INTRODUCTION


In this paper, an attempt is made to determine the temperature distribution, displacement function and thermal stresses of a thin rectangular plate when the boundary conditions are known. Integral transform techniques are used to obtain the solution of the problem.

STATEMENT OF THE PROBLEM-I

Consider a thin rectangular plate occupying the space D: \(-a \leq x \leq a, -b \leq y \leq b, -h \leq z \leq h\) with the known boundary conditions. Finite Marchi-Fasulo transform technique is used to find the solution of the problem.

Abstract: This paper is concerned with steady-state as well as transient thermoelastic problem in which we need to determine the temperature distribution, displacement function and thermal stresses of a thin rectangular plate when the boundary conditions are known. Integral transform techniques are used to obtain the solution of the problem.

Keywords: Thin rectangular plate, transient problem, thermoelastic problem, thermal stresses, integral transform.
where \( T(x, y, z) \) denotes the temperature of a rectangular beam satisfy the following differential equation as Tanigawa et al. [31] is
\[
\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} g(x, y, z) = 0
\]
where \( k \) is the thermal conductivity of the material, subject to the boundary conditions:
\[
T(x, y, z) + k \left[ \frac{\partial T(x, y, z)}{\partial x} \right]_{x=a} = f_i(y, z) \\
T(x, y, z) + k \left[ \frac{\partial T(x, y, z)}{\partial y} \right]_{y=b} = f_j(x, z) \\
T(x, y, z) + k \left[ \frac{\partial T(x, y, z)}{\partial z} \right]_{z=c} = f_k(x, y, z)
\]
\[\left[ T(x, y, z) \right]_{x=a} = f_2(x, y, z) \]
\[\left[ T(x, y, z) \right]_{y=b} = f_3(x, y, z) \]
\[\left[ T(x, y, z) \right]_{z=c} = f_4(x, y, z, t) \]
The stress components in terms of \( U(x, y, z, t) \) as Tanigawa et al. [31] are given by
\[
\sigma_{xx} = \frac{\partial U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2} \\
\sigma_{yy} = \frac{\partial^2 U}{\partial x^2} + \frac{\partial U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2} \\
\sigma_{zz} = \frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2}
\]
Equations (2.1) to (2.14) constitute the mathematical formulation of the problem under consideration.

**SOLUTION OF THE PROBLEM**

By applying Marchi-Fasulo transform defined in [12] w.r.t. \( x \) and \( y \) successively, we get
\[
\frac{\partial^2 T}{\partial z^2} - \mu T = \Psi
\]
where, \( \mu = (\alpha^2 + \mu_m^2) \)
\[
\Psi = \frac{P(a)}{k_1} f_2 - \frac{P(a)}{k_2} f_1 + \frac{Q(b)}{k_3} f_4 - \frac{Q(b)}{k_4} f_3 + \xi
\]
Equation (3.1) is a second order differential equation whose solution is given by
\[
T = Ae^{\mu z} + Be^{-\mu z} + \Pi
\]
\[
P.I. = 1 \quad D^2 - \mu = \Pi
\]
where, Using equations (2.10) and (2.11) in equation (3.2) we get
\[
T = \left[ f_2 + \frac{\Pi}{2} \right] \sinh(2\mu h) + \left[ f_3 + \frac{\Pi}{2} \right] \cosh(2\mu h)
\]
Further applying inversion of Marchi-Fasulo transform to the equation (3.3) one obtains the expression for temperature distribution as
\[
T(x, y, z) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{P_n(x)}{\lambda_n} \frac{Q_n(y)}{\mu_m} \Omega
\]
where, \( m, n \) are the positive integers. 
\[
\Omega = \left[ f_2 + \frac{\Pi}{2} \right] \cosh(2\mu h) + \left[ f_3 + \frac{\Pi}{2} \right] \cosh(2\mu h)
\]
Substituting equation (3.2) in equation (2.4), we get 
\[
U = -\alpha E \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{P_n(x)}{\lambda_n} \frac{Q_n(y)}{\mu_m} \Omega
\]
Substituting equation (3.4) in equations (2.1) - (2.3), the displacement components are obtained as
\[
u_x = \lambda \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \left[ \frac{Q_n(y)}{\mu_m} \Omega - \frac{Q_n(y)}{\mu_m} \Omega \right]
\]
\[
u_y = \lambda \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \left[ \frac{P_n(x)}{\lambda_n} \Omega - \frac{P_n(x)}{\lambda_n} \Omega \right]
\]
\[
u_z = \lambda \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \left[ \frac{P_n(x)}{\lambda_n} \Omega - \frac{P_n(x)}{\lambda_n} \Omega \right]
\]

**DETERMINATION OF STRESS FUNCTION**

Substituting the value of Airy’s stress function \( U(x, y, z) \) from equation (3.2) in the equations (2.13) to (2.14) one obtain the stress functions as,
\[
\sigma_{xx} = -\alpha E \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{P_n(x)}{\lambda_n} \frac{Q_n(y)}{\mu_m} \Omega + \frac{P_n(x)}{\lambda_n} \Omega
\]
\[
\sigma_{yy} = -\alpha E \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{P_n(x)}{\lambda_n} \frac{Q_n(y)}{\mu_m} \Omega + \frac{P_n(x)}{\lambda_n} \Omega
\]
\[
\sigma_{zz} = -\alpha E \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{P_n(x)}{\lambda_n} \frac{Q_n(y)}{\mu_m} \Omega + \frac{P_n(x)}{\lambda_n} \Omega
\]

**SPECIAL CASE**

Set \( f_i(x, y) = (x^2 + ay)(y^2 + ay)(-h) \)
\[
\begin{align*}
\frac{P_n(x)}{k_1} f_2 &= \frac{P_n(x)}{k_2} f_1 + \frac{Q_n(b)}{k_3} f_4 - \frac{Q_n(b)}{k_4} f_3 + \frac{g}{k} \\
\frac{P_n(x)}{k_1} f_2 &= \frac{P_n(x)}{k_2} f_1 + \frac{Q_n(b)}{k_3} f_4 - \frac{Q_n(b)}{k_4} f_3 + \frac{g}{k} \\

\begin{cases}
\frac{a_n \cos \theta}{a_n} - \cos \theta \sin \theta \sin \theta \\
\frac{b_n \cos \theta}{b_n} - \cos \theta \sin \theta \sin \theta
\end{cases}
\end{align*}
\]
\[
\begin{align*}
\begin{cases}
\frac{a_n \cos \theta}{a_n} - \cos \theta \sin \theta \sin \theta \\
\frac{b_n \cos \theta}{b_n} - \cos \theta \sin \theta \sin \theta
\end{cases}
\end{align*}
\]
\[
\begin{align*}
\begin{cases}
\frac{a_n \cos \theta}{a_n} - \cos \theta \sin \theta \sin \theta \\
\frac{b_n \cos \theta}{b_n} - \cos \theta \sin \theta \sin \theta
\end{cases}
\end{align*}
\]
Substitute these values in equations (3.4) to (3.5) one obtains

\[
T(x, y, z) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{P_n(x) Q_m(y)}{\lambda_n} \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} + \frac{\partial^2}{\partial x^2} \right) \times (-h) + Z_{\alpha} \\
\sinh \mu_z(z-h) \times \sinh \mu_z(z+h)
\]

\[
(5.3)
\]

\[
U = -\lambda E \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{P_n(x) Q_m(y)}{\lambda_n} \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} + \frac{\partial^2}{\partial x^2} \right) \times (h) + Z_{\alpha} \\
\sinh \mu_z(z-h) \times \sinh \mu_z(z+h)
\]

\[
(5.4)
\]

### NUMERICAL RESULTS

Set \( a = 1 \text{m}, b = 2 \text{m}, h = 2 \text{m}, t = 1 \text{sec} \) and \( k = 0.86 \) in equation (5.3) to (5.4), we obtain

\[
T(x, y, z) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{P_n(x) Q_m(y)}{\lambda_n} \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} + \frac{\partial^2}{\partial x^2} \right) \times (-2) + Z_{\alpha} \\
\sinh \mu_z(z-2) \times \sinh \mu_z(z+2)
\]

\[
(6.1)
\]

\[
U = -\lambda E \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{P_n(x) Q_m(y)}{\lambda_n} \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} + \frac{\partial^2}{\partial x^2} \right) \times (2) + Z_{\alpha} \\
\sinh \mu_z(z-2) \times \sinh \mu_z(z+2)
\]

### STATEMENT OF THE PROBLEM-II

Consider a thin rectangular plate occupying the space \( D: -a \leq x \leq a, -b \leq y \leq b, -h \leq z \leq h \). The displacement components \( u_x, u_y, u_z \) in the x and y and z directions respectively as Tanigawa et al. [31] are

\[
u_x = \int \frac{1}{E} \left( \frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2} + \frac{\partial^2 U}{\partial x^2} \right) \times \sinh \mu_z(x) dx
\]

\[
u_y = \int \frac{1}{E} \left( \frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2} + \frac{\partial^2 U}{\partial x^2} \right) \times \sinh \mu_z(y) dy
\]

\[
u_z = \int \frac{1}{E} \left( \frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2} + \frac{\partial^2 U}{\partial x^2} \right) \times \sinh \mu_z(z) dz
\]

where \( E, v, \) and \( \lambda \) are the young’s modulus, Poisson’s ratio and the linear coefficient of the thermal expansion of the material of the beam respectively and \( U(x, y, z, t) \) is the Airy’s stress functions which satisfy the differential equation as Tanigawa et al. [31] is

\[
\left( \frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2} \right) \times T(x, y, z, t)
\]

\[
(7.4)
\]

\[
\text{where } T(x, y, z, t) \text{ denotes the temperature of a rectangular beam satisfy the following differential equation as Tanigawa et al. [31] is}
\]

\[
\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + g(x, y, z, t) = \frac{1}{\alpha} \frac{\partial T}{\partial t}
\]

\[
(7.5)
\]

where \( k \) is the thermal conductivity and \( \alpha \) is the thermal diffusivity of the material, subject to the initial and boundary conditions:

\[
T(x, y, z, 0) = F(x, y, z)
\]

\[
(7.6)
\]

\[
T(x, y, z, t) + k_1 \left[ \frac{\partial T(x, y, z, t)}{\partial x} \right]_{y=a} = f_1(y, z, t)
\]

\[
(7.7)
\]

\[
T(x, y, z, t) + k_2 \left[ \frac{\partial T(x, y, z, t)}{\partial y} \right]_{x=b} = f_2(x, z, t)
\]

\[
(7.8)
\]

\[
T(x, y, z, t) + k_3 \left[ \frac{\partial T(x, y, z, t)}{\partial z} \right]_{y=b} = f_3(x, z, t)
\]

\[
(7.9)
\]

\[
T(x, y, z, t) + k_4 \left[ \frac{\partial T(x, y, z, t)}{\partial y} \right]_{z=b} = f_4(x, y, t)
\]

\[
(7.10)
\]

\[
T(x, y, z, t) + k_5 \left[ \frac{\partial T(x, y, z, t)}{\partial z} \right]_{x=b} = f_5(x, y, t)
\]

\[
(7.11)
\]
\[
\left[ T(x,y,z,t) + k_n \frac{\partial T(x,y,z,t)}{\partial t} \right]_{t = \eta} = f_0(x,y,t)
\]  
(7.12)

The stress components in terms of \(U(x, y, z, t)\) as Tanigawa et al. [31] are given by

\[
\sigma_x = \frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} - \frac{\partial^2 U}{\partial z^2}
\]  
(7.13)

\[
\sigma_y = \frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} - \frac{\partial^2 U}{\partial z^2}
\]  
(7.14)

\[
\sigma_z = \frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} - \frac{\partial^2 U}{\partial z^2}
\]  
(7.15)

Equations (7.1) to (7.15) constitute the mathematical formulation of the problem under consideration.

**SOLUTION OF THE PROBLEM**

By applying Marchi-Fasulo transform defined in [12] w.r.t. \(x\), \(y\) and \(z\) successively, we get

\[
\frac{\partial T}{\partial t} + \alpha \rho T = \psi
\]  
(8.1)

Where \(\rho = \left( \lambda t^2 + \mu m^2 + \eta n^2 \right) \)

\[
\psi = \left( \frac{\psi}{k_1} \right) f_i - \frac{P(a)}{k_2} f_i + \frac{Q(a)}{k_3} f_i - \frac{R(a)}{k_4} f_i + \frac{\psi}{k_5} f_i + \frac{g}{k}
\]

This is linear equation.

Further using their inverses in equation (8.1), one obtains the expression for temperature distribution as

\[
T(x, y, z, t) = e^{\alpha \rho \psi t} \sum_{l=1}^{\infty} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} P_l(x) Q_m(y) R_n(z)
\]

\[
\times \left[ \frac{F_1}{\mu_m \eta_n} + \int_0^t \psi e^{\alpha \rho \psi \tau} d\tau \right]
\]  
(8.2)

where \(l, m, n\) are the positive integers.

Substituting equation (8.2) in equation (2.4) we get

\[
U = -\lambda \alpha \rho \psi \sum_{l=1}^{\infty} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} P_l(x) Q_m(y) R_n(z)
\]

\[
\times \left[ \frac{F_1}{\mu_m \eta_n} + \int_0^t \psi e^{\alpha \rho \psi \tau} d\tau \right]
\]  
(8.3)

Substituting equation (3.3) in equations (8.1) - (8.3), the displacement components are obtained as

\[
u_x = \lambda \int_{-a}^a \frac{e^{\alpha \rho \psi t}}{\lambda t \mu_n \eta_n} \left[ \frac{F_1}{\mu_m \eta_n} + \int_0^t \psi e^{\alpha \rho \psi \tau} d\tau \right] dx
\]

\[
\times \left[ P_l(x) Q_m(y) R_n(z) - P_l(x) Q_m(y) R_n(z) - P_l(x) Q_m(y) R_n(z)
\right.

\[
\left. + P_l(x) Q_m(y) R_n(z) \right] dy
\]  
(8.4)

\[
u_y = \lambda \int_{-a}^a \frac{e^{\alpha \rho \psi t}}{\lambda t \mu_m \eta_n} \left[ \frac{F_1}{\mu_m \eta_n} + \int_0^t \psi e^{\alpha \rho \psi \tau} d\tau \right] dy
\]

\[
\times \left[ P_l(x) Q_m(y) R_n(z) - P_l(x) Q_m(y) R_n(z) - P_l(x) Q_m(y) R_n(z)
\right.

\[
\left. + P_l(x) Q_m(y) R_n(z) \right] dx
\]  
(8.5)

\[
u_z = \lambda \int_{-a}^a \frac{e^{\alpha \rho \psi t}}{\lambda t \mu_m \eta_n} \left[ \frac{F_1}{\mu_m \eta_n} + \int_0^t \psi e^{\alpha \rho \psi \tau} d\tau \right] dz
\]

\[
\times \left[ P_l(x) Q_m(y) R_n(z) - P_l(x) Q_m(y) R_n(z) - P_l(x) Q_m(y) R_n(z)
\right.

\[
\left. + P_l(x) Q_m(y) R_n(z) \right] dx
\]  
(8.6)

**DETERMINATION OF STRESS FUNCTION**

Substituting the value of Airy’s stress function \(U(x,y,z,t)\) from equation (8.2) in the equations (7.13) to (7.15) one obtains the stress functions as,

\[
\sigma_{xx} = -\lambda \alpha \rho \psi \sum_{l=1}^{\infty} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{P_l(x) Q_m(y) R_n(z)}{\lambda t \mu_m \eta_n} \left[ \frac{F_1}{\mu_m \eta_n} + \int_0^t \psi e^{\alpha \rho \psi \tau} d\tau \right]
\]

\[
\times \left[ Q_n''(y) R_l(z) + Q_n(y) R_l''(z) \right]
\]  
(9.1)

\[
\sigma_{yy} = -\lambda \alpha \rho \psi \sum_{l=1}^{\infty} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{Q_n(y) R_m(z)}{\lambda t \mu_m \eta_n}
\]

\[
\times \left[ \frac{F_1}{\mu_m \eta_n} + \int_0^t \psi e^{\alpha \rho \psi \tau} d\tau \right]
\]

\[
\times \left[ P_l''(x) R_l(z) + P_l(x) R_l''(z) \right]
\]  
(9.2)

\[
\sigma_{zz} = -\lambda \alpha \rho \psi \sum_{l=1}^{\infty} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{R_n(z)}{\lambda t \mu_m \eta_n}
\]

\[
\times \left[ \frac{F_1}{\mu_m \eta_n} + \int_0^t \psi e^{\alpha \rho \psi \tau} d\tau \right]
\]

\[
\times \left[ P_l''(x) Q_n(y) + P_l(x) Q_n''(y) \right]
\]  
(9.3)

**SPECIAL CASE**

Set \(f(x,y,t) = (e^{-\alpha \rho \psi t})(x+a)^2(y+b)^2(z+h)^2\),

\[
F(x,y,z) = (e^{-\alpha \rho \psi t})(x+a)^2(y+b)^2(z+h)^2
\]

\[
\Rightarrow \frac{F(x,y,z)}{a^2} = (e^{-\alpha \rho \psi t}) \left[ \frac{a^2 \cos^2(a, a) - \cos(a, a) \sin(a, a)}{a^2} \right]
\]

\[
\times \left[ b_h \cos^2(h, b) - \cos(h, b) \sin(h, b) \right]
\]

\[
\times \left[ h_n^2 \cos^2(h, h) - \cos(h, h) \sin(h, h) \right] \]

\[
\times \left[ \frac{h_n^2}{h_n^2} \right]
\]  
(10.1)

Substituting this value in equations (8.2)- (9.3) we get

\[
T(x, y, z, t) = e^{\alpha \rho \psi t} \sum_{l=1}^{\infty} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} P_l(x) Q_m(y) R_n(z)
\]

\[
\times \left[ \frac{F_1}{\lambda t \mu_m \eta_n} + \int_0^t \psi e^{\alpha \rho \psi \tau} d\tau \right]
\]

\[
\times \left[ \left[ a_i \cos^2(a, a) - \cos(a, a) \sin(a, a) \right] \right]
\]

\[
\times \left[ b_h \cos^2(h, b) - \cos(h, b) \sin(h, b) \right]
\]

\[
\times \left[ h_n^2 \cos^2(h, h) - \cos(h, h) \sin(h, h) \right]
\]

\[
\times \left[ \frac{h_n^2}{h_n^2} \right]
\]  
(10.2)

\[
U = -\lambda \alpha \rho \psi \sum_{l=1}^{\infty} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} P_l(x) Q_m(y) R_n(z)
\]

\[
\times \left[ \left( e^{-\alpha \rho \psi t} \right) \right]
\]

\[
\times \left[ \left[ a_i \cos^2(a, a) - \cos(a, a) \sin(a, a) \right] \right]
\]

\[
\times \left[ b_h \cos^2(h, b) - \cos(h, b) \sin(h, b) \right]
\]

\[
\times \left[ h_n^2 \cos^2(h, h) - \cos(h, h) \sin(h, h) \right]
\]

\[
\times \left[ \frac{h_n^2}{h_n^2} \right]
\]  
(10.3)

\[
u_x = \lambda \int_{-a}^a \frac{e^{\alpha \rho \psi t}}{\lambda t \mu_m \eta_n} \left( e^{-\alpha \rho \psi t} \right) \left[ \left[ a_i \cos^2(a, a) - \cos(a, a) \sin(a, a) \right] \right]
\]
\[ u_y = \frac{b}{a} \cdot \frac{\psi_{eop} \cdot r}{d} \left\{ \left( e^{-i} \right) \times \left[ \frac{a \cos^2(a,a) - \cos(a,a) \sin(a,a)}{a_i^2} \right] \right\} \]

\[ \sigma_{xx} = -\lambda E \cdot e^{\psi_{eop} \cdot r} \sum_{i=1}^{\infty} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{P_l(x)}{\lambda_{i} \cdot \mu_{m} \cdot \eta_{n}} \]

\[ \sigma_{xy} = -\lambda E \cdot e^{\psi_{eop} \cdot r} \sum_{i=1}^{\infty} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{Q_m(y)}{\lambda_{i} \cdot \mu_{m} \cdot \eta_{n}} \]

\[ \sigma_{zz} = -\lambda E \cdot e^{\psi_{eop} \cdot r} \sum_{i=1}^{\infty} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{R_n(z)}{\lambda_{i} \cdot \mu_{m} \cdot \eta_{n}} \]

**NUMERICAL RESULTS**

Set \( a = 1 \), \( b = 2 \), \( h = 2 \), \( t = 1 \) sec and \( k = 0.86 \) in equation (10.1)-(10.9), we obtain

\[ T(x, y, z, t) = e^{\psi_{eop} \cdot r} \sum_{i=1}^{\infty} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{P_l(x)}{\lambda_{i} \cdot \mu_{m} \cdot \eta_{n}} \]

\[ \left( e^{-i} \right) \times \left[ \frac{a \cos^2(a,a) - \cos(a,a) \sin(a,a)}{a_i^2} \right] \]

\[ \left( e^{-i} \right) \times \left[ \frac{b \cos^2(2b_m) - \cos(2b_m) \sin(2b_m)}{b_i^2} \right] \]

\[ \left( e^{-i} \right) \times \left[ \frac{h \cos^2(2h_n) - \cos(2h_n) \sin(2h_n)}{h_i^2} \right] \]

\[ U = -\lambda E \cdot e^{\psi_{eop} \cdot r} \sum_{i=1}^{\infty} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{P_l(x)}{\lambda_{i} \cdot \mu_{m} \cdot \eta_{n}} \]

\[ \left( e^{-i} \right) \times \left[ \frac{a \cos^2(a,a) - \cos(a,a) \sin(a,a)}{a_i^2} \right] \]

\[ \left( e^{-i} \right) \times \left[ \frac{b \cos^2(2b_m) - \cos(2b_m) \sin(2b_m)}{b_i^2} \right] \]

\[ \left( e^{-i} \right) \times \left[ \frac{h \cos^2(2h_n) - \cos(2h_n) \sin(2h_n)}{h_i^2} \right] \]
\[ u_x = \frac{\lambda}{2} \int_{-\frac{h}{2}}^{\frac{h}{2}} e^{\nu y} \mathcal{F}(y) dy \]

\[ \left( e^{-1} \right) \times \left[ a_i \cos(a_i) - \cos(a_i) \sin(a_i) \right] \]

\[ \left( e^{-1} \right) \times \left[ b_m \cos^2(b_m) - \cos(2b_m) \sin(2b_m) \right] \]

\[ \left( e^{-1} \right) \times \left[ h_n \cos^2(2h_n) - \cos(2h_n) \sin(2h_n) \right] + \int_0^\infty \psi e^{\nu y} dt' \]

\[ \sigma_{xx} = -\lambda E \psi e^{\nu y} \sum_{l=1}^\infty \sum_{m=1}^\infty \frac{P_l(x)}{\lambda_i \mu_m \eta_n} \]

\[ \left( e^{-1} \right) \times \left[ a_i \cos(a_i) - \cos(a_i) \sin(a_i) \right] \]

\[ \left( e^{-1} \right) \times \left[ b_m \cos^2(b_m) - \cos(2b_m) \sin(2b_m) \right] \]

\[ \left( e^{-1} \right) \times \left[ h_n \cos^2(2h_n) - \cos(2h_n) \sin(2h_n) \right] + \int_0^\infty \psi e^{\nu y} dt' \]

\[ \left( e^{-1} \right) \times \left[ Q_n''(y)R_n(z) + P_n(x)R_n''(z) \right] \]

\[ \sigma_{yy} = -\lambda E \psi e^{\nu y} \sum_{l=1}^\infty \sum_{m=1}^\infty \frac{Q_m(y)}{\lambda_i \mu_m \eta_n} \]

\[ \left( e^{-1} \right) \times \left[ a_i \cos(a_i) - \cos(a_i) \sin(a_i) \right] \]

\[ \left( e^{-1} \right) \times \left[ b_m \cos^2(b_m) - \cos(2b_m) \sin(2b_m) \right] \]

\[ \left( e^{-1} \right) \times \left[ h_n \cos^2(2h_n) - \cos(2h_n) \sin(2h_n) \right] + \int_0^\infty \psi e^{\nu y} dt' \]

\[ \left( e^{-1} \right) \times \left[ P_l(x)R_n(z) + P_l(x)R_n''(z) \right] \]

\[ \sigma_{zz} = -\lambda E \psi e^{\nu y} \sum_{l=1}^\infty \sum_{m=1}^\infty \frac{R_n(z)}{\lambda_i \mu_m \eta_n} \]

\[ \left( e^{-1} \right) \times \left[ a_i \cos(a_i) - \cos(a_i) \sin(a_i) \right] \]

\[ \left( e^{-1} \right) \times \left[ b_m \cos^2(b_m) - \cos(2b_m) \sin(2b_m) \right] \]

\[ \left( e^{-1} \right) \times \left[ h_n \cos^2(2h_n) - \cos(2h_n) \sin(2h_n) \right] + \int_0^\infty \psi e^{\nu y} dt' \]

\[ \left( e^{-1} \right) \times \left[ E(x)Q_n'(y) + E(x)Q_n''(y) \right] \]

CONCLUSION

In both the problems, the temperature distribution, displacement function and thermal stresses of a thin rectangular plate have been derived, with the aid of finite Marchi-Fasulo transform technique when the stated boundary conditions are known. The results are obtained in terms of Bessel's function in the form of infinite series. The series solutions converge provided if we take sufficient number of terms in the series. The numerical results are calculated and depicted graphically. The results that are obtained can be applied to the design of useful structures or machines in engineering applications.
Graph 4: Displacement component vs. x

Graph 5: Stress function vs. x

Graph 6: Stress function vs. x

Graph 7: Temperature distribution vs x

Graph 8: Airy’s stress function vs x

Graph 9: Displacement component vs x
Graph 10: Displacement component vs x

Graph 11: Displacement component vs x

Graph 12: Stress function vs x

Graph 13: Stress function vs x

Graph 14: Stress function vs x

REFERENCES

[10] Ishihara, Noda and Tanigawa, “Theoretical analysis of thermoelastic plastic deformation of a circular plate due to


