development of this charger was to charge the batteries as quickly as possible but without any damage to these. Input- output data consisting of 561 points.

For this charger, the two input variables used to control the charging rate (Ct) are absolute temperature of the batteries (T) and its temperature gradient (dT/dt). Maximum charging current can be 8C where C is capacity of battery. In case of 2 AA battery with a capacity of 500 mAh, charging current is 500 * 8 = 4A. The input and output variables identified for rapid Ni-Cd battery charger along with their universe of discourse are listed in table 5.

Table 5 shows the results obtained after applying the new algorithm and fuzzy c-means algorithm to create membership function for input/output of rapid Ni-Cd battery charger problem with 5 tests run for each. Figure 1 shows the performance of proposed algorithm and its comparison with Fuzzy C-Means algorithm.

The results obtained showed that the convergence of the proposed algorithm using initial cluster center location and number of clusters being specified by learning vector quantization is better than the other algorithms. This shows that the algorithm of combination using both the number and initial cluster center locations provide the better rate of convergence with modified shapes of the extreme membership functions.

Table 4 Proposed Algorithms

1) Take first input pattern as first cluster center.
2) Calculate the Euclidean distance between input vector and cluster center.
3) Find the cluster closest to the input as:

$$\left\| X^p - C_k \right\| < \left\| X^p - C_j \right\|$$

Where j=1,2,……….,M and j ≠k. And M is number of allocated clusters.
4) Once the closest cluster k has been determined, the distance $\| X^p - C_k \|$ must be tested against the threshold distance ρ as

i $\| X^p - C_k \| < \rho$ the input pattern assigned n^{th} cluster.
ii $\| X^p - C_k \| > \rho$ a new cluster is allocated to p

5) Update the cluster center using eq^n(10)
6) Go to step 2 with new input pattern. If no new input pattern then go to Step 7.
After getting the number of clusters and initial cluster center locations from above steps
7) Calculate the partition matrix using eq^n(8).
8) Update the cluster centers using eq^n(7).
9) If $\left\| U^{(r+1)} - U^{(r)} \right\| \le \varepsilon_L$, stop; otherwise set r = r+1 and return to step 7.
10) Find the cluster centers v_1 and v_2, where v_1 is the cluster center with minimum distance from first data point and v_2 is the cluster center with maximum distance from first data point.
11) For v_1:
If μ_{11}=0.0 then add a cluster in the beginning, let it be called cluster number "0" such that
$\mu_{0k} = 1 - \mu_{1k}$ for $x_k < v_1$;
and the number of clusters are given as z=c+1
else $\mu_{0k} = 1$ for $x_k < v_1$
For v_2:
If μ_{cn}=0.0 then add one more cluster at the end, let it be called cluster "z" such that
$\mu_{zk} = 1 - \mu_{ck}$ for $x_k > v_2$;
and the number of clusters are given as z=c+1
else $\mu_{zk} = 1$ for $x_k > v_2$
Where k = 1,2,……..,n

Table 5 Input and Output variables for rapid Ni-Cd battery charger along with their universe of discourse

INPUT VARIABLES	**MINIMUM VALUE**	**MAXIMUM VALUE**
Temperature (T)[^{0}C]	0	50
Temperature Gradient (dT/dt)[^{0}C/10sec]	0	1
OUTPUT VARIABLE		
Charging Rate (Ct)[A]	0	8C

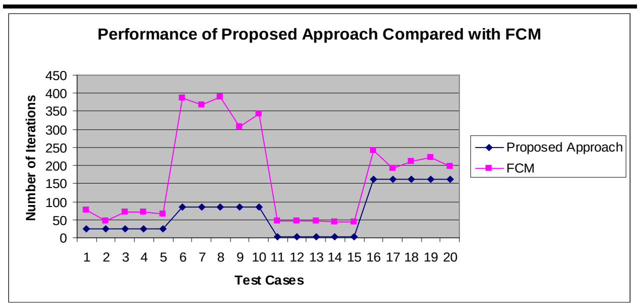

Fig. 1 Performance of proposed approach

Table 6 Comparison of Fuzzy C-Means Algorithm and the Proposed Algorithm

Data Set	***Threshold Value***	***No. of Clusters***	***Acceptable limit***	***Test Run***	***No. of Iterations***	
					By New Algorithm	***By FCM***
Temperature (T)	10	3	1.0e-025	1	24	78
				2	24	48
				3	24	70
				4	24	72
				5	24	67
	5	6	1.0e-025	1	85	387
				2	85	368
				3	85	391
				4	85	308
				5	85	344
Temperature Gradient (dT/dt)	0.3	2	1.0e-0.25	1	2	47
				2	2	46
				3	2	48
				4	2	44
				5	2	45
Charging Current (Ct)	0.9	5	1.0e-0.25	1	162	241
				2	162	192
				3	162	210
				4	162	223
				5	162	198

V CONCLUSION

From the above experiment it is seen that the proposed clustering algorithm does not require specifying the number of clusters beforehand and randomly initializing the cluster partition matrix. The learning vector quantization provides both of these and then the modified C-Means algorithm automatically gives the optimum solution. The membership functions generated using the proposed algorithm represent the real time conditions more efficiently and the control actions generated using these clusters provide a smooth variation at parameter extremes. The performance of the proposed algorithm is better in terms of convergence also.

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