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Group of Continuous Time Recurrent Neural Networks

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Abstract: This paper explain a different type process for absolute exponential stability (AEST) of a group of continuous time recurrent neural networks with locally Lipschitz continuous and monotone non decreasing activation function. The result extends and improves the existing the analysis of absolute stability (ABST) and absolute exponential stability (AEST).

Keywords: Neural network; absolute exponential stability; global exponentially stability.

I. INTRODUCTION

The main force lies in information that an absolute stability (ABST) or absolutely exponential stability (AEST) neural network can meet globally asymptotically to a single equilibrium with any activation function in a proper given group and any other network parameters. This type neural network property is used for solving many optimization problems. The optimization neural networks are devoid of the hollow suboptimal answer for any variety of the activation function.

A quantitative analysis for globally exponentially stability (GES) [1]-[2] known the union performances of neural network. This method appears at a solution with a specified accuracy.

ABST or AEST discover for continuous time recurrent neural networks. There researchers have to restrain the connection weight matrix and activation function of neural network. The group of sigmoid activation function proved symmetric or non inhibitory lateral connection weight matrix of neural network model. The necessary and sufficient condition of this model for ABST neural network [3] and[4]. The ABST results are extended to the AEST ones in [5] and [6] respectively. In inference is raised [7]. The group of partially Lipschitz continuous and monotone non decreasing activation function for current AEST result is given is [8].

Here we describe with AEST of continuous time recurrent neural network with locally Lipschitz continuous and monotone non decreasing activation function. These type of result expand and improve the presented ABST with AEST ones in the literature

II. PERLIMINARIES

Consider group of continuous time recurrent neural network model as follows

$$\dot{x} = -D_i x + w_i\, g(y) + u\,, \qquad x(0) \tag{1}$$

$$\dot{y} = -D_j y + w_j\, f(x) + v\,, \qquad y(0) \tag{2}$$

Where

$x = (x_1, x_2 \dots \dots x_n)^T \in R^n, y = (y_1, y_2 \dots \dots y_m)^T \in R^m$ is the state vector, $D_i = dig\,(d_1, d_2 \dots \dots d_n) \in R^{nxn}$, $D_j = dig\,(d_1, d_2 \dots \dots d_m) \in R^{mxm}$, is diagonal matrix with d_{ij} >0 $w_i = [w_{ij}] \in R^{nxn}$, $w_j = [w_{ji}] \in R^m$ is connection weight matrix

$u = (u_1, u_2 \dots \dots u_n)^T \in R^n$ $v = (v_1, v_2 \dots \dots v_m)^T \in R^m$ is a input vector and,

$g(y) = (g_1(y), g_2(y) \dots \dots g_m(y))^T, f(x) = (f_1(x), f_2(x) \dots \dots f_n(x))^T$, is a nonlinear vector valued activation function from R^n to R^n and R^m to R^m .

In this paper let LL denote the Locally Lipschiz Continuous (l.l.c) and monotone non decreasing activation function that is for any $x_{j0}, y_{i0} \in$ R there exit $\varepsilon_{j0}, \delta_{i0} > 0$ and a constant $l_{j0}, k_{i0} >_0$ (i=1,2,3…n), (j=1,2,3…m) such that

$\forall \ \theta_1, \theta_2, \rho_1 \in [x_{j0} - \varepsilon_{j0}, x_{j0} + \varepsilon_{j0}], \rho_2 \in [y_{i0} - \delta_{i0}, y_{i0} + \delta_{i0}]$ and $\theta_1 \neq \rho_1, \theta_2 \neq \rho_2$

$$0 \leq \frac{g_i(\theta_2) - g_i(\rho_2)}{\theta_2 - \rho_2} \leq k_{i0} \qquad 0 \leq \frac{f_j(\theta_1) - f_j(\rho_1)}{\theta_1 - \rho_1} \leq l_j \tag{3}$$

Let PL denote the group of partially Lipschiz continuous (p.l.c) and monotone non decreasing activation function [9] that is for any $\rho_1, \rho_2 \in R$ there exist $l_j(\rho_1), k_i(\rho_2) > 0$, $(i = 1,2,3 \ldots n)$, (j=1,2,3…m) such that $\theta_1, \theta_2 \in R$ and $\theta_1 \neq \rho_1, \theta_2 \neq \rho_2$

$$0 \leq \frac{g_i(\theta_2) - g_i(\rho_2)}{\theta_2 - \rho_2} \leq k_i(\rho_2), \quad 0 \leq \frac{f_j(\theta_1) - f_j(\rho_1)}{\theta_1 - \rho_1} \leq l_j(\rho_1)$$

Let GL denote the group of globally Locally Lipschiz continuous (g.l.l.c) and monotone non decreasing activation function.

Here $GL \subset PL$ and $GL \subset LL$. As for connection between PL and LL, the presented continuous activation function in the literature being to LL but many not be PL such as $g_i(y) = y^3, f_j(x) = x^3$ and $g_i(y) = \max(y^3, 0), f_j(x) = \max(x^3, 0)$

III. MAIN RESULT

This part will be prove that $w \in M_0$ is a sufficient condition for AEST of common neural network (1)-(2) with its activation function in LL. We proved and improve the existing result ABST and AEST.

Theorem- The neural network (1)-(2) has a unique equilibrium continuous and monotone non decreasing activation function g any $u \in R^n$ and any positive diagonal matrix D if and only if $w_i, w_j \in p_0$ by [10, Leema 2]

Proof- The neural network (1)-(2) has a unique equilibrium denoted by x^*, y^* for $-w_i, -w_j \in p_0$. Here we let

$$z_1 = (z_{11}, z_{12} \ldots \ldots z_{1n})^T = x - x^*, z_2 = (z_{21}, z_{22} \ldots \ldots z_{2m})^T = y - y^*.$$

Then the neural network (!)-(2) can be changed into the following corresponding system with a unique equilibrium at z = 0.

$$\dot{z_1} = -D_i z_1 + w_i g_i(z_2), \qquad z_1(0) = x - x^*$$
$$\dot{z_2} = -D_j z_2 + w_j f_j(z_1), \qquad z_2(0) = y - y^* \qquad (3)$$

Where $f(z_1) = (f_1(z_{11}), f_2(z_{12}) \ldots \ldots f_n(z_{1n}))^T = f(z_1 + x^*) - f(x^*) \in LL$, $g(z_2) = (g_2(z_{21}), g_2(z_{22}) \ldots\ldots g(z_{2m}))^T = g(z_2 + y^*) - g(y^*) \in LL$ and $f(0) = 0, g(0) = 0$ from which we have three useful properties

Property A: There exist positive function $l_j(z_j)$ (j=1,2,3…m),and $k_i(z_i)(i = 1,2,3 \ldots n)$ such that

$$0 \leq \frac{g_i(z_i)}{z_i} \leq k_i(z_i) \quad \forall (z_i) \in R\{0\} \qquad (4)$$
$$0 \leq \frac{f_j(z_j)}{z_j} \leq l_j(z_j) \quad \forall (z_J) \in R\{0\}$$

Property B:

There exist positive constant l_{j0}, k_{i0} such that

$$0 \leq D^+ g_{i(S_2)} \leq k_{i0}$$
$$0 \leq D^+ f_{j(S_1)} \leq l_{j0} \qquad (5)$$

Where $S_1 \in [-a_j, a_j], S_2 \in [-a_i, a_i]$ where $[-a_j, a_j], [-a_i, a_i]$ Is any given bounded interval i = 1,2…n, j = 1,2…m.

Property C: There exist positive constant l_j $(j = 1,2,3 \ldots m), k_{i0}, (i = 1,2,3 \ldots n)$ such that

$$0 \leq \frac{g_i(z_i)}{z_i} \leq k_{i0} \quad \forall (z_i) \in [-a_i, a_i] - \{0\}$$
$$0 \leq \frac{f_j(z_j)}{z_j} \leq l_{j0} \quad \forall (z_J) \in [-a_j, a_j] - \{0\}$$

(6)

where $[-a_j, a_j], [-a_i, a_i]$ is any given bounded interval.

Property B as follows $f_{j(S_1)}, g_{i(S_2)}$ is l.l.c and monotone non decreasing for any $S_1^* \in [-a_j, a_j], S_2^* \in [-a_i, a_i]$ $\exists\ \varepsilon_{j0} > 0$ $\delta_{i0} > 0$ and a constant $k_{i0} > 0, l_{j0} > 0$ such that $\forall\ \theta_1, \theta_2, \rho_1 \in [S_1^* - \varepsilon_j, S_2^* + \varepsilon_j], \rho_2 \in [S_1^* - \delta_i, S_2^* + \delta_i], \theta_1 \neq \rho_1, \theta_2 \neq \rho_2$

$$\text{we have } 0 \leq \frac{g_i(\theta_2) - g_i(\rho_2)}{\theta_2 - \rho_2} \leq k_i, \quad 0 \leq \frac{f_j(\theta_1) - f_j(\rho_1)}{\theta_1 - \rho_1} \leq l_j$$

(7)

Next by contradiction we will show that there exist a positive constant k_{i0}, such that

$$0 \leq \frac{g_i(S_3) - g_i(S_4)}{S_3 - S_4} \triangleq k_i(S_3, S_4) \leq k_{i0}, \forall S_3, S_4 \in [-a_i, a_i]$$
$$0 \leq \frac{f_j(S_1) - f_j(S_2)}{S_1 - S_2} \triangleq l_j(S_1, S_2) \leq l_{j0}, \forall S_1, S_2 \in [-a_j, a_j]$$

(8)

Suppose (8) does not hold. Then we may select six sequence $\{N_j\}, \{S_{1j}\}, \{S_{2j}\}, \{N_i\}, \{S_{3i}\}, \{S_4\}$ such that $0 < N_1 < l_j(S_{11}, S_{21}) < N_2 < l_j(S_{12}, S_{22}) < \cdots N_j < l_j(S_{1j}, S_{2j}) < N_{j+1} < l_j(S_{1j+1}, S_{2j+1}) < \cdots \lim_{j \to +\infty} N_j = +\infty$ and $\lim_{j \to +\infty} l_j(S_{1j}, S_{2j}) = +\infty$ and $0 < N_1 < k_i(S_{31}, S_{41}) < N_2 < k_i(S_{32}, S_{42}) < \cdots N_i < k_i(S_{3i}, S_{4i}) < N_{i+1} < k_i(S_{3i+1}, S_{4i+1}) < \cdots \lim_{i \to +\infty} N_i = +\infty$ and $\lim_{i + \infty} k_i(S_{3i}, S_{4i}) = +\infty$ where each $S_{1j}, S_{2j} \in [-a_j, a_j]$ and $S_{3i}, S_{4i} \in [-a_i, a_i]$ and $S_{1j} \neq S_{2j}, S_{3i} \neq S_{4i}$ since each $S_{1j}, S_{2j} \in [-a_j, a_j]$ and $S_{3i}, S_{4i} \in [-a_i, a_i]$ there must exist two subsequence of each part $\{S_{1nj}\} \subset \{S_{1j}\}, \{S_{2nj}\} \subset \{S_{2j}\}$, $\{S_{3mi}\} \subset \{S_{3i}\}$, $\{S_{4mi}\} \subset \{S_{4i}\}$ such that

$$\lim_{j \to +\infty} S_{1nj} = S_1^* \in [-a_j, a_j]$$
$$\lim_{j \to +\infty} S_{2nj} = S_2^* \in [-a_j, a_j]$$

$\lim_{i \to +\infty} S_{3mi} = S_3^* \in [-a_i, a_i], \lim_{i \to +\infty} S_{4mi} = S_4^* \in [-a_i, a_i]$ so $\lim_{j \to +\infty} l_j(S_{1nj}, S_{2nj}) = l_j(S_1^*, S_2^*) = +\infty$ $\lim_{i \to +\infty} k_i(S_{3mi}, S_{4mi}) = k_i(S_3^*, S_4^*) = +\infty$ if $S_1^* = S_2^*, S_3^* = S_4^*$ then there exist some integer i^*, j^* such that $S_{1nj}, S_{2nj} \in [S_1^* - \varepsilon_j, S_2^* + \varepsilon_j]$, $S_{3mi}, S_{4mi} \in [S_3^* - \delta_i, S_4^* + \delta_i]$ when $i^* \geq i^*$, $j^* \geq j^*$. in view of (7) we can derived $\forall i^* \geq i^*, j^* \geq j^*$

$$0 \leq \frac{g_i(S_{3mi}) - g_i(S_{4mi})}{S_{3mi} - S_{4mi}} \triangleq k_i(S_{3mi}, S_{4mi}) \leq k_i$$
$$0 \leq \frac{f_j(S_{1nj}) - f_j(S_{2nj})}{S_{1nj} - S_{2nj}} \triangleq l_j(S_{1nj}, S_{2nj}) \leq l_j$$

Which contradicts $l_j(S_{1nj}, S_{2nj}) \to \infty$ as $j \to \infty$, and $k_i(S_{3mi}, S_{4mi}) \to \infty$ as $i \to \infty$, $S_1^* \neq S_2^*$ $S_3^* \neq S_4^*$. Based on continuity of function $f_{j(.)}$ on $[-a_j, a_j]$, $g_{i(.)}$ on $[-a_i, a_i]$,

clearly $\frac{g_i(s_3^*)-g_i(s_4^*)}{(S_3^*-S_4^*)}$, $\frac{f_j(s_1^*)-f_j(s_2^*)}{(S_1^*-S_2^*)}$ is finite; that is $l_j(S_1^*, S_2^*)$, $k_i(S_3^*, S_4^*)$ is finite. Contradicting $l_j(S_1^*, S_2^*) = +\infty$, $k_i(S_3^*, S_4^*) = +\infty$, therefore there exist a positiove

constant l_{j0}, k_{i0} such that (8) is true consequently property B is true by nothing that (8) is equivalent to (5).

Lemma 1 [11, Th.2]: Let $g \in LL$ if $w \in D_0$, then the neural network (1)-(2) has a unique GAS equilibrium for x* and $u \in R^n$

Theorem : If $w_i, w_j \in D_0$ then the neural network (1)-(2) with its activation function in the class of LL is AEST.

Proof : We know that $w \in D_0 \subset I_0 \subset P_0$, we consider the equal model (3) only. In view of lemma 1, model (3)is GAS at $z_1, z_2 = 0$, and consequently there exist constant $a_i > 0, a_j > 0$ such that $|z_i(t)| \le a_i, |z_j(t)| \le a_j \; \forall t \ge 0$, i=1,2……n, j=1,2……m, According to properties B and we have (5) and (6) since $w_i, w_j \in D_0$ there exist a positive diagonal matrix $p_i = dig(p_1, p_2 \ldots\ldots p_n)$ $p_j = dig(p_1, p_2 \ldots\ldots p_m)$,

such that $[p_i w_i]^s \le 0$, $[p_j w_j]^s \le 0$. Let $z = z_1 + z_2$ then $\dot{z} = \dot{z}_1 + \dot{z}_2$ we define a differentiable function

$$v(z) = v(\dot{z}_1 + \dot{z}_2) = \sum_{i=1}^{n} \int_0^{z_i} p_i\, g_i(s)\, ds + \sum_{j=1}^{m} \int_0^{z_j} p_j\, f_j(s)\, ds \tag{9}$$

obviously $v(z) \ge 0$ by [12], calculating the time derivative v(z) beside the positive half trajectory of (3) give up.

$$\frac{dv(z)}{dt} = -G^T(z_2)p_i\,[-D_i(z_1) + w_i(z_2)] + F^T(z_1)p_j\,[-D_j(z_2) + w_j F(z_1)]$$

$$= -G^T(z_2)p_i D_i(z_1) + G^T(z_2)p_i w_i(z_2) - F^T(z_1)p_j D_j(z_2) + F^T(z_1)p_j w_j F(z_1)$$

$$= -G^T(z_2)p_i D_i z_1 + G^T(z_2)[p_i w_i]^s G(z_2) - F^T(z_1)p_j D_j z_2 + F^T(z_1)[p_j w_j]^s F(z_1)$$

$$\le -G^T(z_2)p_i D_i z_1 \le -\dim\, G^T(z_2)p_i(z_2)$$

$$-F^T(z_1)p_j D_j z_2 \le -\dim F^T(z_1)\, p_j(z_1)$$

$$= -\dim \sum_{i=1}^{n} p_i\, g_i(z_2)z_i - \dim \sum_{j=1}^{m} p_j\, f_j(z_1)z_j$$

$$\le -\dim \sum_{i=1}^{n} p_i \int_0^{z_i} g_i(s)\, ds - \dim \sum_{j=1}^{m} p_j \int_0^{z_j} f_j(s)\, ds$$

$$= -\dim v(z_1) - \dim v(z_2)$$

And so $v(z) \le v(z(0)\exp(-\dim t)$ for all $t \ge 0$ by (9)we have

$$v(z(0)) = \sum_{i=1}^{n} \int_0^{z_i(0)} p_i\, g_i(s)\, ds + \sum_{j=1}^{m} \int_0^{z_j(0)} p_j\, f_j(s)\, ds$$

$$\le \sum_{i=1}^{n} p_i\, g_i(z_i(0))z_i(0) + \sum_{j=1}^{m} p_j\, f_j(z_j(0))z_j(0)$$

$$\le \textstyle\sum_{i=1}^{n} p_i\, k_{i0}\, z_i^2(0) + \sum_{j=1}^{m} p_j\, l_{j0}\, z_j^2(0) \qquad \text{[from [6]]}$$

$$\le \|z(0)\|^2 \sum_{i=1}^{n} p_i\, k_{i0} + \|z(0)\|^2 \sum_{j=1}^{m} p_j\, l_{j0} \tag{10}$$

Define continuous function

$$E_i(\alpha) = \textstyle\int_0^{u} g_i(s)\, ds - g_i^{\,2}(\alpha)/(4k_{i0})$$

$$E_j(\beta) = \textstyle\int_0^{\beta} f_j(s)\, ds - f_j^{\,2}(\beta)/(4l_{i0})$$

Where

$\alpha \in [-a_i, a_i]$, $\beta \in [-a_j, a_j]$, $i = 1,2 \ldots\ldots n$, $j = 1,2 \ldots\ldots m$

Then terms of (6) we readily obtain

$$D^+E_i(\alpha) = g_i(\alpha)\left(1 - \frac{1}{2k_{i0}} D^+ g_i(\alpha)\right)\begin{cases} \ge 0 & a_i \ge \alpha > 0 \\ \le 0 & -a_i \le \alpha < 0 \\ = 0 & \alpha = 0 \end{cases}$$

$$D^+E_j(\beta) = f_j(\alpha)\left(1 - \frac{1}{2l_{j0}} D^+ f_j(\beta)\right)\begin{cases} \ge 0 & a_j \ge \beta > 0 \\ \le 0 & -a_j \le \beta < 0 \\ = 0 & \beta = 0 \end{cases}$$

Implying $E_i(\alpha) \ge E_i(0) = 0$ and $E_j(\beta) \ge E_j(0) = 0$ and for all $\alpha \in [-a_i, a_i]$, $\beta \in [-a_j, a_j]$ thus $\int_0^{\alpha} g_i(s)\, ds \ge g_i^{\,2}(\alpha)/(4k_{i0})$, $\int_0^{\beta} f_j(s)\, ds \ge f_j^{\,2}(\beta)/(4l_{j0})$ for all $i = 1,2 \ldots\ldots n$, $j = 1,2 \ldots\ldots m$, $\alpha \in [-a_i, a_i]$, $\beta \in [-a_j, a_j]$ and

$$v(z) \ge \int_0^{z_i} p_i\, g_i(s)\, ds + \int_0^{z_j} p_j\, f_j(s)\, ds \ge \frac{p_i}{4k_{i0}} g_i^{\,2}(z_i)c + \frac{p_j}{4l_{j0}} f_j^{\,2}(z_j)\, c \tag{11}$$

Based on (10) it is seen that

$$|g_i(z_i)| \le \sqrt{\frac{4k_{i0}}{p_i}\alpha(z_1)} \le \|z(0)\| \sqrt{\frac{4k_{i0}}{p_i}\textstyle\sum_{i=1}^{n} p_i\, k_{i0}} \; \exp\left(\frac{-d_{min}t}{2}\right) \;\; \forall t \ge x$$

$$|f_j(z_j)| \le \sqrt{\frac{4l_{j0}}{p_j}\beta(z_2)} \le \|z(0)\| \sqrt{\frac{4l_{j0}}{p_j}\textstyle\sum_{j=1}^{m} p_j\, l_{j0}} \; \exp\left(\frac{-d_{min}t}{2}\right) \;\; \forall t \ge x$$

So from (3) we have

$$D^+|z_i(t)| \le -d_i|z_i(t)| + \sum_{i=1}^{n}|w_{ij}||g_i(z_i(t))| \le -d_i|z_i(t)| + \|z(0)\| \sum_{i=1}^{n}|w_{ij}| \sqrt{\frac{4k_{i0}}{p_i}\sum_{k_1=1}^{n} p_{k_1}\, k_{k_{10}}} \; \exp\left(\frac{-d_{min}\,t}{2}\right)$$

$$D^+|z_j(t)| \le -d_j|z_j(t)| + \sum_{j=1}^{m}|w_{ji}||f_j(z_j(t))| \le -d_j|z_j(t)|$$

$$+\|z(0)\|\sum_{j=1}^{m}|w_{ji}|\sqrt{\frac{4l_{j0}}{p_j}\sum_{k_2=1}^{m}p_{k_2}\,k_{k_{20}}}\ \exp\left(\frac{-d_{min}\,t}{2}\right)$$

$i = 1,2 \ldots \ldots n, j = 1,2 \ldots \ldots m$ based on lemma $3|z_i(t)| = (1,2 \ldots \ldots n), (j = 1,2 \ldots \ldots m)$ isGES at the convergence rate of at least $\dim/2$. Here $z = 0$ is the GES equilibrium of model 3 that is x*, y* is a GES equilibrium of neural network (1). These resources that the neural network (1) with its activation function is the class of LL is AEST the proof is complete.

IV. CONCLUSION

We have shown that Bidirectional associative neural network for group of continuous time recurrent neural network has a unique equilibrium point. Under certain on weight matrix this network with its activation functions on his AEST. This obtained AEST result actually improves for group of continuous time recurrent neural network literature.

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