Thermoelastic Problems of a Hollow Cylinder and its Thermal Stresses

INTRODUCTION


FORMULATION OF THE PROBLEM-I

Consider a hollow cylinder as shown in the figure 1. The material of the cylinder is isotropic, homogenous and all properties are assumed to be constant. We assume that the cylinder is of a small thickness and its boundary surfaces remain traction free. The initial temperature of the cylinder is the same as the temperature of the surrounding medium, which is kept constant.

The displacement function $\phi(r, z)$ satisfying the differential equation as Khobragade [9-11] is

$$\frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r} \frac{\partial \phi}{\partial r} + \frac{\partial^2 \phi}{\partial z^2} = \left(1 + \nu \right) \frac{1}{1 - \nu} \mu T$$

with $\phi = 0$ at $r = a$ and $r = b$
where \( \nu \) and \( a_i \) are Poisson ratio and linear coefficient of thermal expansion of the material of the cylinder respectively and \( T(r,z) \) is the heating temperature of the cylinder satisfying the differential equation as Khobragade [9-11] is

\[
\left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) + \frac{\partial^2 T}{\partial z^2} \right] + \frac{g(r,z)}{k} = 0
\]

(2.3)

where \( \kappa = K / \rho c \) is the thermal diffusivity of the material of the cylinder, \( K \) is the conductivity of the medium, \( c \) is its specific heat and \( \rho \) is its calorific capacity (which is assumed to be constant) respectively, subject to the boundary conditions

\[
M_r(T,1,\bar{K}_1,a) = F_1(z), \quad \text{for all} \quad -h \leq z \leq h, \quad (2.4)
\]

\[
M_r(T,1,\bar{K}_2,b) = F_0(z), \quad \text{for all} \quad -h \leq z \leq h, \quad (2.5)
\]

\[
M_z(T,1,k_3,-h) = F_3(r), \quad \text{for all} \quad a \leq r \leq b, \quad (2.6)
\]

\[
M_z(T,1,k_4,h) = G(r), \quad \text{for all} \quad a \leq r \leq b, \quad (2.7)
\]

being: \( M_{\vartheta}(f, \bar{K}, \tilde{K}, s) = (\bar{K} f + \tilde{K} \tilde{f})_{\vartheta=\varphi} \)

where the prime (') denotes differentiation with respect to \( \vartheta \), radiation constants are \( \bar{K} \) and \( \tilde{K} \) on the curved surfaces of the plate respectively.

The radial and axial displacement \( U \) and \( W \) satisfy the uncoupled thermoelastic equation as Khobragade [9-11] are

\[
\nabla^2 U - \frac{U}{r^2} + (1 - 2\nu)^{-1} \frac{\partial e}{\partial r} = 2 \left( \frac{1 + \nu}{1 - 2\nu} \right) a_i \frac{\partial T}{\partial r}
\]

(2.8)

\[
\nabla^2 W + (1 - 2\nu)^{-1} \frac{\partial e}{\partial z} = 2 \left( \frac{1 + \nu}{1 - 2\nu} \right) a_i \frac{\partial T}{\partial z}
\]

(2.9)

where

\[
e = \frac{\partial U}{\partial z} - \frac{U}{r} + \frac{\partial W}{\partial r}
\]

(2.10)

\[
U = \frac{\partial \varphi}{\partial z},
\]

(2.11)

\[
W = \frac{\partial \vartheta}{\partial z}
\]

(2.12)

The stress functions are given by

\[
\tau_{zz}(a,z) = 0, \tau_{zz}(b,z) = 0, \tau_{rr}(r,0) = 0
\]

(2.13)

\[
\sigma_z(a,z) = p_i, \quad \sigma_r(b,z) = -p_0, \quad \sigma_z(r,0) = 0
\]

(2.14)

where \( p_i \) and \( p_0 \) are the surface pressure assumed to be uniform over the boundaries of the cylinder. The stress functions are expressed in terms of the displacement components by the following relations as Khobragade [9-11] are

\[
\sigma_r = (\lambda + 2\nu) \frac{\partial U}{\partial r} + \lambda \left( \frac{U}{r} + \frac{\partial W}{\partial z} \right)
\]

(2.15)

\[
\sigma_z = (\lambda + 2\nu) \frac{\partial W}{\partial z} + \lambda \left( \frac{U}{r} + \frac{\partial U}{\partial r} + \frac{W}{r} \right)
\]

(2.16)

\[
\sigma_\theta = (\lambda + 2\nu) \frac{U}{r} + \lambda \left( \frac{\partial U}{\partial r} + \frac{\partial W}{\partial z} \right)
\]

(2.17)

\[
\tau_{rz} = G \left( \frac{\partial W}{\partial r} + \frac{\partial U}{\partial z} \right)
\]

(2.18)

where \( \lambda = 2G\nu(1-2\nu) \) is the lème’s constant, \( G \) is the shear modulus and \( U, W \) are the displacement components, ease of use.

**SOLUTION OF THE OF THE PROBLEM**

Applying Marchi-Fasulo transform on equation (2.1), we get

\[
\frac{\partial^2 \tilde{T}}{\partial r^2} + \frac{1}{r} \frac{\partial \tilde{T}}{\partial r} - \lambda_n^2 \tilde{T} = \Psi
\]

(2.19)

where \( \Psi = -\frac{P_0(h)}{k_3} F_3(r) - \frac{P_0(h)}{k_4} G(r) - \frac{g(r,z)}{k} \)

Equation (2.6) is a Bessel’s equation whose solution yields

\[
\tilde{T} = A I_0(\lambda_n r) + B K_0(\lambda_n r) + F(r)
\]

(2.20)

where \( F(r) \) is the P.I.

As \( r \to 0, K_0 \to \infty \), But \( \tilde{T} \) is finite.

\[
\therefore \quad B = 0
\]

(2.21)

\[
A = \frac{\tilde{F}_1(0) - k_1 F'(a) - F(a)}{I_0(\lambda_n a) + k_1 I_0'(\lambda_n a)}
\]

(2.22)

\[
\therefore \quad \tilde{T} = \tilde{F}_1(z) - k_1 F'(a) - F(a) + \frac{A k_1 I_0'(\lambda_n a)}{I_0(\lambda_n a) + k_1 I_0'(\lambda_n a)}
\]

(2.23)

Applying inverse Marchi-Fasulo transform an equation (3.3) we get,

\[
T = \sum_{n=1}^{\infty} \frac{P_n(z)}{\lambda_n} \left[ \tilde{F}_1(z) - k_1 F'(a) - F(a) \right] I_0(\lambda_n r) + F(r)
\]

(2.24)

**DETERMINATION OF DISPLACEMENT AND STRESS COMPONENTS**

Substituting the value of temperature distribution from (3.4) in equation (2.1) one obtains the thermo elastic displacement function \( \phi(r,z) \) as

\[
\phi = r^2 \left( \frac{1 + \nu}{1 - 2\nu} \right) a_i
\]

Figure 1: Geometry of the problem
\[ \times \sum_{n=1}^{\infty} P_n(z) \frac{F_1(z) - k_1 F'(a) - F(a)}{I_0(\lambda_0 a) + k_1 I'_0(\lambda_0 a)} \] (4.1)

The stress components are
\[ U = \frac{a_t}{4} \left\{ \frac{1}{1-v} \sum_{n=1}^{\infty} P_n(z) \frac{F_1(z) - k_1 F'(a) - F(a)}{I_0(\lambda_0 a) + k_1 I'_0(\lambda_0 a)} \right\} \]
\[ \times \left[ r^2 I_0(\lambda_0 r) + 2r I_0(\lambda_0 r) \right] + r^2 F(r) \right\} \]
\[ W = r^2 \left( \frac{1+v}{1-v} \right) \frac{a_t}{4} \sum_{n=1}^{\infty} P_n(z) \frac{F_1(z) - k_1 F'(a) - F(a)}{I_0(\lambda_0 a) + k_1 I'_0(\lambda_0 a)} \cdot F(r) \right\} \]

(4.2)

(4.3)

**DETERMINATION OF STRESS FUNCTION**

Substituting the value of (4.2) and (4.3) in equations (2.15)- (2.18) one obtains the thermal stresses as

\[ \sigma_\tau = \frac{a_t}{4} \left\{ \frac{1}{1-v} \sum_{n=1}^{\infty} \frac{1}{\lambda_0} \left[ F_1(z) - k_1 F'(a) - F(a) \right] \right\} \]
\[ \times \left[ \frac{r^2 I_0(\lambda_0 r) + 2r I_0(\lambda_0 r)}{l_0(\lambda_0 a) + k_1 I'_0(\lambda_0 a)} \right] \]
\[ + \lambda \left( \frac{r^2 I_0(\lambda_0 r) + 2r I_0(\lambda_0 r)}{l_0(\lambda_0 a) + k_1 I'_0(\lambda_0 a)} \right) \]
\[ + P_n(z) \frac{r^2 F(r) + 2F(r)}{l_0(\lambda_0 a) + k_1 I'_0(\lambda_0 a)} \]
\[ + P_n(z) \frac{r^2 F(r) + 2F(r)}{l_0(\lambda_0 a) + k_1 I'_0(\lambda_0 a)} \]

(5.1)

\[ \sigma_\nu = \frac{a_t}{4} \left\{ \frac{1}{1-v} \sum_{n=1}^{\infty} \frac{1}{\lambda_0} \left[ F_1(z) - k_1 F'(a) - F(a) \right] \right\} \]
\[ \times \left[ \frac{r^2 I_0(\lambda_0 r) + 2r I_0(\lambda_0 r)}{l_0(\lambda_0 a) + k_1 I'_0(\lambda_0 a)} \right] \]
\[ + \lambda \left( \frac{r^2 I_0(\lambda_0 r) + 2r I_0(\lambda_0 r)}{l_0(\lambda_0 a) + k_1 I'_0(\lambda_0 a)} \right) \]
\[ + P_n(z) \frac{r^2 F(r) + 2F(r)}{l_0(\lambda_0 a) + k_1 I'_0(\lambda_0 a)} \]
\[ + P_n(z) \frac{r^2 F(r) + 2F(r)}{l_0(\lambda_0 a) + k_1 I'_0(\lambda_0 a)} \]

(5.2)

\[ \sigma_\xi = \frac{a_t}{4} \left\{ \frac{1}{1-v} \sum_{n=1}^{\infty} \frac{1}{\lambda_0} \left[ F_1(z) - k_1 F'(a) - F(a) \right] \right\} \]
\[ \times \left[ \frac{r^2 I_0(\lambda_0 r) + 2r I_0(\lambda_0 r)}{l_0(\lambda_0 a) + k_1 I'_0(\lambda_0 a)} \right] \]
\[ + \lambda \left( \frac{r^2 I_0(\lambda_0 r) + 2r I_0(\lambda_0 r)}{l_0(\lambda_0 a) + k_1 I'_0(\lambda_0 a)} \right) \]
\[ + P_n(z) \frac{r^2 F(r) + 2F(r)}{l_0(\lambda_0 a) + k_1 I'_0(\lambda_0 a)} \]
\[ + P_n(z) \frac{r^2 F(r) + 2F(r)}{l_0(\lambda_0 a) + k_1 I'_0(\lambda_0 a)} \]

(5.3)

\[ \tau_{\xi\nu} = \frac{a_t}{2} \left\{ \frac{1}{1-v} \sum_{n=1}^{\infty} P_n(z) \left[ F_1(z) - k_1 F'(a) - F(a) \right] \right\} \]
\[ \times \left[ r^2 I_0(\lambda_0 r) + 2r I_0(\lambda_0 r) \right] + r^2 F(r) \right\} \]

(5.4)

**SPECIAL CASE**

Set
\[ f(r, t) = (1 - e^{-t}) \Delta (r - r_0) \]

(6.1)

Applying finite transform defined in Marchi Zgrablich [2] to the equation (32) one obtains
\[ \tilde{f}(n, t) = (1 - e^{-t}) r_0 S_0(k_1, k_2, \mu_n r_0) \]

(6.2)

Substituting the value of (32) in the equations (21) to (31) one obtains
\[ T = \sum_{n=1}^{\infty} P_n(z) \frac{F_1(z) - k_1 F'(a) - F(a)}{I_0(\lambda_0 a) + k_1 I'_0(\lambda_0 a)} \right\} \]

(6.3)

**NUMERICAL RESULTS, DISCUSSION AND REMARKS**

To interpret the numerical computation we consider material properties of low carbon steel (AISI 1119), which can be used for medium duty shafts, studs, pins, distributor cams, cam shafts, and universal joints having mechanical and thermal properties \( \kappa = 13.97 \mu m/s^2 \) \( \nu = 0.29 \) \( \lambda = 51.9 \ W/(m-K) \) and \( a_t = 14.7 \mu m/m^0 \ C \).

Setting the physical parameter with \( a = 0.5 \) , \( b = 1 \) and \( h = 3 \).

**FORMULATION OF THE PROBLEM-II**

Consider a hollow cylinder as shown in the figure 1. The material of the cylinder is isotropic, homogenous and all properties are assumed to be constant. We assume that the cylinder is of a small thickness and its boundary surfaces remain traction free. The initial temperature of the cylinder is the same as the temperature of the surrounding medium, which is kept constant.

The displacement function \( \phi(r, z, t) \) satisfying the differential equation as Khobragade [9-11] is

\[ \frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r} \frac{\partial \phi}{\partial r} + \frac{\partial^2 \phi}{\partial z^2} = 0 \]

(8.1)

with \( \phi = 0 \) at \( r = a \) and \( r = b \)

(8.2)

where \( \nu \) and \( a_t \) are Poisson ratio and linear coefficient of thermal expansion of the material of the cylinder respectively and \( T(r, z, t) \) is the heating temperature of the cylinder at time \( t \) satisfying the differential equation as Khobragade [9-11] is

\[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) + \frac{\partial^2 T}{\partial z^2} + \frac{g(r, z, t)}{K} = \frac{1}{\alpha} \frac{\partial T}{\partial t} \]

(8.3)

where \( \kappa = K / \rho \) is the thermal diffusivity of the material of the cylinder, \( K \) is the conductivity of the medium, \( C \) is its specific heat and \( \rho \) is its calorific capacity (which is assumed to be constant) respectively, subject to the initial and boundary conditions

\[ M_1(T, 1, 0, 0) = F \]

for all \( a \leq r \leq b \), \( -h \leq z \leq h \)

(8.4)

\[ M_1(T, 1, k_1, a) = F_1(z, t), \text{ for all } -h \leq z \leq h, t > 0 \]

(8.5)

\[ M_1(T, 1, k_2, b) = F_2(z, t), \text{ for all } -h \leq z \leq h, t > 0 \]

(8.6)

\[ M_2(T, 1, k_3, h) = F_3(r, t), \text{ for all } a \leq r \leq b, t > 0 \]

(8.7)

\[ M_2(T, 1, k_4, h) = G(r, t), \text{ for all } a \leq r \leq b, t > 0 \]

(8.8)

where \( M_\phi(f, k, \kappa, \gamma, s) = (K f + \tilde{F}) \gamma_{a,s} \)

where the prime (’ ) denotes differentiation with respect to \( \phi \), radiation constants are \( \kappa \) and \( \kappa \) on the curved surfaces of the plate respectively.
The radial and axial displacement \( U \) and \( W \) satisfy the uncoupled thermoelastic equation as Khobragade [9-11] are

\[
\nabla^2 U - \frac{U}{r^2} + (1-2\nu)^{-1} \frac{\partial e}{\partial r} = 2 \left( \frac{1+\nu}{1-2\nu} \right) a_i \frac{\partial T}{\partial r}
\]

(8.9)

\[
\nabla^2 W + (1-2\nu)^{-1} \frac{\partial e}{\partial z} = 2 \left( \frac{1+\nu}{1-2\nu} \right) a_i \frac{\partial T}{\partial z}
\]

(8.10)

where

\[
e = \frac{\partial U}{\partial r} + \frac{U}{r} + \frac{\partial W}{\partial z}
\]

(8.11)

\[
U = \frac{\partial \phi}{\partial r},
\]

(8.12)

\[
W = \frac{\partial \phi}{\partial z}
\]

(8.13)

The stress functions are given by

\[
\tau_{rr}(a, z, t) = 0, \quad \tau_{rz}(b, z, t) = 0, \quad \tau_{zz}(r, 0, t) = 0
\]

(8.14)

\[
\sigma_r(a, z, t) = p_i, \quad \sigma_r(b, z, t) = -p_o, \quad \sigma_z(r, 0, t) = 0
\]

(8.15)

where \( p_i \) and \( p_o \) are the surface pressure assumed to be uniform over the boundaries of the cylinder. The stress functions are expressed in terms of the displacement components by the following relations as Khobragade [9-11] are

\[
\sigma_r = (\lambda + 2G) \frac{\partial U}{\partial r} + \lambda \left( \frac{U}{r} + \frac{\partial W}{\partial z} \right)
\]

(8.16)

\[
\sigma_z = (\lambda + 2G) \frac{\partial W}{\partial z} + \lambda \left( -\frac{U}{r} + \frac{\partial W}{\partial z} \right)
\]

(8.17)

\[
\sigma_\theta = G \left( \frac{\partial W}{\partial r} + \frac{\partial U}{\partial z} \right)
\]

(8.18)

\[
\tau_{\theta r} = G \frac{\partial r}{\partial r} + \frac{\partial U}{\partial z}
\]

(8.19)

where \( \lambda = 2G \nu/(1-2\nu) \) is the Lame’s constant, \( G \) is the shear modulus and \( U, W \) are the displacement components. Equations (8.1)- (8.19) constitute the mathematical formulation of the problem under consideration.

**SOLUTION OF THE OF THE PROBLEM-II**

Applying transform defined in [18] to the equations (8.3), (8.4) and (8.6) over the variable \( \rho' \) having \( p = 0 \) with responds to the boundary conditions of type (8.5) and then Marchi-Fasulo transform, one obtains

\[
\bar{T}^*(n, z, s) = e^{-\alpha p^2 t} \left[ \bar{F}^* + \int_0^t \psi e^{\alpha p^2 t'} dt' \right]
\]

(9.1)

where constants involved \( \bar{T}^*(n, z, s) \) are obtained by using boundary conditions (8.6). Finally applying the inversion theorems of transform defined in [18] and inverse Marchi-Fasulo transform, one obtains the expressions of the temperature distribution \( T(r, z, t) \) for heating processes as

\[
T(r, z, t) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} P_m(z) S_0(k_1, k_2, \mu_n r) \mu_n^2 \lambda_m
\]

\[
\times e^{-\alpha p^2 t} \left[ \bar{F}^* + \int_0^t \psi e^{\alpha p^2 t'} dt' \right]
\]

(9.2)

where \( n \) is the transformation parameter as defined in appendix, \( m \) is the Marchi-Fasulo transform parameter.

**DETERMINATION OF DISPLACEMENT AND STRESS FUNCTION**

Substituting the value of temperature distribution from (9.2) in equation (8.1) one obtains the thermoelastic displacement \( \phi(r, z, t) \) as

\[
\phi(r, z, t) = -\frac{r^2 a_i}{4(1-\nu)} \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} P_m(z) \left[ 2n S_0(k_1, k_2, \mu_n r) + \mu_n \lambda_m S_0(k_1, k_2, \mu_n r) \right]
\]

\[
\times e^{-\alpha p^2 t} \left[ \bar{F}^* + \int_0^t \psi e^{\alpha p^2 t'} dt' \right]
\]

(10.1)

Using (10.1) in the equations (8.11) and (8.12) one obtains

\[
U = \frac{a_i (1+\nu)}{4(1-\nu)} \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} P_m(z) [ 2n S_0(k_1, k_2, \mu_n r) + \mu_n \lambda_m S_0(k_1, k_2, \mu_n r) ]
\]

\[
\times e^{-\alpha p^2 t} \left[ \bar{F}^* + \int_0^t \psi e^{\alpha p^2 t'} dt' \right]
\]

(10.2)

\[
W = \frac{r^2 a_i (1+\nu)}{4(1-\nu)} \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} P_m(z) S_0(k_1, k_2, \mu_n r)
\]

\[
\times e^{-\alpha p^2 t} \left[ \bar{F}^* + \int_0^t \psi e^{\alpha p^2 t'} dt' \right]
\]

(10.3)

Substitution the value of (10.2), (10.3) in (8.16) to (8.19) one obtains the stress functions as

\[
\sigma_r = \frac{a_i (1+\nu)}{4(1-\nu)} \sum_{m,n=1}^{\infty} \Phi_{mn} P_m (z).
\]
\[
\begin{align*}
&\left[\lambda + 2G\left(2S_0\mu_n + r^2S_0\mu_n + S_02r + 2S_0 + \frac{G}{r}(k_1, k_2, \mu_n)\right)(rS_0\mu_n + 2S_0)\right] \\
&+ \lambda r^2S_0\left(k_1, k_2, \mu_n\right) \times P^m_n
\end{align*}
\] (10.4)

\[
\sigma_z = \frac{a_1}{4} \left[1 + \frac{1}{\lambda} \right] \sum_{m,n=1}^{\infty} \Phi_{mn} \left[\left(r^2S_0\left(k_1, k_2, \mu_n\right)\right)P^m_n + \frac{P_n(z)}{\lambda}\right] \\
\left(2S_0\mu_n + r^2S_0\mu_n^2 + S_02r + 2S_0 + \frac{1}{r}(rS_0\mu_n + 2S_0)\right)
\] (10.5)

\[
\sigma_n = \frac{a_1}{4} \left[1 + \frac{1}{\lambda} \right] \sum_{m,n=1}^{\infty} \Phi_{mn} \left[\frac{P_n(z)}{\lambda} \left(r^2S_0\mu_n + 2S_0\right) \right] \\
+ \lambda \left[2S_0\mu_n + r^2S_0\mu_n^2 + S_02r + 2S_0\right] + \lambda r^2S_0\left(k_1, k_2, \mu_n\right) P^m_n
\] (10.6)

\[
\tau_{rl} = P_m \frac{a_1}{4} \left[1 + \frac{1}{\lambda} \right] \sum_{m,n=1}^{\infty} \Phi_{mn} \left(2G_0^2\mu_n + 4G_0\right)
\] (10.7)

Where \( \Phi_{mn} = \frac{e^{-\alpha p^2 t}}{\mu_n^2 \lambda_m^2} \left[F^* + \int_0^t \psi e^{\alpha p^2 t'} dt' \right] \)

**SPECIAL CASE**

Set \( f(r, t) = (1 - e^{-t}) \delta(r - r_0) \) (11.1)

Applying finite transform defined in Marchi Zgrablich [18] to the equation (11.1) one obtains

\[
\tilde{f}_n(t) = (1 - e^{-t}) \mu_0 S_0(k_1, k_2, \mu_n r_0)
\] (11.2)

Substituting the value of (11.2) in the equations (9.2) one obtains

\[
T(r, z, t) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} P_m(z) S_0(k_1, k_2, \mu_n r) \\
\times e^{-\alpha p^2 t} \left[F^* + \int_0^t \psi e^{\alpha p^2 t'} dt' \right]
\] (11.3)

**NUMERICAL RESULTS**

Set \( a = 0.5, b = 1 \) and \( h = 3 \) \( \text{sec} \) \( k_1 = 0.25, k_2 = 0.25 \) in equation (11.3) we get

\[
T(r, z, t) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} P_m(z) S_0(0.25, 0.25, \mu_n r) \\
\times e^{-\alpha p^2 t} \left[F^* + \int_0^t \psi e^{\alpha p^2 t'} dt' \right]
\] (12.1)

**NUMERICAL RESULTS, DISCUSSION AND REMARKS**

To interpret the numerical computation we consider material properties of low carbon steel (AISI 1119), which can be used for medium duty shafts, studs, pins, distributor cams, cam shafts, and universal joints having mechanical and thermal properties

\[\kappa = 13.97 \text{[} \mu\text{m/s}^2 \] \( \nu = 0.29, \lambda = 51.9 \text{[} W/(m-K) \] \} and \( a_j = 14.7 \mu\text{m/m}^0 C \).

**CONCLUSION**

In this paper, we modify the conceptual idea proposed by Khobragade et al. [174] for hollow cylinder and the temperature distributions, displacement and stress functions at the edge \( z = h \) occupying the region of the cylinder \( a \leq r \leq b, -h \leq z \leq h \) have been obtained with the known boundary conditions. We develop the analysis for the temperature field by introducing the transformation defined by Zgrablich et al., finite Fourier sine transform and Laplace transform techniques with boundaries conditions of radiations type. The series solutions converge provided we take sufficient number of terms in the series. Since the thickness of cylinder is very small, the series solution given here will be definitely convergent. Assigning suitable values to the parameters and functions in the series expressions can derive any particular case. The temperature, displacement and thermal stresses that are obtained can be applied to the design of useful structures or machines in engineering applications.
REFERENCES


