Volume 1, No. 3, Sept-Oct 2010
International Journal of Advanced Research in Computer Science
RESEARCH PAPER
Available Online at www.ijarcs.info

ISSN No. 0976-5697

Nonlinear Observer Design for Nonlinear Pendulum Systems

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Abstract: This paper investigates the nonlinear observer design for nonlinear pendulum systems. Explicitly, Sundarapandian's theorem (2002) for observer design for nonlinear systems is used to solve the problem of local exponential observer design for nonlinear pendulum systems. In this paper, we derive results for exponential observer design for pendulum systems for three cases, viz. (a) no damping, (b) linear damping and (c) quadratic damping. Numerical examples and simulations of nonlinear observer design for nonlinear pendulum systems are shown to illustrate the results and validate the proposed observer design for nonlinear pendulum systems.

Keywords: Nonlinear Pendulum, Observer Design, Nonlinear Observers, Exponential Observers, Stability, Nonlinear Systems.

I. INTRODUCTION

In the control systems design, it is often necessary to construct estimates of state variables, which are not available for direct measurement. In such cases, the state vector of the control system can be approximately reconstructed by building an observer which is driven by the available outputs and inputs of the original control system. Local observer design for nonlinear control systems is one of the central problems in the control systems literature.

The problem of designing observers for linear control systems was first introduced by Luenberger ([1], 1966) and that for nonlinear control systems was proposed by Thau ([2], 1973). Over the past three decades, significant attention has been paid in the control systems literature to the construction of observers for nonlinear control systems.

A necessary condition for the existence of an exponential observer for nonlinear control systems was obtained by Xia and Gao ([3], 1988). Explicitly, in [3], Xia and Gao showed that an exponential observer exists for the nonlinear system only if the linearization of the nonlinear system is detectable.

On the other hand, sufficient conditions for nonlinear observers have been obtained in the control systems literature from an impressive variety of points of view. Kou, Elliott and Tarn ([4], 1975) obtained conditions for the existence of exponential observers using Lyapunov-like method. In ([5]-[10]), suitable coordinate transformations were found under which a nonlinear control system is transferred into a canonical form, where the observer design is carried out. In [11], Kazantzis and Kravaris obtained results on nonlinear observer design using Lyapunov auxiliary theorem. In ([12]-[13]), Tsinias derived sufficient Lyapunov-like conditions for the existence of asymptotic observers for nonlinear systems. A harmonic analysis approach was proposed by Celle *et al.* ([14], 1989) for the synthesis of nonlinear observers.

Necessary and sufficient conditions for the existence of local exponential observers for nonlinear control systems were obtained using differential geometric techniques by Sundarapandian ([15], 2002). Krener and Kang ([16], 2003) introduced a new method for the design of observers for nonlinear systems using backstepping.

In this paper, we shall use Sundarapandian's theorem (2002) for observer design for nonlinear systems to solve the problem of designing observers for nonlinear pendulum systems. The observer design for pendulum systems is very important in applications because pendulum systems are classical examples of stable systems widely studied in the literature.

This paper is organized as follows. Section II reviews the definition of nonlinear observers and the results of observability and observers. Section III details the design of nonlinear observers for pendulum systems for the three cases, *viz.* (a) no damping, (b) linear damping and (c) quadratic damping. Numerical examples and simulations of nonlinear observer design for pendulum systems are also contained in this section. Finally, Section IV provides the conclusions of this paper.

II. REVIEW OF OBSERVERS FOR NONLINEAR SYSTEMS

By the concept of a *state observer* or *state estimator* for a nonlinear system, it is meant that from the observation of certain states of the system considered as outputs or indicators, it is desired to estimate the state of the whole system as a function of time. Mathematically, observers for nonlinear systems are defined as follows.

Consider the nonlinear system described by

$$\dot{x} = f(x) \tag{1a}$$

$$y = h(x) \tag{1b}$$

where $x \in R^n$ is the state and $y \in R^p$ the output. It is assumed that $f : R^n \to R^n$, $h : R^n \to R^p$ are C^1 maps and for some $x^* \in R^n$, the following hold:

$$f(x^*) = 0, \; h(x^*) = 0.$$

Note that the solutions x^* of the equation $f(x) = 0$ are called the *equilibrium points* of (1a).

Definition 1. The nonlinear system (1) is called **locally observable** at the equilibrium x^* over a given time interval $[0, T]$, if there exists $\varepsilon > 0$ such that for any two different solutions $x(t)$ and $\overline{x}(t)$ of the system (1a) with

$$| x(t) - x^* | < \varepsilon \text{ and } | \overline{x}(t) - x^* | < \varepsilon \text{ for } t \in [0, T],$$

the observed functions $h \circ x$ and $h \circ \overline{x}$ are different, *i.e.* there exists some $\tau \in [0, T]$ such that

$$(h \circ x)(\tau) \neq (h \circ \overline{x})(\tau). \quad \blacksquare$$

For the formulation of a sufficient condition for local observability of the nonlinear system (1), consider the linearization of (1) at the equilibrium x^* given by

$$\dot{x} = Ax \tag{2a}$$

$$y = Cx \tag{2b}$$

where

$$A = \left[\frac{\partial f}{\partial x} \right]_{x = x^*} \quad \text{and} \quad C = \left[\frac{\partial h}{\partial x} \right]_{x = x^*}.$$

Theorem 1. (Lee and Markus, [17], 1971)

If the observability matrix for the linear system (2) given by

$$O(C, A) = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{bmatrix}$$

has rank n, then the nonlinear system (1) is locally observable at x^*. $\blacksquare$

Definition 2. (**Hurwitz Matrices**)

An $n \times n$ matrix A is called **Hurwitz** if all eigenvalues of A have negative real parts. $\blacksquare$

Next, the definition of nonlinear observers for the given nonlinear system (1) is given. Basically, an observer for a nonlinear system is a state estimator.

Definition 3. (Sundarapandian, [15], 2002)

A C^1 dynamical system described by

$$\dot{z} = g(z, y), \qquad (z \in R^n) \tag{3}$$

is a local asymptotic (respectively, exponential) observer for the nonlinear system (1) if the composite system (1) and (3) satisfies the following two requirements:

(i) If $z(0) = x(0)$, then $z(t) = x(t), \quad \forall t \geq 0.$

(ii) There exists a neighbourhood V of the equilibrium x^* of R^n such that for all $z(0), x(0) \in V$, the error $e(t) = z(t) - x(t)$ decays asymptotically (resp., exponentially) to zero. $\blacksquare$

Theorem 2. (Sundarapandian, [15], 2002)

Suppose that the nonlinear system (1) is Lyapunov stable at the equilibrium x^* and that there exists a matrix K such that $A - KC$ is Hurwitz. Then the dynamical system defined by

$$\dot{z} = f(z) + K\left[y - h(z) \right] \tag{4}$$

is a local exponential observer for the nonlinear system (1). $\blacksquare$

Remark 1. If the estimation error e is defined as

$$e = z - x,$$

then the estimation error is governed by the dynamics

$$\dot{e} = f(x + e) - f(x) - K\left[h(x + e) - h(x) \right] \tag{5}$$

Linearizing the error dynamics (5) at x^*, we obtain the linear system

$$\dot{e} = Ee, \qquad \text{where } E = A - KC. \tag{6}$$

If (C, A) is observable, ie. if the observability matrix $O(C, A)$ has full rank, then the eigenvalues of $E = A - KC$ can be arbitrarily assigned in the complex plane. Since the linearization of the error dynamics (5) is governed by the system matrix $E = A - KC$, it follows that when (C, A) is observable, then a local exponential observer of the form (4) can be always found so that the transient response of the error decays quickly with any desired speed of convergence. $\blacksquare$

III. NONLINEAR OBSERVERS FOR THE NONLINEAR PENDULUM SYSTEMS

In this section, we discuss the nonlinear observer design for nonlinear pendulum systems.

Consider the classical simple pendulum model ([18]-[19]), which is illustrated in Figure 1.

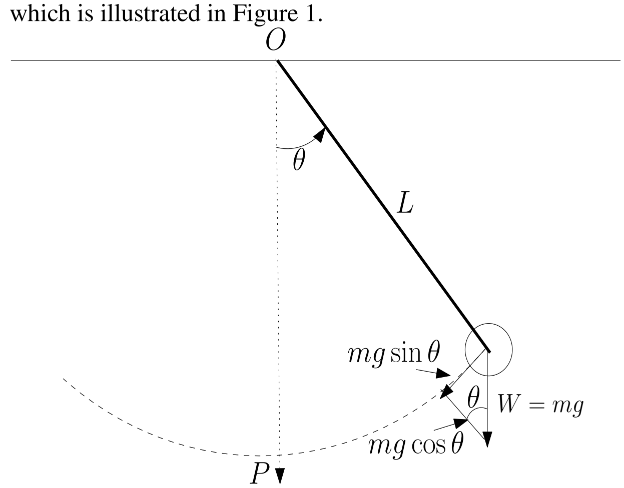

Figure 1. Simple Pendulum

Let m denote the mass of the bob and L the length of the rod. Let θ denote the angle suspended by the rod and the vertical axis through the pivot point. The pendulum is free to swing in the vertical plane and the bob of the pendulum moves in a circle of radius L.

Using Newton's second law of motion, the equation of motion of the pendulum in the tangential direction can be easily obtained as

$$mL\ddot{\theta} = -mg \sin\theta - L\, Q(\dot{\theta}) \tag{7}$$

where $Q(\dot{\theta})$ is the damping force.

Using the phase variables $x_1 = \theta, x_2 = \dot{\theta}$, the pendulum equation (2) can be written in state-space form as

$$\begin{aligned}\dot{x}_1 &= x_2 \\ \dot{x}_2 &= -\frac{g}{L}\sin x_1 - \frac{1}{m}Q(x_2)\end{aligned} \tag{8}$$

We also suppose that the angular displacement θ is available for measurement. Thus, we consider the output y as

$$y = x_1 \tag{9}$$

In this section, we shall use Sundarapandian's result (Theorem 2) for solving the nonlinear observer design problem for the pendulum model described in (8)-(9) for the following cases:

(a) No damping, *i.e.* $Q(x_2) = 0$.

(b) Linear damping, *i.e.* $Q(x_2) = kx_2$.

(c) Quadratic damping, *i.e.* $Q(x_2) = k\, x_2 \mid x_2 \mid$.

[Here, k is the damping constant.]

A. **Observer Design for Pendulums with No Damping**

In this case, the pendulum model is described by

$$\begin{aligned}\dot{x}_1 &= x_2 \\ \dot{x}_2 &= -\frac{g}{L}\sin x_1 \\ y &= x_1\end{aligned} \tag{10}$$

The system dynamics in (10) has a Lyapunov stable equilibrium at $(x_1, x_2) = (0,0)$. Also, the linearization matrices for the system (10) are given by

$$C = \begin{bmatrix} 1 & 0 \end{bmatrix} \quad \text{and} \quad A = \begin{bmatrix} 0 & 1 \\ -\frac{g}{L} & 0 \end{bmatrix}.$$

The observability matrix for this system is

$$\boldsymbol{O}(C, A) = \begin{bmatrix} C \\ CA \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

which has full rank. Thus, by Theorem 1, we obtain the following result for the pendulum system with no damping.

Theorem 3. The pendulum system with no damping described by (10) is locally observable near $(x_1, x_2) = (0,0)$. ■

Next, we note that the pendulum system (10) has a Lyapunov stable equilibrium at $(x_1, x_2) = (0,0)$.

Thus, by Sundarapandian's result (Theorem 2), we derive the following result which gives a formula for the construction of nonlinear exponential observer for the pendulum system with no damping.

Theorem 4. The pendulum system with no damping described by (10) has a local exponential observer given by

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} = \begin{bmatrix} z_2 \\ -\frac{g}{L}\sin z_1 \end{bmatrix} + K\left[y - z_1 \right] \tag{11}$$

where $K = \begin{bmatrix} k_1 \\ k_2 \end{bmatrix}$ is a matrix chosen so that $A - KC$ is Hurwitz. Since (C, A) is observable, a gain matrix K can be found so that the error matrix

$$E = A - KC$$

has arbitrarily assigned set of eigenvalues with negative real parts. ■

Example 1. Consider the pendulum model (10) with $L = 2g$. In this case, the plant equations (10) simplify to

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} x_2 \\ -0.5\sin x_1 \end{bmatrix} \tag{12}$$
$$y = x_1$$

The system linearization matrices are

$$C = \begin{bmatrix} 1 & 0 \end{bmatrix} \quad \text{and} \quad A = \begin{bmatrix} 0 & 1 \\ -0.5 & 0 \end{bmatrix}.$$

Note that the pair (C, A) is observable.

Using the Ackermann formula for the observability gain matrix ([20], p.822), we can choose K so that the error matrix $E = A - KC$ has the eigenvalues $\{-2, -2\}$.

A simple calculation using MATLAB yields

$$K = \begin{bmatrix} 4.0 \\ 3.5 \end{bmatrix}.$$

By Theorem 4, a local exponential observer for the pendulum plant (12) near the equilibrium $(x_1, x_2) = (0,0)$ is given by

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} = \begin{bmatrix} z_2 \\ -0.5\sin z_1 \end{bmatrix} + \begin{bmatrix} 4.0 \\ 3.5 \end{bmatrix}\left[y - z_1 \right]. \tag{13}$$

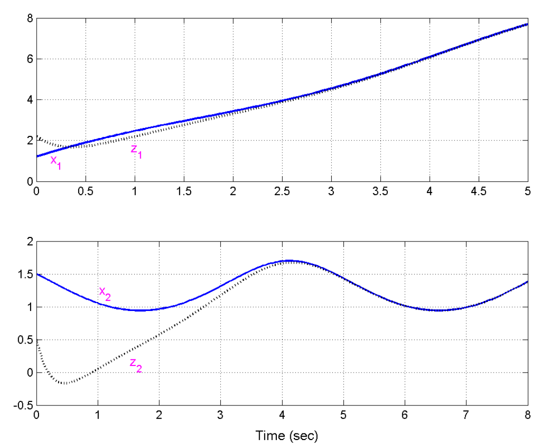

Figure 2. Exponential Observer the Pendulum (12)

Figure 2 depicts the exponential convergence of the observer states z_1 and z_2 of the system (13) to the states x_1 and x_2 of the plant (12). For simulation, we have taken the initial conditions as

$$x(0) = \begin{bmatrix} 1.2 \\ 1.5 \end{bmatrix} \quad \text{and} \quad z(0) = \begin{bmatrix} 2.2 \\ 0.5 \end{bmatrix}.$$

B. **Observer Design for Pendulums with Linear Damping**

In this case, the pendulum model is described by

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -\frac{g}{L}\sin x_1 - \frac{k}{m}x_2 \\ y &= x_1 \end{aligned} \tag{14}$$

The system dynamics in (14) has an asymptotically stable equilibrium at $(x_1, x_2) = (0, 0)$. Also, the linearization matrices for the system (14) are given by

$$C = \begin{bmatrix} 1 & 0 \end{bmatrix} \quad \text{and} \quad A = \begin{bmatrix} 0 & 1 \\ -\frac{g}{L} & -\frac{k}{m} \end{bmatrix}.$$

The observability matrix for this system is

$$\boldsymbol{O}(C, A) = \begin{bmatrix} C \\ CA \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

which has full rank. Thus, by Theorem 1, we obtain the following result for the pendulum system with linear damping.

Theorem 5. The pendulum system with linear damping described by (14) is locally observable near $(x_1, x_2) = (0, 0)$. ■

Next, we note that the pendulum system (14) has an asymptotically stable equilibrium at $(x_1, x_2) = (0, 0)$. Thus, by Sundarapandian's result (Theorem 2), we derive the following result which gives a formula for the construction of nonlinear exponential observer for the pendulum system with linear damping.

Theorem 6. The pendulum system with linear damping described by (14) has a local exponential observer given by

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} = \begin{bmatrix} z_2 \\ -\frac{g}{L}\sin z_1 - \frac{k}{m} z_2 \end{bmatrix} + K\left[y - z_1 \right] \tag{15}$$

where $K = \begin{bmatrix} k_1 \\ k_2 \end{bmatrix}$ is a matrix chosen so that $A - KC$ is Hurwitz. Since (C, A) is observable, a gain matrix K can be found so that the error matrix

$$E = A - KC$$

has arbitrarily assigned set of eigenvalues with negative real parts. ■

Example 2. Consider the pendulum model (14) with $L = 2g$, $k = 0.2$ and $m = 1$.

In this case, the plant equations (14) simplify to

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} x_2 \\ -0.5\sin x_1 - 0.2 x_2 \end{bmatrix} \tag{16}$$
$$y = x_1$$

The system linearization matrices are

$$C = \begin{bmatrix} 1 & 0 \end{bmatrix} \quad \text{and} \quad A = \begin{bmatrix} 0 & 1 \\ -0.5 & -0.2 \end{bmatrix}.$$

Note that the pair (C, A) is observable.

Using the Ackermann formula for the observability gain matrix ([20], p.822), we can choose K so that the error matrix $E = A - KC$ has the eigenvalues $\{-2, -2\}$.

A simple calculation using MATLAB yields

$$K = \begin{bmatrix} 3.80 \\ 2.74 \end{bmatrix}.$$

By Theorem 6, a local exponential observer for the pendulum plant (16) near the equilibrium $(x_1, x_2) = (0, 0)$ is given by

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} = \begin{bmatrix} z_2 \\ -0.5\sin z_1 - 0.2 z_2 \end{bmatrix} + K\left[y - z_1 \right]. \tag{17}$$

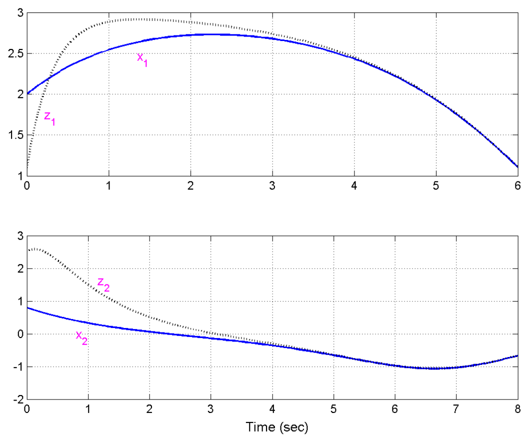

Figure 3. Exponential Observer the Pendulum (16)

Figure 3 depicts the exponential convergence of the observer states z_1 and z_2 of the system (17) to the states x_1 and x_2 of the plant (16). For simulation, we have taken the initial conditions as

$$x(0) = \begin{bmatrix} 2.0 \\ 0.8 \end{bmatrix} \quad \text{and} \quad z(0) = \begin{bmatrix} 1.1 \\ 2.5 \end{bmatrix}.$$

C. Observer Design for Pendulums with Quadratic Damping

In this case, the pendulum model is described by

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -\frac{g}{L}\sin x_1 - \frac{k}{m}x_2\left|x_2\right| \\ y &= x_1 \end{aligned} \tag{18}$$

The system dynamics in (18) has an asymptotically stable equilibrium at $(x_1, x_2) = (0, 0)$. Also, the linearization matrices for the system (18) are given by

$$C = \begin{bmatrix} 1 & 0 \end{bmatrix} \quad \text{and} \quad A = \begin{bmatrix} 0 & 1 \\ -\frac{g}{L} & 0 \end{bmatrix}.$$

The observability matrix for this system is

$$\boldsymbol{O}(C, A) = \begin{bmatrix} C \\ CA \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

which has full rank. Thus, by Theorem 1, we obtain the following result for the pendulum system with quadratic damping.

Theorem 7. The pendulum system with linear damping described by (18) is locally observable near $(x_1, x_2) = (0,0)$. ■

Next, we note that the pendulum system (18) has an asymptotically stable equilibrium at $(x_1, x_2) = (0,0)$. Thus, by Sundarapandian's result (Theorem 2), we derive the following result which gives a formula for the construction of nonlinear exponential observer for the pendulum system with quadratic damping.

Theorem 8. The pendulum system with quadratic damping described by (18) has a local exponential observer given by

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} = \begin{bmatrix} z_2 \\ -\dfrac{g}{L}\sin z_1 - \dfrac{k}{m} z_2 \mid z_2 \mid \end{bmatrix} + K\left[y - z_1\right] \qquad (19)$$

where $K = \begin{bmatrix} k_1 \\ k_2 \end{bmatrix}$ is a matrix chosen so that $A - KC$ is Hurwitz. Since (C, A) is observable, a gain matrix K can be found so that the error matrix

$$E = A - KC$$

has arbitrarily assigned set of eigenvalues with negative real parts. ■

Example 3. Consider the pendulum model (18) with $L = 2g$, $k = 0.2$ and $m = 1$.

In this case, the plant equations (14) simplify to

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} x_2 \\ -0.5\sin x_1 - 0.2 x_2 \mid x_2 \mid \end{bmatrix} \qquad (20)$$

$$y = x_1$$

The system linearization matrices are

$$C = \begin{bmatrix} 1 & 0 \end{bmatrix} \text{ and } A = \begin{bmatrix} 0 & 1 \\ -0.5 & 0 \end{bmatrix}.$$

Note that the pair (C, A) is observable.

Using the Ackermann formula for the observability gain matrix ([20], p.822), we can choose K so that the error matrix $E = A - KC$ has the eigenvalues $\{-1, -1\}$. A simple calculation using MATLAB yields

$$K = \begin{bmatrix} 2.0 \\ 0.5 \end{bmatrix}.$$

By Theorem 8, a local exponential observer for the pendulum plant (12) near the equilibrium $(x_1, x_2) = (0,0)$ is given by

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} = \begin{bmatrix} z_2 \\ -0.5\sin z_1 - 0.2 z_2 \mid z_2 \mid \end{bmatrix} + K\left[y - z_1\right]. \qquad (21)$$

Figure 4 depicts the exponential convergence of the observer states z_1 and z_2 of the system (21) to the states x_1 and x_2 of the plant (20).

For simulation, we have taken the initial conditions as

$$x(0) = \begin{bmatrix} 0.2 \\ 2.6 \end{bmatrix} \text{ and } z(0) = \begin{bmatrix} 2.5 \\ 0.9 \end{bmatrix}.$$

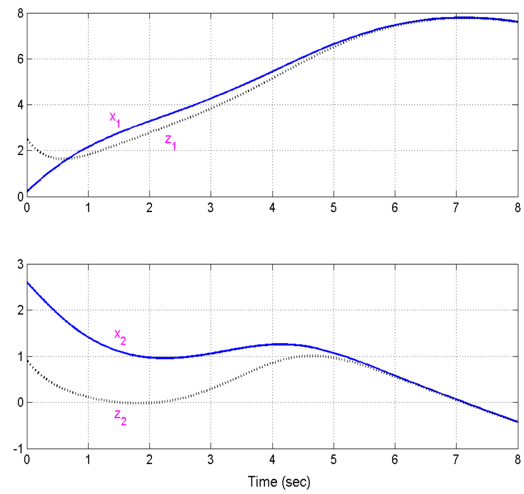

Figure 4. Exponential Observer the Pendulum (20)

IV. Conclusions

For many real problems of science and engineering, pendulum systems are classical examples of stable nonlinear systems. In this paper, methodology based on Sundarapandian's theorem (2002) for nonlinear observer design is suggested for the design of exponential observers for pendulum systems for the three important cases, *viz.* (a) no damping, (b) linear damping and (c) quadratic damping. Numerical examples have been worked out in detail to illustrate the construction of local exponential observers for the pendulum systems for all the three cases of damping.

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