EEG Signals Analysis for motor imagery based on Curvelet Transform

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Abstract: EEG-based brain-computer interface is a computer-based system that provides effective communication and control channels between human brain and computer to carry out a desired action. However, the classification of single-trial EEG signals and controlling a device continuously during motor imagery is a difficult task. In this paper, we propose a feature extraction method for single trial online motor imagery using curvelet transform. These curvelet coefficients were used to extract the characters from the motor imagery EEG and classify the pattern of left and right hand movement imagery by Bayesian analysis with Gaussian model. The performance of motor imagery tested by the eye dataset for BCI competition 2003. The hypothetical results presented highest classification accuracy of 96% and superior information transfer rate is obtained.

Keywords: Electroencephalograph (EEG), Curvelet coefficients, Motor imagery, Bayesian classifier, Gaussian model, Brain-computer interface (BCI).

I. INTRODUCTION

Electroencephalograph based Brain-computer Interface provides a non-muscular communication for the generation of movement related signals to drive an assistive device. Electroencephalograph recordings during motor imagery tasks are frequently used as input signals for brain-computer interfaces [1]. Single-trial identification of motor imagery EEG is one of the key methods in the brain-computer interface. Motor imagery can modify the neuronal activity in the primary sensori motor areas, so as a result it can serve to generate self-induced variations of the EEG. One of the issues in BCI research is the presence of superfluous data in the features of a given dataset, which not only increases the processing time but also reduces the accuracy of the classifiers [2]. Classification of Electroencephalography signal is an open area of research in Brain-computer interfacing. The classifiers detect the different mental states produced by a subject to control an external prosthesis. Brain Computer Interface techniques are used to assist disabled people to translate brain signals to control commands imitating peculiar human thoughts based on Electroencephalography signal processing [3].

We presented a novel method for motor imagery. We mingle the curvelet transform with Gaussian model to excerpt features extra efficient for non-stationary EEG signals. This paper is organized as follows: In the next section, we discuss EEG in BCI research is the presence of superfluous data in the features of a given dataset, which not only increases the processing time but also reduces the accuracy of the classifiers [2]. Classification of Electroencephalography signal is an open area of research in Brain-computer interfacing. The classifiers detect the different mental states produced by a subject to control an external prosthesis. Brain Computer Interface techniques are used to assist disabled people to translate brain signals to control commands imitating peculiar human thoughts based on Electroencephalography signal processing [3].

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II. EEG SIGNAL MODEL

The EEG signals demonstrate the electrical activity of the person brain. They are extremely non-linear, non-Gaussian and non-stationary in natural world and may hold useful information concern brain state. However, it is complicated to get valuable data from these signals straightly in time domain. The transmission of the brain cells take place through electrical impulses. It is calculated by placing the electrodes on the scalp of the object. The poetical nerve cell inhibitor and excitedly postsynaptic potentials set up the EEG signals. These postsynaptic potentials analyze in the cortex and enlarge to the scalp plane where they are registered as EEG. A classic EEG signal, deliberate from the scalp, amplitude scope from 10 μV to 100 μV.
Electrodes placed on the scalp for the EEG signals are registered purpose. There are two types of EEG registrations: (i) Bi-polar: Bipolar electrodes provide the voltage disparity between two scalp electrodes (ii) Mono-polar: Mono-polar registrations pick up the voltage disparity between a dynamic electrode on the scalp and a pose electrode on the ear lobe. The electric sources inside the brain produce electrical and magnetic fields that can be modeled by the Maxwell Equations. The electric current is assumed to be of the form 

\[ K(x) = \sigma(x) E(x) + K_i(x) \]

Where \( \sigma \) is the conductive function, \( \sigma E \) is the microscopic electric field and \( ji \) is the impressed current (microscopic level). During an epileptic seizure, spikes can be monitored along the EEG signals. They are mainly generated by the impressed current. Due to the high speed of dissemination of the electric waves inside the head, there is no lag in the data acquisition by the EEG recorder. Hence, in order to find the position of the effected current, we consider the EEG data instant at which one of the spikes attain its highest amplitude. Therefore, a time-frequency Maxwell equation may be used to model the relationship between the electrical potential and the impressed present \( K_i \) (Hamalainen et al., 1993).

\[
\nabla \cdot \left( \sigma(x) \nabla u(x) \right) = \nabla \cdot K_i(x) \quad x \in H
\]

\[
\frac{\partial u(x)}{\partial t} = 0, \quad x \in \partial H \quad \text{Subject to} \quad [u]_{T_i} = 0, \quad [\sigma(x) \frac{\partial u}{\partial n}]_{T_i} = 0.
\]

Where \( H \) represents the head having different compartments with transition surfaces named \( T_i \) and \([\ ]\) denotes the variation between the values of the functions within the brackets through the indicated surface.

III. RELATED WORK

Electroencephalograph based Brain-computer Interface provides a non-muscular communication to drive assistive devices using movement related signals, generated from the motor activation areas of the brain. Electroencephalogram recordings during motor imagery tasks are often used as input signals for brain-computer interfaces. Single-trial identification of motor imagery EEG is one of the key techniques in the brain-computer interface. The dimensions of the feature vector play an important role in BCI, which not only increases the computational time but also reduces the accuracy of the classifiers. Classification of Electroencephalography signal is an open area of research in brain-computer interfacing. The classifiers detect the different mental states generated by a subject to control an external prosthesis. There have been several algorithms developed so far for processing EEG signals. The operations include, but are not limited to, time-domain, frequency-domain, spatial-domain analysis, and multi way processing. Also, several algorithms have been developed to show the brain activity from images reconstructed from only the EEGs. The following research articles are revived and helpful for the present work:

Zhu Chen & Ah Chung presents a feature extraction method for the application of an artificial neural network technique. In this concept different classes of EEG signals were applied: obsessive compulsive disorder, schizophrenia, and common. The classification is a three-layered feed forward network used architecture of artificial neural network in which appliance the back propagation of error learning algorithm. The wavelet transform provides a potentially dominant method for preprocessing EEG signals classified priority wise[5].

Effectively eradication EEG data features is the key point in Brain Computer Interface technology. Deep Learning algorithm was applied, EEG data classified based on Motor Imagery task. For the classification of left and right hand motor imagery, primarily, based on certain single channel, a weak classifier was trained by deep belief net then hire the thought of Ada-boost algorithm to unite the trained weak classifiers as a more commanding one. The presentation of the DBN was tested with different combinations of hidden units and hidden layers on multiple subjects.

Single trial recognition of motor imagery EEG is one of the key techniques in the brain computer interface. To get better accuracy of classification and reduce the algorithm time, targeting at motor imagery EEG of four kinds of motion, Peng Lu, Daoren Yuan introduced a single-trial recognition algorithm of MI EEG based on HHT and SVM. Firstly, MI EEG is decomposed into 8-order intrinsic mode function and fringe \( R \) by empirical mode decomposition. Secondly, Hilbert spectrum is got by Hilbert transformation. 6-order IMF is extracted AR model parameter. The acquired 6-order AR parameter and the feature quantity of 29 power spectral density included in the 4-32 Hz EEGs constitute a 35 dimensional feature vector. So support vector machine is used to classify [6].

Umut Orhan Hekim & Mahmut Ozer proposed a multi-layer perception neural network based classification model as a diagnostic decision support system in the epilepsy treatment. The decomposition of EEG signals into frequency sub-bands by discrete wavelet transform. The group of wavelet coefficients was using the K-means algorithm for all frequency sub-bands. The probability distributions of EEG signals processed according to sharing of wavelet coefficients to the clusters, and then used as inputs to the MLPNN model. Finally, the proposed model resulted in satisfactory classification accuracy rates [7].

EEG signals includes inventing signal properties that describe EEG activity in such a way that they show the greatest difference between the clusters of EEG signals that are classified and classic fourier transform already triumph in stationary signals. However, EEG signal consist of non-stationary features and it is not right to strictly apply fourier transform to such type of signals. The implementation of a wavelet transform decomposes a two-dimensional ECG signal...
(think of an image) in a set of coefficients associated with different directions and scales. Each analysis of curvelet has its own time duration, time location and frequency band. The curvelet coefficient derive from the curvelet transformation corresponds to assessment of the ECG components in this time segment and frequency band.

The most common use feature extraction methods for EEG signal analysis are autoregressive Models, power spectral density, wavelet transform and Curvelet transform. Discrete curvelet transform has good signal compression properties; it is applicable for many real signals and it is also computationally efficient. For these reasons it is used for many purposes including image compression, noise reduction, numerical integration and pattern recognition. Curvelet transform is a multi-directional and multi-scale transform. Its source functions are pointer shaped and have high directional nervousness and anisotropy. First, describe x as space position parameter, w as frequency domain parameter and (r, p) as polar frequency domain in the 2-dimensional space \( \mathbb{R}^2 \). Z(r) and V(r) are flat non-negative “radius window” and “corner window” correspondingly, and they must satisfy [9]:

\[
\sum_{j=\infty}^{\infty} Z^2(2^j r) = 1, \quad r \in \left( \frac{3}{4}, \frac{3}{2} \right)
\]

\[
\sum_{j=\infty}^{\infty} V^2(2^j r) = 1, \quad r \in \left( -\frac{1}{2}, \frac{1}{2} \right)
\]

For all scales \( j=0 \), define its Fourier frequency domain window:

\[
U_j(r, \theta) = z^{2j} Z(2^j r) V\left(\frac{2^{j/2} \pi}{2\pi}\right)
\]

Frequency domain, curvelet transform is defined as product of \( \varphi_{j,l,k} \) and \( f \in L^2(\mathbb{R}^2) \):

\[
C(j,l,k) = \frac{1}{(2\pi)^2} \int f(\omega) \varphi_{j,l,k}(\omega) d\omega
\]

Curvelet also include components on coarse and fine scale, the same as wavelet theory. Introduce a low-pass window \( W_0 \), which satisfies [10]:

\[
|Z(r)|^2 + \sum_{j>0} |Z(r)|^2 = 1, \quad (k,k_z) \in z
\]

Curvelets differ from wavelet and associated systems, and it takes the form of basic elements, which exhibit a very high directional compassion and are extremely anisotropic. In two dimensions, for instance, curvelets are more suitable for high directional compassion and are extremely anisotropic. In curvelets differ from wavelet and associated systems, which satisfies [9]:

\[
\sum_{j=\infty}^{\infty} Z^2(2^j r) = 1, \quad r \in \left( \frac{3}{4}, \frac{3}{2} \right)
\]

\[
\sum_{j=\infty}^{\infty} V^2(2^j r) = 1, \quad r \in \left( -\frac{1}{2}, \frac{1}{2} \right)
\]

For each \( j \geq j_0 \), a frequency window \( U_j \) in the Fourier domain is defined by the support of \( Z \) and \( V \), the radial and angular windows, applied with scale-dependent window widths in each direction

\[
U_j(r, \theta) = z^{2j} Z(2^j r) V\left(\frac{2^{j/2} \pi}{2\pi}\right)
\]

\[
\varphi_{j,l,k}(x) = \varphi_j(R \omega - x \omega_{j,l,k})
\]

Where \( R \omega \) is the rotation by \( \omega \) radians and \( R^\theta \theta \) its inverse (also its transpose),

\[
R\theta = \left( \begin{array}{cc} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{array} \right)
\]

For a given function \( f \in L^2(\mathbb{R}^2) \), the curvelet coefficients is defined by

\[
C(j,l,k) = \frac{1}{(2\pi)^2} \int f(\omega) \varphi_{j,l,k}(\omega) d\omega
\]

Digital curvelet transforms can also be operated in the frequency domain, and it will be helpful to apply Plancherel’s theorem and state the inner product as the integral over the frequency plane

\[
C(j,l,k) = \frac{1}{(2\pi)^2} \int f(\omega) \varphi_{j,l,k}(\omega) d\omega
\]

B. Feature extraction using Gaussian distribution

Signals produced from random processes usually have a bell shaped probability density function is called normal distribution and it is also known as a gauss distribution. It is usually represented by

\[
X \sim N(\mu, \sigma^2)
\]

It attains its maximum value of \( \frac{1}{\sqrt{\pi \sigma^2}} \) at

\[
x = \mu, \quad \sigma = 2
\]

\[
\mu = E[X] = \int_{\infty}^{\infty} x f(x) dx
\]

\[
\sigma^2 = E[(X- \mu)^2] = \int_{-\infty}^{\infty}(x- \mu)^2 f(x) dx
\]

To evaluate the probability in eqn (1), the error function, \( \text{erf}(x) \), which is related with \( N(0, 1) \) is

\[
\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_{0}^{x} \exp(-y^2) dy
\]
In fact, with a change of variables, eqn (2) may rewritten as

\[
\Pr \{ X \leq x_1 \} = \begin{cases} 
0.5 - \text{erf}(\mu - x_1) / \sigma & \text{for } x_1 \leq \mu \\
0.5 + \text{erf}(\mu - x_1) / \sigma & \text{for } x_1 \geq \mu 
\end{cases}
\]

distributed random data where distribution follows that normal (Gaussian) curve.

![Normal Probability distribution](image)

![Uniform Probability distribution](image)

Figure 1. Normal and uniform probability

We will produce a huge data set of points from a known Gaussian distribution. Mathcad has a built in function, RNORM (N, μ, σ) which preceedes a vector of N data points, where the points are pull from a Gaussian distribution with mean μ and standard deviation σ.

C. Feature classification using Bayesian Analysis

Naive Bayes classifiers is a simple and effective technique for classifiers algorithm. It is depends on Bayes theory with powerful independence expectation, i.e a naive Bayes classifier assumes that the presence or absence of a precise feature is independent to the presence/absence of any other feature, given the class variable. Based on class conditional density assessment and class prior probability, the posterior probability, P(\(\gamma \mid \delta\)) of a test data point can be borrowed considering a Gaussian distribution with mean \(\mu\) and standard deviation \(\sigma\).

The conditional Probability that an incident \(\gamma\) will happen, given that \(\delta\) has occurred and stand by P (A\(\backslash\)B) is define by

\[
P(\gamma \mid \delta) = \frac{P(\gamma \cap \delta)}{P(\delta)} ; P(\delta) > 0 \quad (1)
\]

The Probability of the immediate occurrence of two events \(\gamma\) and \(\delta\) is equal to the product of probability of \(\gamma\) and the conditional Probability of \(\delta\) on the expectation that \(\gamma\) existed. From (1) & (2)

i.e.  \(P(\gamma \cap \delta) = P(\gamma) \cdot P(\delta \mid \gamma) = P(\delta) \cdot P(\gamma \mid \delta) \quad (2)\)

\[
P(\gamma / \delta) = P(\gamma) \cdot P(\delta / \gamma) / P(\delta) \quad \ldots \quad (3)
\]

If \((\gamma \cap \delta)\) and \((\gamma \cap \delta')\) are equally elite tasks then by axiomatic definition

\[
P(\gamma) = P(\delta) \cdot P(\gamma / \delta) + P(\delta') \cdot P(\gamma / \delta') \quad (4)
\]

From (3) & (4)

\[
P(\gamma / \delta) = \frac{P(\delta) \cdot P(\gamma / \delta)}{P(\delta) \cdot P(\gamma / \delta) + P(\delta') \cdot P(\gamma / \delta')} \quad \ldots \quad (5)
\]

If \(\delta 1, \delta 2, \delta 3 \ldots \ldots \ldots\) \(\delta n\) are \(n\) mutually exclusive events of which one of the event occur then

\[
P(\gamma) = \sum_{i=1}^{n} P(\delta_i) \cdot P(\gamma / \delta_i) \quad \ldots \quad (6)
\]

Bayes’ theorem provides a way to calculating the posterior probability, \(P(\gamma / \delta)\) from \(P(\delta), P(\delta / \gamma)\) and \(P(\gamma)\).

The naive bayesian classifier works as follows:

Input: D, a training set of associated class labels and their tuples.

\(\gamma\): (\(\gamma1, \gamma2, \ldots, \gamma n\)) is an \(n\)-dimensional attribute vector. A: (A1, A2, \ldots, An) \(n\) measurements made on the tuple from \(n\) attributes, respectively.

Output: \(\delta : (\delta 1, \delta 2, \ldots, \delta m)\) be \(m\) no of classes.

Method:

Let \(\gamma\) is given tuple; the classifier will predict that \(\gamma\) belongs to the class having the maximum posterior probability, conditioned on \(\gamma\).

If the class prior probabilities are not known, then it is commonly assumed that the classes are equally.

\(\text{i.e.} \quad P(\delta 1) = P(\delta 2) = \ldots = P(\delta m)\)

The class prior probabilities may be estimated by \(P(\delta i) = |\delta i| / |D|\), where \(|\delta i, D|\) is the number of training tuples of class \(\delta i\) in D.

To reduce computation in evaluating \(P(\gamma / \delta i)\), the naive assumption of class-conditional independence is made. Thus, \(P(\gamma / \delta i) = \prod_{k=1}^{n} P(\gamma_i / \delta_i)\)

For continuous attributes, we compute \(P(\gamma / \delta i)\), by considering a Gaussian distribution with a mean \(\mu\), and standard deviation \(\sigma\).

\[
P(\gamma_j \leq Y_j < Y_j + \Delta / Y = y_j = \frac{\gamma_j^2 + \delta f(x; \mu, \sigma 2)}{\Delta} , \text{where } \Delta \text{ is a petite constant.}
\]

To foresee the class label of X, \(P(\delta i / \gamma)\) is appraise for each class \(\delta i\).

The classifier foresee that the class label of tuple \(\gamma\) is the class \(\delta i\) if and only i

\[
P(\delta i / \gamma) \cdot P(\delta i) > P(\delta j / \gamma) \cdot P(\delta j) \quad \text{for } 1 \leq j \leq m, j \neq i.
\]
In other words, the foresee class label $\delta_i$ for which $P(\delta_i/\gamma)P(\delta_i)$ is the highest.

The core benefit of Naive Bayes is that it merely craves a little amount of training data to except the parameters essential for classification. It also show high accuracy and speed when applied to large databases.

V. PROPOSED WORK

The flow of processing single-trial motor imagery EEG signal analysis based on curvelet transform sing is shown Figure-1. First, EEG signals are passed to a temporal filter in temporal domain. These signals are break down into the frequency sub bands using discrete curvelet transform. Statistical features are extorting from the sub bands to stand for the sharing of curvelet coefficients. Next, the characteristics of curvelet coefficients are used as input vector. Finally Bayesian analysis based on Gaussian allocation was employ to classify characteristics into various categories that display the left or right hand movement imagery.

![Figure 2: Flow of processing single-trial motor imagery EEG signal analysis](image)

The step-by-step procedure used for processing EEG signals using curvelet transform is:

**Algorithm: Signal analysis based on Curvelet Transform**

**Task:** Apply the curvelet transform on EEG signals for BCI.

**Parameter:** Eye data set X and Bayesian classifier.

**Initialization:** $Y = \text{SMOOTH}(X, \text{SPAN})$

1. Decompose the signal into Frequency sub band $F_j$ with block size $S_j$.
   \[
   \sum_{j=-\infty}^{\infty} Z2 (2jr) = 1, \ r \in \left(\frac{3}{4}, \frac{3}{2}\right)
   \]
   \[
   \sum_{j=-\infty}^{\infty} V2 (2jr) = 1, \ r \in \left(-\frac{1}{2}, \frac{1}{2}\right)
   \]

2. Compute the curvelet coefficients of X with J scales, get $\{C_1, \ldots, C_j, D_j\}$ by
   \[
   U_j(r, \Theta) = 2 \frac{3j}{4} Z(2-j, r \Theta) V\left(\frac{2j/2\Theta}{2\pi}\right)
   \]

3. Apply the digital Curvelet transform to each block; get the curvelet coefficients at scale j
   \[
   C(j, l, k) = (f, \varphi_{j,l,k}(Y)) = \int f(X) \varphi_{j,l,k}(Y) \, d(Y)
   \]

4. Bayesian analysis based on Gaussian distribution was utilized to classify features into different categories
   \[
   P(\delta_i/\gamma) = \frac{P(\delta_i)P(\gamma/\delta_i)}{\sum_{i=1}^{\infty} P(\gamma)P(\gamma/\delta_i)}
   \]

**Output:** The curvelet transform on EEG signal of X gives a lower classification error rate and Higher information transfer rate.
VI. RESULTS AND DISCUSSION

In this paper, we presented the implementation of curvelet transform for EEG signals. Here, two-dimensional ECG signals are decomposed into a set of coefficients associated with different directions and scales. The combination of statistical curvelet and Gaussian coefficients were selected as inputs of Bayesian classifier. Bayesian analysis was utilized to classify features into different classes that represent the left or right hand movement imagery and experimental results showed in the below figures. Also the results indicate that method of combining DCT with Gaussian model are able of extracting more useful information from the simultaneously acquired motor imagery EEG. The performance was tested by the eye dataset for BCI competition 2003. Statistical results shows maximum classification accuracy of 95% and higher information transfer rate is achieved.

![Figure 3. Parametric Modeling of EEG Data for the Identification of left or right hand movement imagery tasks.](image)

VII. CONCLUSION

EEG signals analysis is accurate, simple and reliable enough to use in brain computer interface. In this paper, we proposed, a curvelet based motor imagery feature extraction method based on curvelet transform. Feature extraction method for a single trial online motor imagery using curvelet transform. These curvelet coefficients were used to extract the characters from the motor imagery EEG and classify the pattern of left and right hand movement imagery by Bayesian analysis with Gaussian model. The achievement of motor imagery tested by the eye dataset for BCI competition 2003. The hypothetical results presented highest classification accuracy of 96% and superior information transfer rate is obtained.

![Figure 3. Parametric Modeling of EEG Data for the Identification of left or right hand movement imagery tasks.](image)

VIII. ACKNOWLEDGEMENT

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IX. REFERENCES


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