MULTIVALUED POSITIVE BOOLEAN DEPENDENCIES BY GROUPS IN THE DATABASE MODEL OF BLOCK FORM

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I. INTRODUCTION

In recent years, research to expand the relational data model has been interested by many scientists around the world. Following this research direction, there are some proposed database models such as: Multidimensional data model \([1],[2],[3]\), data block \([4],[5]\), data warehouse \([6],[7]\)… the database model of block form \([8]\).

In a database model of block form, the concepts: blocks, block diagrams, slices, relational algebra over blocks, functional dependencies, closures of index attribute set ... have been studied \([8]\). However, the study of extended logical dependencies in this data model is limited, many types of dependencies have not been studied. This article wants to propose and study the properties of a new type of logical dependency in a database model of block form: that is multivalued positive Boolean dependencies by groups.

II. THE DATABASE MODEL OF BLOCK FORM

II.1 The block, slice of the block

**Definition II.1** \([8]\)

Let \( R = (id; A_1, A_2,..., A_n) \) is a finite set of elements, where id is non-empty finite index set, \( A_i \) (i=1..n) are attributes. Each attribute \( A_i \) (i=1..n) there is a corresponding value domain \( \text{dom}(A_i) \). A block \( r \) on \( R \) is denoted \( r(R) \). For each \( x \in id \) we denoted \( r(R_{x}) \) is a block with \( R_x = (\{x\}; A_1, A_2,..., A_n) \) such that:

\[
x \in r(R_x) \iff t_y = \{ t^k : i \in \text{id} \} \}
\]

Then, block is denoted \( r(R) \) or \( r(id; A_1, A_2,..., A_n) \), if without fear of confusion we simply denoted \( r \).

**Definition II.2** \([8]\)

Let \( R = (id; A_1, A_2,..., A_n) \), \( r(R) \) is a block over \( R \). For each \( x \in id \) we denoted \( r(R_{x}) \) is a block with \( R_x = (\{x\}; A_1, A_2,..., A_n) \) such that:

\[
t_x \in r(R_x) \iff t_x = \{ t^k : i \in \text{id} \}
\]

Then \( r(R_{x}) \) is called a slice of the block \( r(R) \) at point \( x \).

II.2 Functional dependencies

Here, for simplicity we use the notation:

\[
X^{(m)} = (x; A_i) ; \ x^{(0)} = \{x \in id\}, \ A_i \subseteq \text{dom}(A_i)
\]

and \( X^{(m)} (x \in id, i = 1..n) \) is called an index attribute of block scheme \( R = (id; A_1; A_2;...; A_n) \).

**Definition II.3** \([8]\)

Let \( R = (id; A_1, A_2,..., A_n) \), \( r(R) \) is a block over \( R \) and \( X, Y \subseteq \bigcup_{x \in \text{id}} \) then \( X \rightarrow Y \) is a notation of functional dependency. A block \( r \) satisfies \( X \rightarrow Y \) if \( \forall t_1, t_2 \in r \) such that \( t_1(x) = t_2(x) \) then \( t_1(Y) = t_2(Y) \).

**Definition II.4** \([9]\)

Let block scheme \( \alpha = (R,F) \), \( R = (id; A_1, A_2,..., A_n) \), \( F \) is the set of functional dependencies over \( R \). Then, the closure of \( F \) denoted \( F' \) is defined as follows:

\[
F' = \{ X \rightarrow Y : F \implies X \rightarrow Y \}
\]

If \( X = \{x^{(m)} \} \subseteq \text{id}^{(m)}, Y = \{y^{(k)} \} \subseteq \text{id}^{(k)} \) then we denoted functional dependency \( X \rightarrow Y \) is simply \( x^{(m)} \rightarrow y^{(k)} \).

The block \( r \) satisfies \( x^{(m)} \rightarrow y^{(k)} \) if \( \forall t_1, t_2 \in r \) such that \( t_1(x^{(m)}) = t_2(x^{(m)}) \), then \( t_1(y^{(k)}) = t_2(y^{(k)}) \).

Henceforth, for convenience, we used notation for subsets of functional dependencies on \( R \):

\[
F_h = \{ X \rightarrow Y : X = \bigcup_{x \in \text{id}} x^{(m)}, Y = \bigcup_{y \in \text{id}} y^{(k)}, A, B \subseteq \{1,2,...,n\}, x \in \text{id} \}, \\
F_h = F_h \bigcup_{x \in \text{id}} x^{(m)} = \{ X \rightarrow Y : X \in F_h, X, Y \subseteq \bigcup_{x \in \text{id}} x^{(m)} \}
\]
We choose the operations and basic multivalued logic formulas as follows:

(i) Each value in $B = \{b_1, b_2, ..., b_k\}$, including $k$ values in $[0;1]$, $k \geq 2$, is a multivalued logical formula. The functions $I_b, b \in B$, are defined as follows:

$$I_b(x) = 1 \text{ if } x = b \quad \text{and} \quad I_b(x) = 0 \text{ if } x \neq b.$$ 

The functions $I_b, b \in B$, are called generalized negative functions.

Definition III.2 [11]

Let $P = \{x_1, x_2, ..., x_n\}$ be a finite set of Boolean variables, $B$ is a set of Boolean values. Then the multivalued boolean formulas (CTBDT) also known as multivalued logic formulas are constructed as follows:

(i) Each value in $B$ is a CTBDT.

(ii) Each variable in $P$ is a CTBDT.

(iii) Each function $I_b, b \in B$ is a CTBDT.

(iv) If $a$ is a multivalued boolean formula then $a$ is a CTBDT.

(v) If $a$ and $b$ are CTBDT then $a \land b$, $a \lor b$, and $\neg a$ are CTBDT.

(vi) Only formulas created by rules from (i) – (v) are CTBDT.

We denoted $MVL(P)$ as a set of CTBDT building on the set of variables $P = \{x_1, x_2, ..., x_n\}$ and the set of values $B = \{b_1, b_2, ..., b_k\}$, including $k$ values in $[0;1]$, $k \geq 2$.

Definition III.3 [11]

We define $a \rightarrow b$ equivalent to $CTBDT$ ($\neg a$)$\lor b$ and then: $a \rightarrow b = \max (1-a, b)$.

Definition III.4 [11]

Each vector of elements $v = \{v_1, v_2, ..., v_n\}$ in space $B^n = B \times B \times ... \times B$ is called a value assignment. Thus, with each CTBDT $f \in MVL(P)$ we have $f(v) = f(v_1, v_2, ..., v_n)$ is the value of formula $f$ for $v$ value assignments.

We understand the symbol $X \subseteq P$ at the same time performing for the following subjects:

- An attribute set in $P$.
- A set of logical variables in $P$.
- A multivalued Boolean formula is the logical union of variables in $X$.

On the other hand, if $X = \{B_1, B_2, ..., B_k\} \subseteq P$, we denote:

$\wedge X = B_1 \land B_2 \land ... \land B_k$ called the associational form.

$\lor X = B_1 \lor B_2 \lor ... \lor B_k$ called the recruitmental form.

For each finite set CTBDT $F = \{f_1, f_2, ..., f_m\}$ in $MVL(P)$, we consider $F$ as a formatted formula $F = f_1 \land f_2 \land ... \land f_m$. Then we have:

$$F(v) = f_1(v) \land f_2(v) \land ... \land f_m(v).$$

III.2 Table of values and truth tables

With each formula $f$ on $P$, table of values for $f$, denote that $V_f$ contains $n+1$ columns, with the first $n$ columns containing the values of the variables in $U$, and the last column contains the values of $f$ for each values signment of the corresponding row. Thus, the value table contains $k^n$ row, $n$ is the element number of $P$, $k$ is the element number of $B$.

Definition III.5 [11]

Let $m \in [0;1]$, truth table with $m$ threshold of $f$ or the m-truth table of $f$, denoted $T_{f,m}$ is the set of assignments $v$ such that $f(v)$ satisfies value not less than $m$: $T_{f,m} = \{v \in B^n | f(v) \geq m\}$.

Then the m-truth table $T_{f,m}$ of finite sets of formulas $F$ on $P$, is the intersection of the m-truth tables of each member formula in $F$.

$$T_{f,m} = \bigcap_{f \in F} T_{f,m}.$$ 

We have: $v \in T_{f,m}$ necessary and sufficient are $\forall f \in F: f(v) \geq m$.

III.3 Logical deduction

Definition III.6 [11]

Let $f, g$ are two CTBDT and value $m \in B$. We say formula $f$ derives formula $g$ from threshold $m$ and denoted $f \models_g m$ if $T_{f,m} \subseteq T_{g,m}$. We say $f$ and $g$ are two $m$-equivalent formulas, denoted $f \equiv_m g$ if $T_{f,m} = T_{g,m}$.

With $F, G$ in $MVL(P)$ and value $m \in [0;1]$, we say $F$ derives $G$ from threshold $m$, denoted $F \models_m G$, if $T_{F,m} \subseteq T_{G,m}$.

Moreover, we say $F$ and $G$ are $m$-equivalents, denoted $F \equiv_m G$ if $T_{F,m} = T_{G,m}$.

III.4 Multivalued positive Boolean formula

Definition III.7 [11]

Formula $f \in MVL(P)$ is called a multivalued positive Boolean formula (CTBDT) if $f(e) = 1$ with $e$ is the unit value assignment: $e = (1, 1, ..., 1)$, we denoted $MVP(P)$ is the set of all multivalued positive Boolean formulas on $P$.

IV. RESEARCH RESULTS

IV. The multivalued truth block by groups of the data block
Definition IV.1
Let \( R = \{ id; A_1, A_2, \ldots, A_n \} \), \( r(R) \) is a block over \( R \), we convention that each value domain \( d_i \) of attribute \( A_i \) (is also of index attribute \( x^{(i)}, x \in id, 1 \leq i \leq n \)), contains at least \( p \) elements. Then, with each value domain \( d_i \), we consider the mapping \( \beta_i : (d_i)^p \rightarrow \mathbb{B} \), satisfies the following properties:

(i) Reflectivity: \( \forall a \in (d_i)^p \), \( \beta(a) = 1 \), if in a contains at least two identical components.

(ii) Commutation: \( \forall a \in (d_i)^p \), \( \beta(a) = \beta(a') \), where \( a' \) is permutation of \( a \).

(iii) Sufficiency: \( \exists m \in \mathbb{B}, \exists a \in (d_i)^p \), \( \beta(a) = m \).

Example IV.1
Let \( R = \{ (1, 2), A_1, A_2 \} \), then the index attribute of \( R \) are \( U = \{ 1^{(1)}, 1^{(2)}, 2^{(1)}, 2^{(2)} \} \), with:

- \( A_1 \): Weight of the ball (C: high, K: quite high, M: average, S: low).
- \( A_2 \): Color of the ball (D: red, V: yellow, X: blue, N: brown).

\( r \) is a block over \( R \), includes 4 elements: \( t_1, t_2, t_3, t_4 \) as follows:

\[
\begin{align*}
t_1 &= (1^{(1)}, 1^{(2)}, 1^{(2)}, 1^{(2)}) = \text{C, C, C} = \text{D} \\
t_2 &= (1^{(1)}, 1^{(2)}, 1^{(2)}, 2^{(2)}) = \text{M, M, V} = \text{V} \\
t_3 &= (1^{(2)}, 2^{(2)}, 1^{(2)}, 2^{(2)}) = \text{S, M, N} = \text{S} \\
t_4 &= (1^{(2)}, 2^{(2)}, 1^{(2)}, 2^{(2)}) = \text{K, K, N} = \text{N}
\end{align*}
\]

Then we have the elements \( a_1, a_2, a_3, a_4, \ldots \) of the truth block \( T_p \) as follows:

\[
\begin{align*}
a_1 &= \text{1(C, M, S)} = 0.5 \\
a_2 &= \text{1(M, S, C)} = 0.5 \\
a_3 &= \text{1(V, V, X)} = 0.5 \\
a_4 &= \text{1(V, V, X)} = 0.5 \\
a_5 &= \text{1(V, V, X)} = 0.5 \\
\end{align*}
\]

Definition IV.2
Let \( R = \{ id; A_1, A_2, \ldots, A_n \} \), \( r(R) \) is a block over \( R \), each value domain \( d_i \) of attribute \( A_i \) (is also of index attribute \( x^{(i)}, x \in id, 1 \leq i \leq n \)), contains at least \( p \) elements, \( \beta_i \) is an evaluation on groups containing \( p \) values of \( d_i \), satisfying reflection and commutative properties. Equality relation is a separate case of this relation.

Example IV.2:
With the given block in the example IV.1, \( r \) is a block of 4 elements: \( t_1, t_2, t_3, t_4 \) as follows:

\[
\begin{align*}
t_1 &= \text{C} \in \{ C, V, X, N \} = 0.5 \\
t_2 &= \text{V} \in \{ C, V, X, N \} = 0.5 \\
t_3 &= \text{S} \in \{ C, V, X, N \} = 0.5 \\
t_4 &= \text{N} \in \{ C, V, X, N \} = 0.5 \\
\end{align*}
\]

Then we have the elements \( a_1, a_2, a_3, \ldots \) of the truth block \( T_p \) as follows:

\[
\begin{align*}
a_1 &= \text{1(C, M, S)} = 0.5 \\
a_2 &= \text{1(M, S, C)} = 0.5 \\
a_3 &= \text{1(V, V, X)} = 0.5 \\
a_4 &= \text{1(V, V, X)} = 0.5 \\
a_5 &= \text{1(V, V, X)} = 0.5 \\
\end{align*}
\]
dependency by groups is a multivalued positive Boolean formula in

\[ \text{MVP}(U) \text{ with } U = \bigcup_{i=1}^{n} \{i\}^{\alpha_i} \]

Let value \( m \in \mathbb{B} \), we say block \( r \) is m-satisfying by groups the multivalued positive Boolean dependency by groups (PTBDTTNB) \( f \) and denoted \( r^p(f,m) \) if \( T_{p} \subseteq T_{f,m} \).

The block \( r \) is m-satisfying by groups set PTBDTTNB \( F \) and denoted \( r^p(F,m) \) if \( r \) is m-satisfying by groups all \( f \) in \( F \):

\[ r^p(F,m) \iff \forall f \in F: r^p(f,m) \iff T_{p} \subseteq T_{f,m} \]

If \( r^p(F,m) \) then we say set PTBDTTNB \( F \) is m-right by groups in the block \( r \).

**Proposition IV.1**

Let \( R = (id; A_1,A_2,...,A_n) \), \( r(R) \) is a block on \( R \), then:

i) If \( r \) is m-satisfying by groups the multivalued positive Boolean dependency by groups \( F: r^p(f,m) \) then \( r^p(F,m), \forall x \in id. \)

ii) If \( r \) is m-satisfying by groups set of multivalued positive Boolean dependency by groups \( F: r^p(f,m) \) then \( r^p(F,m), \forall x \in id. \)

**Proposition IV.2**

Let \( R = (id; A_1,A_2,...,A_n) \), \( r(R) \) is a block on \( R \), then:

i) If \( r^p(f,m), \forall x \in id. \) then \( r \) is m-satisfying by groups the multivalued positive Boolean dependency by groups \( F: r^p(f,m) \).

ii) If \( r^p(F,m), \forall x \in id. \) then \( r \) m-satisfying by groups set of multivalued positive Boolean dependency by groups \( F: r^p(F,m) \).

**Theorem IV.1**

Let \( R = (id; A_1,A_2,...,A_n) \), \( r(R) \) is a block on \( R \), then:

i) Under the assumption we have \( r^p(f,m) \Rightarrow T_{p} \subseteq T_{f,m} \).

ii) Under the assumption \( r^p(F,m), \forall x \in id. \) then \( r \) is m-satisfying by groups the multivalued positive Boolean dependency by groups \( F: r^p(F,m) \).

**Theorem IV.2**

Let \( R = (id; A_1,A_2,...,A_n) \), \( r(R) \) is a block on \( R \), then:

i) Under the assumption we have \( r^p(f,m) \Rightarrow T_{p} \subseteq T_{f,m} \).

ii) Under the assumption \( r^p(F,m), \forall x \in id. \) then \( r \) m-satisfying by groups set of multivalued positive Boolean dependency by groups \( F: r^p(F,m) \).

**Consequence IV.1**

Let \( R = (id; A_1,A_2,...,A_n) \), \( r(R) \) is a block on \( R \), then:

i) Under the assumption we have \( r^p(f,m) \Rightarrow T_{p} \subseteq T_{f,m} \).

ii) Under the assumption \( r^p(F,m), \forall x \in id. \) then \( r \) m-satisfying by groups set of multivalued positive Boolean dependency by groups \( F: r^p(F,m) \).
Let $R = (\text{id}; A_1, A_2, \ldots, A_n)$, $r(R)$ is a block over $R$, $U = \bigcup_{i=1}^{n} \text{id}$, $m \in \mathbb{B}$, each value domain $d_i$ of attribute $A_i$ (is also of index attribute $x^{(i)}_i$, $x \in \text{id}, 1 \leq i \leq n$), contains at least $p$ ($p \geq 2$) elements, $\beta_i$ are evaluations on groups containing $p$ value of the index attribute $x^{(i)}_i$, $x \in \text{id}, 1 \leq i \leq n$. With $\Sigma$ is the subset PTBDTTNB on $U$, we denote $(\Sigma, m)^{\bullet}$ is the set of all PTBDTTNB m-deduced from $\Sigma$, in other words:

$$(\Sigma, m)^{\bullet} = \{ f \mid f \in \text{MVP(U)} \}, \Sigma^{\bullet} = \{ f \mid f \in \text{MVP(U)} \}, T_m^{\bullet} = T_{\Sigma, m}^{\bullet}.$$ 

**Definition IV.4.**

Let $R = (\text{id}; A_1, A_2, \ldots, A_n)$, $r(R)$ is a block over $R$, $U = \bigcup_{i=1}^{n} \text{id}$, $m \in \mathbb{B}$, each value domain $d_i$ of attribute $A_i$ (is also of index attribute $x^{(i)}_i$, $x \in \text{id}, 1 \leq i \leq n$), contains at least $p$ ($p \geq 2$) elements, $\beta_i$ are evaluations on groups containing $p$ value of the index attribute $x^{(i)}_i$, $x \in \text{id}, 1 \leq i \leq n$. Then, we denote $\text{NMBD}(r,m)$ is the set of all PTBDTTNB m-right by groups in block $r$, means:

$\text{NMBD}(r,m) = \{ f \mid f \in \text{MVP(U)} \}, \text{NMBD}(r,m, m)^{\bullet}$. 

**Theorem IV.3.**

Let $R = (\text{id}; A_1, A_2, \ldots, A_n)$, $r(R)$ is a block over $R$, $U = \bigcup_{i=1}^{n} \text{id}$, $m \in \mathbb{B}$, each value domain $d_i$ of attribute $A_i$ (is also of index attribute $x^{(i)}_i$, $x \in \text{id}, 1 \leq i \leq n$), contains at least $p$ ($p \geq 2$) elements, $\beta_i$ are evaluations on groups containing $p$ value of the index attribute $x^{(i)}_i$, $x \in \text{id}, 1 \leq i \leq n$. Then, we have:

$\text{NMBD}(r,m, m)^{\bullet} = \text{NMBD}(r,m)$.

**Proof**

By definition, we have:

$\text{NMBD}(r,m, m)^{\bullet} = \{ f \mid f \in \text{MVP(P)} \}, \text{NMBD}(r,m, m)^{\bullet} = \{ f \mid f \in \text{MVP(U)} \}, T_{\text{NMBD}(r,m, m)^{\bullet}} \subseteq T_{\Sigma, m}^{\bullet}$. 

We infer: $\text{NMBD}(r,m, m)^{\bullet} \supseteq \text{NMBD}(r,m)$

(3)

On the other hand, suppose we have: $g \in \text{NMBD}(r,m, m)^{\bullet}$. We need proof $g \in \text{NMBD}(r,m)$. Indeed, the hypothesis:

$g \in \text{NMBD}(r,m, m)^{\bullet} = \{ f \mid f \in \text{MVP(U)} \}, T_{\text{NMBD}(r,m, m)^{\bullet}} \subseteq T_{\Sigma, m}^{\bullet}$. 

Which by definition of $\text{NMBD}(r,m, m)^{\bullet}$ we have:

$T_p \subseteq T_{\text{NMBD}(r,m, m)^{\bullet}} \Rightarrow T_p \subseteq T_{\Sigma, m}^{\bullet}$, block $r$ is m-satisfying by groups PTBDTTNB $g$.

From there we have: $g \in \text{NMBD}(r,m)$. 

$\Rightarrow (\text{NMBD}(r,m), m)^{\bullet} \subseteq \text{NMBD}(r,m)$

(4)

From (3) and (4) we have:

$(\text{NMBD}(r,m), m)^{\bullet} = \text{NMBD}(r,m)$.

**Consequence IV.3**

Let $R = (\text{id}; A_1, A_2, \ldots, A_n)$, $r(R)$ is a block over $R$, $U = \bigcup_{i=1}^{n} \text{id}$, $m \in \mathbb{B}$, each value domain $d_i$ of attribute $A_i$ (is also of index attribute $x^{(i)}_i$, $x \in \text{id}, 1 \leq i \leq n$), contains at least $p$ ($p \geq 2$) elements, $\beta_i$ are evaluations on groups containing $p$ value of the index attribute $x^{(i)}_i$, $x \in \text{id}, 1 \leq i \leq n$. Then, on $r$ we have:

$(\text{NMBD}(r,m), m)^{\bullet} = \text{NMBD}(r,m)$.

**Consequence IV.4**

Let $R = (\text{id}; A_1, A_2, \ldots, A_n)$, $r(R)$ is a block over $R$, $U = \bigcup_{i=1}^{n} \text{id}$, $m \in \mathbb{B}$, each value domain $d_i$ of attribute $A_i$ (is also of index attribute $x^{(i)}_i$, $x \in \text{id}, 1 \leq i \leq n$), contains at least $p$ ($p \geq 2$) elements, $\beta_i$ are evaluations on groups containing $p$ value of the index attribute $x^{(i)}_i$, $x \in \text{id}, 1 \leq i \leq n$. Then, we find a multivalued Boolean formula $h$ so that: $T_p \subseteq T_{\Sigma, m}^{\bullet}$.

**Theorem IV.4**

Let $R = (\text{id}; A_1, A_2, \ldots, A_n)$, $r(R)$ is a block over $R$, $U = \bigcup_{i=1}^{n} \text{id}$, $m \in \mathbb{B}$, each value domain $d_i$ of attribute $A_i$ (is also of index attribute $x^{(i)}_i$, $x \in \text{id}, 1 \leq i \leq n$), contains at least $p$ ($p \geq 2$) elements, $\beta_i$ are evaluations on groups containing $p$ value of the index attribute $x^{(i)}_i$, $x \in \text{id}, 1 \leq i \leq n$. Then, we have:

$T_p = T_{(\text{NMBD}(r,m), m)^{\bullet}}$. 

**Proof**

According to the definition of the set PTBDTTNB $\text{NMBD}(r,m)$ we have: if $f \in \text{NMBD}(r,m)$ block $r$ is m-satisfying by groups PTBDTTNB $f \Rightarrow T_p \subseteq T_{\Sigma, m}$.

From the properties of the relationship between Boolean formulas and truth blocks, with truth block $T_p$, we have found a multivalued Boolean formula $h$ so that: $T_{h,m} = T_p$.

On the other hand, because $e \in T_p$, $h \in T_{h,m}$ so $h$ is a multivalued positive Boolean formula.

From the equality: $T_p = T_{h,m}$, we deduce that block $r$ is m-satisfying by groups PTBDTTNB $h$, means:

$h \in \text{NMBD}(r,m)$.

So infer: $\text{NMBD}(r,m)^{\bullet} = h$. Hence we have:

$T_{\text{NMBD}(r,m)^{\bullet}} \subseteq T_{h,m}$ which means: $T_{\text{NMBD}(r,m)^{\bullet}} \subseteq T_p$. 

From the definition of $\text{NMBD}(r,m)$ we have:

$T_p \subseteq T_{\text{NMBD}(r,m)^{\bullet}}$ 

(5)

From (5) and (6) we infer: $T_p = T_{\text{NMBD}(r,m)^{\bullet}}$.

**Consequence IV.5**

Let $R = (\text{id}; A_1, A_2, \ldots, A_n)$, $r(R)$ is a block over $R$, $U = \bigcup_{i=1}^{n} \text{id}$, $m \in \mathbb{B}$, each value domain $d_i$ of attribute $A_i$ (is also of index attribute $x^{(i)}_i$, $x \in \text{id}, 1 \leq i \leq n$), contains at least $p$ ($p \geq 2$) elements, $\beta_i$ are evaluations on groups containing $p$ value of the index attribute $x^{(i)}_i$, $x \in \text{id}, 1 \leq i \leq n$. Then, we have:

$T_p = T_{\text{NMBD}(r,m)^{\bullet}}$.
(is also of index attribute \(x^0\), \(x \in \text{id}, 1 \leq i \leq n\)), contains at least \(p\) (\(p \geq 2\)) elements, \(\beta_i\), \(\beta_i\) are evaluations on groups containing \(p\) value of the index attribute \(x^0\), \(x \in \text{id}, 1 \leq i \leq n\). Then we have: if \(\text{id} = \{x\}\) then block \(r\) degenerates into relation and in the relational data model:

\[
T_p^r = T_{\text{NMBD}(r,m),m}.
\]

**Definition IV.6**

Let \(R = (\text{id}; A_1,A_2,...,A_n)\), \(r(R)\) is a block over \(R\), \(U = \bigcup_{i=1}^{\text{id}} m \in B\), each value domain \(d_i\) of attribute \(A_i\) (is also of index attribute \(x^0\), \(x \in \text{id}, 1 \leq i \leq n\)), contains at least \(p\) (\(p \geq 2\)) elements, \(\beta_i\), \(\beta_i\) are evaluations on groups containing \(p\) value of the index attribute \(x^0\), \(x \in \text{id}, 1 \leq i \leq n\). Then we say \(r\) is m-representation by groups set \(\text{PTBDTTNB} \Sigma\) if and only if \(T_p^r = T_{\Sigma,m}\).

Proof

Use the results of the theorem IV.3 and IV.4 for \(\text{PTBDTTNB}\) we have:

\[
\text{NMBD}(r,m),m \supseteq \text{NMBD}(r,m)
\]

and \(T_p^r = T_{\text{NMBD}(r,m),m}\). Then:

Block \(r\) is m-tight representation by groups set \(\text{PTBDTTNB} \Sigma\) if and only if: \(\text{NMBD}(r,m) \supseteq \Sigma \supseteq T_{\text{NMBD}(r,m),m} = T_{\Sigma,m} \supseteq T_p^r = T_{\Sigma,m} \).

So, that, block \(r\) is m-tight representation by groups set \(\text{PTBDTTNB} \Sigma\) if and only if \(T_p^r = T_{\Sigma,m}\).

**Consequence IV.6**

Let \(R = (\text{id}; A_1,A_2,...,A_n)\), \(r(R)\) is a block over \(R\), \(U = \bigcup_{i=1}^{\text{id}} m \in B\), each value domain \(d_i\) of attribute \(A_i\) (is also of index attribute \(x^0\), \(x \in \text{id}, 1 \leq i \leq n\)), contains at least \(p\) (\(p \geq 2\)) elements, \(\beta_i\), \(\beta_i\) are evaluations on groups containing \(p\) value of the index attribute \(x^0\), \(x \in \text{id}, 1 \leq i \leq n\). Then \(r\) is m-tight representation by groups set \(\text{PTBDTTNB} \Sigma\) if and only if \(T_p^r = T_{\Sigma,m} x \in \text{id}\).

**Theorem IV.6**

Let \(R = (\text{id}; A_1,A_2,...,A_n)\), \(r(R)\) is a block over \(R\), \(U = \bigcup_{i=1}^{\text{id}} m \in B\), \(\Sigma\) is set \(\text{PTBDTTNB} \Sigma\) on \(U\), \(\Sigma = \bigcup_{x \in \text{id}} \Sigma_x\).

\(\Sigma_x \neq \emptyset\). Each value domain \(d_i\) of attribute \(A_i\) (is also of index attribute \(x^0\), \(x \in \text{id}, 1 \leq i \leq n\)), contains at least \(p\) (\(p \geq 2\)) elements, \(\beta_i\), \(\beta_i\) are evaluations on groups containing \(p\) value of the index attribute \(x^0\), \(x \in \text{id}, 1 \leq i \leq n\). Then \(r\) is m-tight representation by groups set \(\text{PTBDTTNB} \Sigma\) if and only if \(T_p^r = T_{\Sigma,m} x \in \text{id}\).

**V. CONCLUSIONS**

From a proposed concept are functions that evaluate values on a group with \(p\) elements. The article gave the definition of the multivalued truth block by groups of data blocks. From there build a new type of dependency: it is a multivalued positive Boolean dependency by groups in the database model of block form. From the new concept of dependency is proposed, the authors have stated and proved the equivalent theorem for multivalued positive Boolean dependencies by groups on the block, the necessary and sufficient condition for a block \(r\) is m-tight representation set \(\text{PTBDTTNB} \Sigma\). From these results we can further study the relationship between other types of extended logical dependencies on the data block.

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**VII. REFERENCES**

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