



## Elasticities and Index Analysis of Usual Internet Traffic Share Problem

Diwakar Shukla

Department of Mathematics and Statistics, Sagar University  
Sagar, M.P. India. Associate Member, DST Centre for  
Interdisciplinary Mathematical Science, Banaras Hindu  
University Varanasi, U.P.  
[diwakarshukla@rediffmail.com](mailto:diwakarshukla@rediffmail.com)

Kapil Verma

Department of Computer Science,  
M.P. Bhoj (Open) University,  
Bhopal, M. P, India.  
[kapil\\_mca100@rediffmail.com](mailto:kapil_mca100@rediffmail.com)

Sharad Gangele\*

Department of Computer Science,  
M.P. Bhoj (Open) University,  
Bhopal,  
M. P., India.  
[sharadgangele@gmail.com](mailto:sharadgangele@gmail.com)

Pankaja Singh

Department of Physics & Computer Science,  
Govt. Motilal Vigyan Mahavidhyalaya,  
Bhopal, M.P.  
[pankajasingh@gmail.com](mailto:pankajasingh@gmail.com)

**Abstract:** The elasticities and indices are tools used to study the relative change among variables in a model. The basic markov chain based internet traffic sharing model was first suggested by Naldi [2002]. This paper extends the similar approach in view to elasticity computation and index formation. Mathematical expressions are derived and simulation study is performed. It shows that elasticities of traffic sharing indicates for situations of gain due to an operator.

**Keywords:** Markov chain model, Initial preference, Blocking probability, Call-by-call basis, Internet service providers [operators or ISP], Transition probability matrix.

### I. INTRODUCTION

Internet service providership is a profitable business and a large number of service providers are in competition in the market. Naldi [2002] assumed that there are only two operators and developed a probability model. His model is a basic work which has given the traffic share modeling problem a new look. This paper takes into account the same model and computers elasticity values. Further relative index for traffic share is also computed.

### II. A REVIEW

Naldi [1998 a, b, c & d] examined the impact of traffic measurement and traffic analysis in telecommunication sector with special reference to telephone network. Shukla and Gadewar [2007] examined stochastic model for cell movement in a knockout switch in computer network. In similar contribution, Shukla *et al.* [2007] discussed a stochastic model for space division switches in computer network and its extension is in Shukla *et al.* [2007 b]. Szabo *et al.* [2007] discussed accurate traffic classification in world of wireless mobile and multimedia network. Dainotti *et al.* [2008] presented a detailed discussion on classification network traffic via packet level hidden Markov model. Shukla and Jain [2007 a & b] developed Markov chain model analysis methodology and indexing technique for multilevel queue scheduling in operating system. Shukla and Thakur [2007] presented crime based user analysis in internet traffic sharing under cyber crime. Shukla and Thakur [2008 a, b & c] discussed Markov chain modeling application for Banyan switch network and traffic sharing problem. John *et al.* [2008] presented an analysis for trends

and differences in connection behavior within classes of internet backbone traffic. Shukla, Tiwari and Tiwari [2008] focused on the rest state analysis in internet traffic distribution in multioperator environment. Shukla, Jain and Ojha [2010 a, b & c] discussed markov chain model based analysis of deadlock index and multilevel queue scheduling. Shukla and Thakur [2010 a & b] applied the imputation methods over knowledge discovery and web mining in data warehouse search problems. Shukla, Thakur and Tiwari [2010] discussed stochastic modeling of internet traffic management and in one more similar analysis Tiwari, Thakur and Shukla [2010] presented a view point on cyber crime analysis for multi dimensional effect in computer network. Marnerides *et al.* [2008] discussed the problem related to detection and mitigation of abnormal traffic behavior in autonomic networked environments. Shukla and Singhai [2010] discussed traffic analysis of message flow in three cross bar architecture in space division switches. Shukla, Jain and Choudhary presented a technique for estimation of ready queue processing time under usual group lottery scheduling (GLS) in multiprocessor environment. Some other useful similar contributions are due to Thakur and Shukla [2010].

### III. MARKOV CHAIN MODEL [BY NALDI [2002]]

- Suppose two operators  $O_1$  and  $O_2$  are in competition in a market.
- The user initially chooses one of the two operators with probability  $p$  and  $1-p$  (initial shares) respectively.
- The probability  $p$  relates to all factors that may lead the user to choose one of the two operators as his first choice, including the range of services it offers and past experience.

- D. After each failed connecting attempt the user has two choices: he can either abandon (with probability  $p_A$ ) or switch to the other operator for a new attempt.
- E. Switching between the two operators is performed on a call-by-call basis and depends just on the latest attempts.
- F. During the repeated call attempt process the blocking probability  $L_1$  and  $L_2$  (i.e. the probability that the call attempt through the operator  $O_1$  and  $O_2$  fails) and the probability of abandon  $p_A$  stands constant.

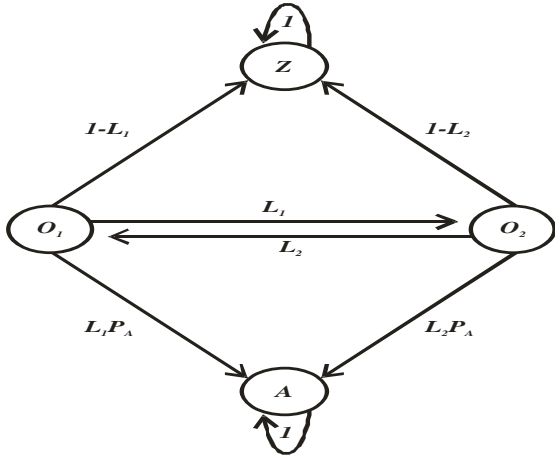


Figure. 1 Markov chain model of the user's behavior [Naldi [2002]]

Let  $\{x^{(n)}, n > 0\}$  be a markov Chain where  $x^{(n)}$  denotes the position of random variable  $X$  over any state  $\{O_1, O_2, Z, A\}$  at the  $n^{th}$  call attempt. The  $Z$  is success state and  $A$  is abandon state. The starting conditions (state distribution before the first call attempt) are [Naldi [2002]]

$$\left. \begin{aligned} P[X^{(0)} = O_1] &= P, \\ P[X^{(0)} = O_2] &= 1 - P, \\ P[X^{(0)} = Z] &= 0, \\ P[X^{(0)} = A] &= 0, \end{aligned} \right\} \dots(3.1)$$

The one-step transition probabilities matrix as stands by Naldi [2002] is:

$$M = \begin{matrix} & \begin{matrix} O_1 & O_2 & Z & A \end{matrix} \\ \begin{matrix} O_1 \\ O_2 \\ Z \\ A \end{matrix} & \begin{bmatrix} 0 & L_1(1-p_A) & 1-L_1 & L_1p_A \\ L_2(1-p_A) & 0 & 1-L_2 & L_2p_A \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{matrix} \dots(3.2)$$

Following results are derived by Naldi [2002]:

$$P[X^{(n)} = O_1] = P[X^{(n-1)} = O_2]L_2(1-p_A) \dots(3.3)$$

$$P[X^{(n)} = O_2] = P[X^{(n-1)} = O_1]L_1(1-p_A) \dots(3.4)$$

After unwrapping the recursions we get the general relationships for  $O_1$

$$\left\{ \begin{aligned} P[X^{(n)} = O_1] &= p\sqrt{(L_1L_2)^n} \cdot (1-p_A)^n, & n \text{ even} \\ P[X^{(n)} = O_1] &= (1-p)L_2\sqrt{(L_1L_2)^{n-1}} \cdot (1-p_A)^n, & n \text{ odd} \end{aligned} \right\} \dots(3.5)$$

for  $O_2$

$$\left\{ \begin{aligned} P[X^{(n)} = O_2] &= (1-p)\sqrt{(L_1L_2)^n} \cdot (1-p_A)^n, & n \text{ even} \\ P[X^{(n)} = O_2] &= pL_1\sqrt{(L_1L_2)^{n-1}} \cdot (1-p_A)^n, & n \text{ odd} \end{aligned} \right\} \dots(3.6)$$

$$P_1^{(n)} = p(1-L_1)(1-p_A)^{n-1}\sqrt{(L_1L_2)^{n-1}}, n \text{ even} \dots(3.7)$$

$$P_2^{(n)} = (1-p)L_2(1-p_A)^{n-1}\sqrt{(L_1L_2)^{n-2}}, n \text{ odd} \dots(3.8)$$

#### IV. ELASTICITY ANALYSIS OF INTERNET TRAFFIC SHARING

In order to obtain the cumulative probability that a call is completed through the operator  $O_1$  within the first  $n$  attempts, we have to sum over the number of attempts:

$$\bar{P}_1 = (1-L_1) \frac{p + (1-p)(1-p_A)L_2}{1-L_1L_2(1-p_A)^2} \dots\dots\dots(4.1)$$

Partially differentiate with respect to  $L_1$  we get

$$f_1(\cdot) = \left( \frac{\partial \bar{P}_1}{\partial L_1} \right)_{p, p_A, L_2} = \frac{\{p + (1-p)(1-p_A)L_2\} \left[ \{L_1L_2(1-p_A)^2 - 1\} + \{(1-L_1)L_2(1-p_A)^2\} \right]}{[1-L_1L_2(1-p_A)^2]^2} \dots(4.2)$$

Partially differentiate with respect to  $L_2$  we get

$$f_2(\cdot) = \left( \frac{\partial \bar{P}_1}{\partial L_2} \right)_{p, p_A, L_1} = \frac{(1-L_1)(1-p_A) \left[ \{1-L_1L_2(1-p_A)^2\} (1-p) + \{p + (1-p)(1-p_A)L_2\} L_1(1-p_A) \right]}{[1-L_1L_2(1-p_A)^2]^2} \dots(4.3)$$

Partially differentiate with respect to  $P_A$  we get

$$f_3(\cdot) = \left( \frac{\partial \bar{P}_1}{\partial P_A} \right)_{L_1, L_2, p} = \frac{(1-L_1)L_2 \left[ \{L_1L_2(1-p_A)^2 - 1\} (1-p) - 2L_1(1-p_A) \right] \{p + (1-p)(1-p_A)L_2\}}{[1-L_1L_2(1-p_A)^2]^2} \dots(4.4)$$

#### V. SIMULATION STUDY

This section contains elasticity and index based analysis of model characteristics with respect to blocking probability.

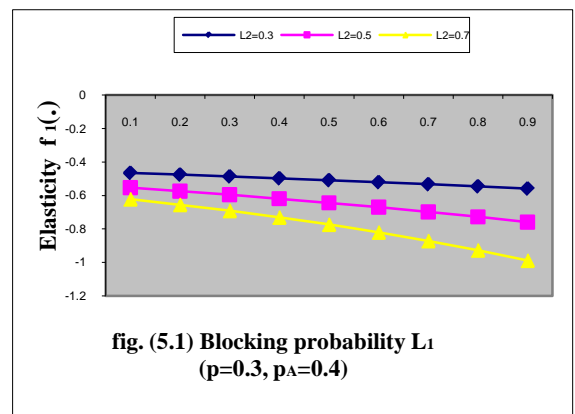
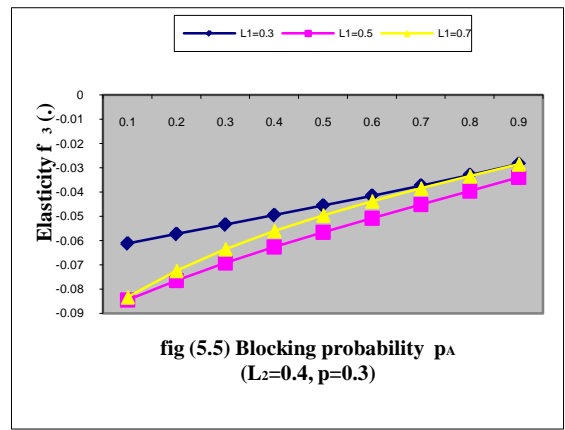
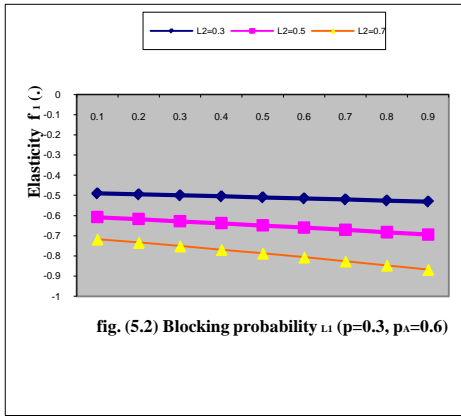


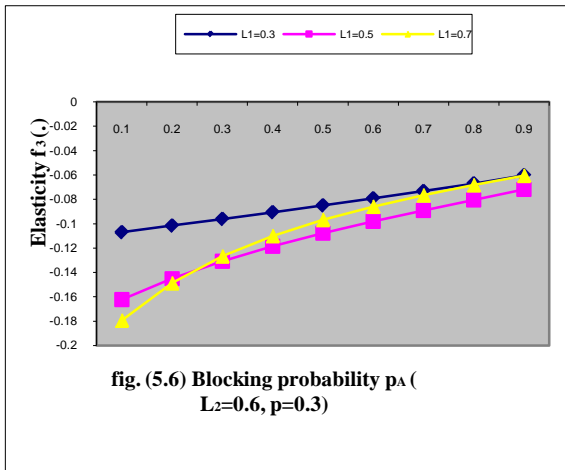
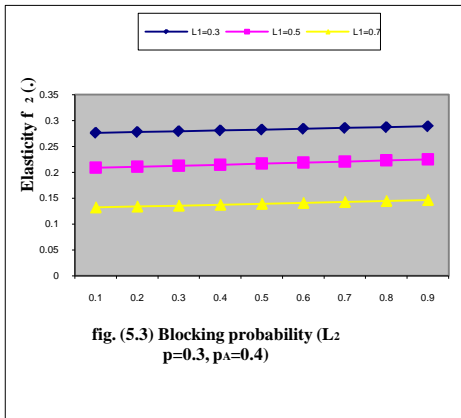
fig. (5.1) Blocking probability  $L_1$  ( $p=0.3, p_A=0.4$ )

The figure: 2 shows that if opponent blocking level is low then elasticity level is almost constant. But when opponent blocking is high, this parameter has reducing elasticity patterns.

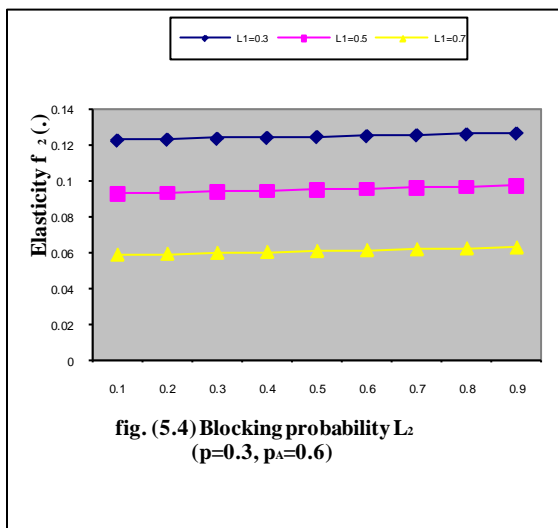


The elasticity pattern with respect to varying parameter  $p_A$  is different than found in earlier cases. Elasticity level constantly increases for traffic sharing over incrementing  $p_A$  (fig. 5.5)

In references to figure 3 when  $p_A$  is high the similar elasticity pattern observed. While elasticity variation with respect  $L_2$  it is found that they are in increasing trends. For lower value of  $L_1$  the elasticity level upper than when  $L_1$  high (fig. 5.3, 5.4).



With high  $L_1$  the increment graph is little lower. When  $L_2$  is high fig. 5.6 then patterns bear a very little change. Overall the elasticity pattern with respect to parameter  $p_A$  remains increasing upward.

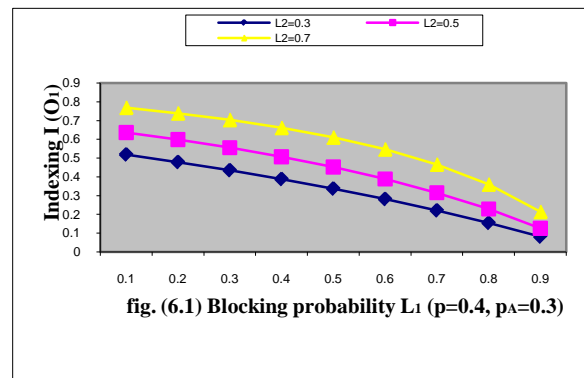


## VI. INDEX BASED ANALYSIS

We defined Operator Popularity Index (OPI) as: For operator  $O_1$

$$I(o_1) = \frac{\bar{P}_1}{P_1 + P_2}$$

$$I(o_1) = \frac{(1-L_1)\{p + (1-p)(1-p_A)L_2\}}{(1-L_1)\{p + (1-p)(1-p_A)L_2\} + (1-L_2)\{(1-p) + pL_1(1-p_A)\}}$$



In view to fig. 6.1 the index for operation has downward trends and when  $L_2$  low, the shifting pattern found words lower side.

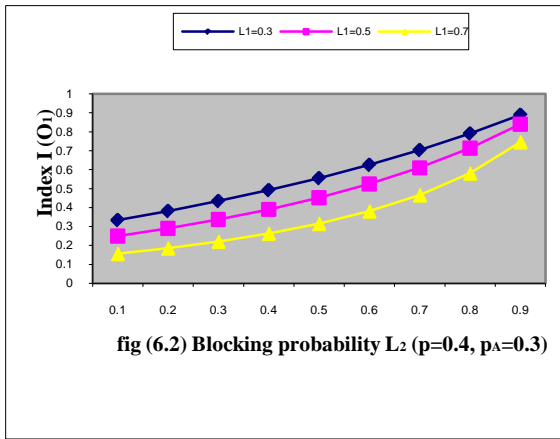


fig (6.2) Blocking probability  $L_2$  ( $p=0.4, p_A=0.3$ )

In contrary to this fig. 6.2 is showing upward trend of index for operator  $O_1$ .

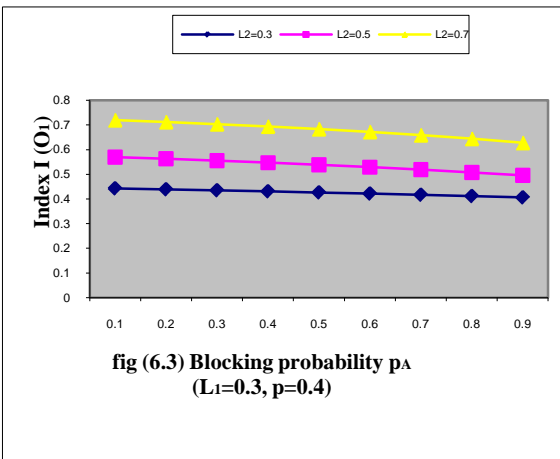


fig (6.3) Blocking probability  $p_A$  ( $L_1=0.3, p=0.4$ )

The fig 6.3 showing a constant pattern of index for operator  $O_1$  it shows that relative traffic sharing of operator  $O_1$  depends on parameters  $L_1, L_2$  and  $p_A$ .

For operator  $O_2$

$$I(o_2) = \frac{\bar{P}_2}{\bar{P}_1 + \bar{P}_2}$$

$$I(o_2) = \frac{(1-L_2)\{(1-p) + pL_1(1-p_A)\}}{(1-L_1)\{p + (1-p)(1-p_A)L_2\} + (1-L_2)\{(1-p) + pL_1(1-p_A)\}}$$

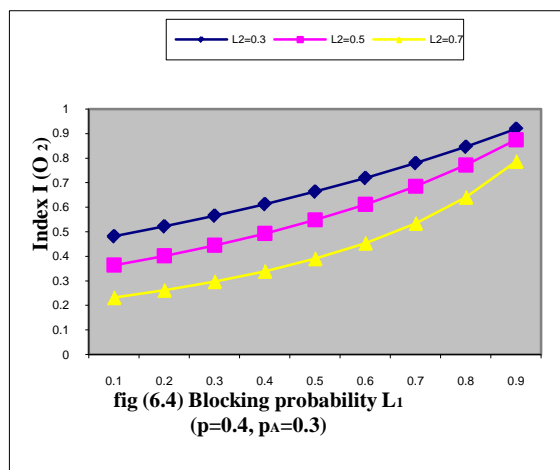


fig (6.4) Blocking probability  $L_1$  ( $p=0.4, p_A=0.3$ )

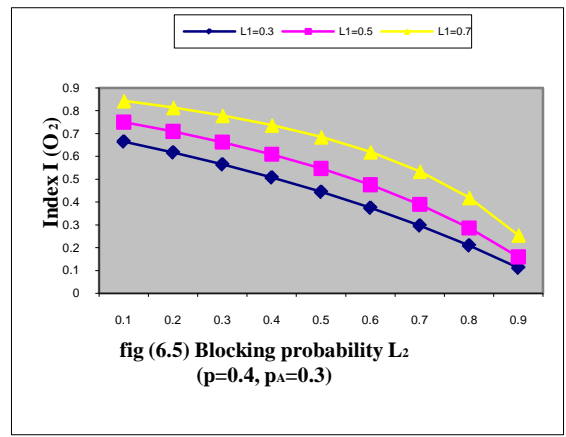


fig (6.5) Blocking probability  $L_2$  ( $p=0.4, p_A=0.3$ )

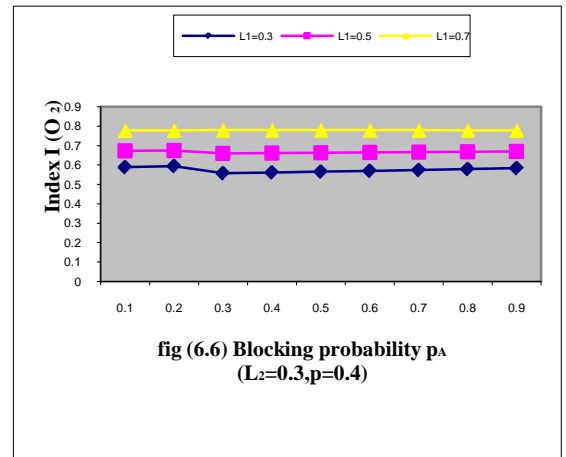


fig (6.6) Blocking probability  $p_A$  ( $L_2=0.3, p=0.4$ )

The similar index analysis observes for operator  $O_2$  by fig. 6.4, 6.5 and 6.6.

## VII. CONCLUSION

The elasticity derivatives are helpful to compare the traffic sharing between two operators. It is found that blocking probabilities are function of these elasticities. The abandon probability also affects the system. If self network blocking is high, the index reduces. It suggests operators to reduce this blocking parameter with respect to other competitor.

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